Constrained adaptive control with transient and steady-state performance guarantees

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Abstract Over the last decades research has been performed in order to improve the transient behavior of adaptive systems. To that end, this paper develops a new adaptive control architecture for uncertain dynamical systems to achieve guaranteed transient performance in the presence of state constraints. For this purpose, we extended a recently developed command governor method. Specifically, the command governor is a dynamical system adjusting the trajectory of a given command in order to follow an ideal reference system in transient time, where this system captures a desired closed-loop dynamical system behavior specified by a control engineer. Our extension enables this method to handle state constraints in the range space of the control input matrix. Alternative approaches for enforcing state constraints outside of the range space are further discussed. Finally, these methods are illustrated for the lateral and the longitudinal motion of an aircraft.

1 Introduction

In control theory, mathematical models, derived from fundamental physical laws, are used for the design of controllers in order to achieve certain stability and performance objectives. These models, however, are subject to uncertainties due to ide-

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alized assumptions, simplifications, nonlinearities, time-varying parameters, measurement noise, and disturbances in order to name a few.

Although fixed-gain robust control design approaches can deal with such uncertainties, they require the knowledge of characterized uncertainty bounds, which may not be trivial to obtain due to practical constraints (e.g. extensive verification and validation procedures). Moreover, in the face of high uncertainty levels, they may fail to satisfy a given system performance requirement. On the other hand, adaptive controllers require less modeling information than fixed-gain robust controllers for coping with these high uncertainty levels, and therefore, they are important candidates for such applications.

Adaptive Controllers can be classified either as direct or indirect [14]. Indirect adaptive controllers employ an estimation algorithm in order to approximate the unknown system and adapt the controller gains. In contrast, direct controllers adapt the controller's feedback gains without requiring any estimation process. In this paper, direct adaptive controllers, in particular model reference adaptive control (MRAC), are considered. MRAC schemes use the error between the output (resp. state) of an ideal reference model, specifically designed in order to achieve a desired closed-loop system behavior, and the output (resp. state) of the uncertain plant in order to update uncertain parameters of the controller online. In particular, the objective of the adaptive controller is to suppress the system's uncertainty and drive the control error to zero. Although this objective can be achieved asymptotically, the uncertain system's output (resp. state) might be far different from the desired output (resp. state) of the ideal reference model in transient time.

It is well known that high learning rates in the adaptive update laws improve the transient performance of the system. However, these high gains might lead to oscillations in the command or amplify unmodeled dynamics, which can lead to instability for real-world applications as discussed in [1], [2], [8]. Hence, often a trade-off between stability and transient performance has to be considered in the design process. In some adaptive approaches, such as [7] and [16], this trade-off is not necessary. However, an upper bound of the uncertainty is required a priori, which might be excessed only in some specific cases. For example, a sudden structural damage might lead to uncertainties much higher than anticipated.

In [17] and [19], a command governor[1] structure has been introduced, which ensures approximate tracking of the desired reference model in transient time along with adaption. The command governor is a dynamical system adjusting the trajectories of a given command in order to follow an ideal reference system in transient time. Since the command governor suppresses the uncertainties rapidly, a smaller learning rate is necessary than in traditional MRAC schemes.

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[1] In the context of handling state and control constraints of dynamical systems, several command governor approaches are studied in the literature (see, e.g., [3], [6] and references therein). Even though the command governor architecture of this paper alters a given command similar to those approaches, our objective is to address the poor transient performance phenomenon of adaptive controllers as applied to nonlinear uncertain dynamical systems, and hence, the proposed architecture signicantly differs from the existing command governor approaches.

In this paper, the idea of the approximate transient tracking of the desired reference model is extended to enforce state constraints on the uncertain dynamical system. Our extension enables this command governor methodology to handle state constraints in the range space of the input matrix. Alternative approaches for enforcing state constraints outside of the range space are further discussed. Finally, these methods are illustrated for the lateral and the longitudinal motion of an aircraft.

2 Notation

The notation used in this paper is fairly standard. Specifically, $\Re$ denotes the set of real numbers, $\Re^n$ denotes the set of $n \times 1$ real column vectors, $\Re^{n \times m}$ denotes the set of $n \times m$ real matrices, $(\cdot)^T$ denotes transpose, $(\cdot)^{-1}$ denotes inverse, and "$\triangleq$" denotes equality by definition. In addition, we write $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix A, $\det(A)$ for the determinant of the Hermitian matrix A, $I_n \in \Re_+^{n \times n}$ for the unity matrix, $\mathrm{tr}(\cdot)$ for the trace operator, $\mathrm{Proj}(\cdot)$ for the projection operator [15], $\mathrm{vec}(\cdot)$ for the column stacking operator, $\|\cdot\|_2$ for the Euclidian norm, $\|\cdot\|_\infty$ for the infinity norm, and $\|\cdot\|_{\mathrm{F}}$ for the Frobenius matrix norm.

3 Problem Formulation

In this section, we present a brief overview of the MRAC scheme augmented with the command governor methodology. Specifically, consider the following nonlinear uncertain dynamical system:

$$\dot{x}(t) = Ax(t) + Bu(t) + D\Delta(x(t)), \quad x(0) = x_0, \tag{1}$$

where $x(t) \in \Re^n$ is the accessible state vector, $u(t) \in \Re^m$, $\Delta(x(t)) : \Re^n \to \Re^m$ is an *uncertainty*, $A \in \Re^{n \times n}$ is a known system matrix, $B \in \Re^{n \times m}$ is an *unknown* control input matrix, and $D \in \Re^{n \times m}$ is a known uncertainty input matrix such that $\det(D^T D) \neq 0$. We assume that the pair (A, D) is controllable and the required properties for the existence and uniqueness of solutions are satisfied in (1).

Assumption 3.1. The uncertainty in (1) can be parameterized as

$$\Delta(x) = W^T \sigma(x), \quad x \in \Re^n, \tag{2}$$

where $W \in \Re^{s \times m}$ is an *unknown* weight matrix and $\sigma : \Re^n \to \Re^s$ is a known basis function of the form $\sigma(x) = [\sigma_1(x), \sigma_2(x), \ldots, \sigma_s(x)]^T$.

Remark 3.1. In Assumption 3.1, we assume that the basis functions $\sigma(x(t))$ is known. For the other case, the parametrization in (2) can be relaxed, for example, by considering $\Delta(x(t)) = W^T \sigma^{nn}(V^T x(t)) + \varepsilon^{nn}$, $x(t) \in D_x$, where $W \in \Re^{s \times m}$ and $V \in \Re^{n \times s}$ are unknown weight matrices, $\sigma^{nn} : D_x \to \Re^s$ is a known vector composed of neural network function approximators, $\varepsilon^{nn}: D_x \to \Re^m$ is an unknown residual error, and D_x is a compact subset of $\Re^n$ [13].

Assumption 3.2. The unknown control input matrix in (1) is parameterized as

$$B = D\Lambda, \tag{3}$$

where $\Lambda \in \Re_+^{m\times m}$ is an *unknown* control effectiveness matrix.

The system (1) is desired to follow the given uniformly continuous bounded command $c(t) \in \Re^n$ through the closed-loop (non-ideal) reference model dynamics proposed in [10], [11]:

$$\dot{x}_m(t) = A_m x_m(t) + B_m c(t) + L e_c(t), \quad x_m(0) = x_{m0}, \quad t \geq 0. \tag{4}$$

In (4), $x_m(t) \in \Re^n$ is the reference model state vector, $A_m \in \Re^{n\times n}$ represents the desired Hurwitz system matrix, and $B_m \in \Re^{n\times m}$ is the command input matrix. $e_c(t) \triangleq x(t) - x_m(t) \in \Re^n$ is the control error and $L = \kappa I_n$ represents a positive definite observer matrix. Notice that $\kappa = 0$ recovers the ideal (undisturbed) reference model. The observer matrix is applied in order to achieve faster error dynamics in comparison to the reference model [10], [11].

Remark 3.2. Even though the formulation given here is based on a closed-loop reference model, as we see later, the augmentation of the MRAC scheme by the command governor allows us to follow the ideal ($\kappa = 0$) reference model in transient time and steady-state.

Now, a feedback law $u(t)$, subject to (2) and (3), is constructed such that the states $x(t)$ asymptotically track the reference model states $x_m(t)$. For this purpose,

$$u(t) = u_n(t) - u_{ad}(t), \tag{5}$$

where $u_n(t) \in \Re^m$ is the nominal feedback control for the certain system and $u_{ad}(t) \in \Re^m$ represents the adaptive feedback control law in order to asymptotically cancel uncertainties of the system. The nominal control is given by

$$u_n(t) = -K_x x(t) + K_c c(t), \tag{6}$$

where $K_x \in \Re^{m\times n}$ is the nominal feedback matrix and $K_c \in \Re^{m\times m}, \det(\mathrm{K_c}) \neq 0$ is the feedforward matrix chosen such that $A - DK_x = A_m$ and $DK_c = B_m$. Application of (5) and (6) on (1) along with the assumptions (2) and (3), yields

$$\dot{x}(t) = A_m x(t) + B_m c(t) + D\Lambda\left(-u_{ad}(t) + W_u^T u_n(t) + W_\sigma^T \sigma(x(t))\right), \tag{7}$$

where $W_\sigma^T = \Lambda^{-1} W^T \in \Re^{m\times s}$ and $W_u^T = [I - \Lambda^{-1}] \in \Re^{m\times m}$. Furthermore, the adaptive feedback law

$$u_{ad}(t) = \hat{W}_u^T u_n(t) + \hat{W}_\sigma^T \sigma(x(t))) \tag{8}$$

is proposed, where $\hat{W}_\sigma^T \in \Re^{m\times s}$ and $\hat{W}_u^T \in \Re^{m\times m}$ represent estimates of the ideal weights W_σ^T and W_u^T, respectively, which are satisfying the weight update laws

$$\dot{\hat{W}}_\sigma = \Gamma_\sigma \sigma(x(t)) e_c^T(t) PD, \quad \hat{W}_\sigma(0) = \hat{W}_{\sigma,0}, \tag{9}$$

$$\dot{\hat{W}}_u = \Gamma_u u_n(t) e_c^T(t) PD, \quad \hat{W}_u(0) = \hat{W}_{u,0}, \tag{10}$$

with the symmetric positive definite learning rates $\Gamma_\sigma \in \Re_+^{s\times s}$ and $\Gamma_u \in \Re_+^{n\times n}$. $P \in \Re_+^{n\times n}$ represents the symmetric positive definite solution of the Lyapunov equation

$$[A_m - L]^T P + P[A_m - L] = -R, \tag{11}$$

where $R \in \Re_+^{n\times n}$ is a symmetric positive definite designer matrix. Furthermore, $\tilde{W}_\sigma(t) \triangleq \hat{W}_\sigma(t) - W_\sigma$ and $\tilde{W}_u(t) \triangleq \hat{W}_u(t) - W_u$ denote the adaptive weight estimation errors. Using (8) in (7), we have

$$\dot{x}(t) = A_m x(t) + B_m c(t) - D\Lambda(\tilde{W}_u^T u_n(t) + \tilde{W}_\sigma^T \sigma(x(t))) \tag{12}$$

The error dynamics are computed by using (4) and (12):

$$\dot{e}_c(t) = [A_m - L] e_c(t) - D\Lambda(\tilde{W}_u^T u_n(t) + \tilde{W}_\sigma^T \sigma(x(t))) \tag{13}$$

It is well known that Lyapunov stability of the adaptive weight estimation errors $\tilde{W}_u(t)$ and $\tilde{W}_\sigma(t)$, and the control error $e_c(t)$ can be shown by application of the Lyapunov function candidate $V = e_c^T(t) P e_c(t) + \mathrm{tr}\left[\tilde{W}_u^T \Gamma_u^{-1} \tilde{W}_u\right] + \mathrm{tr}\left[\tilde{W}_\sigma^T \Gamma_\sigma^{-1} \tilde{W}_\sigma\right]$. Furthermore, it can be proven that $\lim_{t\to\infty} e_c(t) = 0$. Due to space limitations (and similarity to section 5), the proof is omitted.

The MRAC approach with closed-loop reference model, presented above, guarantees only asymptotic convergence of the tracking error to zero. Although the transient performance is improved in comparison to standard MRAC as a consequence of the closed-loop reference model [10], [11], the transient response of the system is still subject to the scaling problem arising with adaptive control (e.g., performance can change with respect to the same commands having different (scaled) amplitudes). That is, in contrast to linear control, the response of adaptive systems is not scalable due to their nonlinear nature. Therefore, a trade-off between good tracking performance, achieved by fast suppression of the uncertainties (high learning rates Γ), and acceptable control inputs $u(t)$ (lower learning rates Γ) has to be considered in the design process. In order to achieve this goal, the so-called command governor (CG, [17], [19]) is used. Basically, the command governor is a parallel linear dynamical system that influences the characteristics of the command $c(t)$, which is applied on the reference model (4) and the nominal control (6). Consequently, the command $c(t)$ consists of two parts,

$$c(t) = c_D(t) + K_c^{-1}\left[D^T D\right]^{-1} D^T g(t), \tag{14}$$

where $c_D(t) \in \Re^m$ is the uniformly continuous bounded desired tracking command and $g(t) \in \Re^n$ is the output of the linear command governor system, given by

$$\dot{\xi}(t) = -\lambda \xi(t) + \lambda e_c(t), \quad \xi(0) = 0, \tag{15}$$

$$g(t) = \lambda\xi(t) + [A_m - L - \lambda I_n]\, e_c(t), \tag{16}$$

with the command governor states $\xi(t) \in \Re^n$ and the command governor gain $\lambda > 0$. Since the command is applied on both, the system and the reference model, the error dynamics (13) do not change. With the error dynamics, it is possible to express the uncertainty as

$$\Lambda\left(\tilde{W}_u^T u_n(t) + \tilde{W}_\sigma^T \sigma(x(t))\right) = [D^T D]^{-1} D^T \{[A_m - L]e_c(t) - \dot{e}_c(t)\}. \tag{17}$$

Using (17) with the augmented command (14) and (16) in (12), yields

$$\dot{x}(t) = A_m x(t) + B_m c_D(t) + D[D^T D]^{-1} D^T \{\lambda\xi(t) - \lambda e_c(t) + \dot{e}_c(t)\}. \tag{18}$$

Transforming the command governor equation (15) into the Laplace domain, results in $\dot{\xi}(s) = \lambda/(s+\lambda)e_c(s)$. Application of this, gives

$$\mathscr{L}\{\lambda\xi(t) - \lambda e_c(t)\} = \lambda(\xi(s) - e_c(s)) = \lambda\left(\frac{\lambda}{s+\lambda} - 1\right) e_c(s)) = -\frac{s}{\frac{s}{\lambda}+1} e_c(s). \tag{19}$$

Obviously, $\lim_{t\to\infty} \lambda(\xi(t) - e_c(t)) = -\dot{e}_c$. As a consequence, for $\lambda \to \infty$ the closed-loop system (18) becomes $\dot{x}(t) = A_m x(t) + B_m c_D(t)$. Conclusively, it can be stated that for a sufficiently large λ, the closed-loop system, consisting of the uncertain system (1), subject to the assumptions (2), (3), the closed-loop reference model (4), the control laws (5), (6), (8) and the command governor, given by (15) and (16), approximately tracks the desired reference model even in transient time

$$\dot{x}(t) \approx A_m x(t) + B_m c_D(t). \tag{20}$$

In contrast to the standard MRAC stability analysis, the command $c(t)$ cannot be assumed to be bounded. Therefore, the boundedness of the command $c(t)$ has to be established by Lyapunov stability analysis. This will be executed along with the stability analysis presented in section 5.

Remark 3.3. In this section, ideal measurements are assumed. The differentiating character of the command governor, displayed in (19), might amplify high-frequency content in the dynamical system. Consequently, robustness modifications have to be applied [18], [19]. If the high-frequency content is due to measurement noise or unmodeled dynamics, the update laws (9) and (10) have to be modified in order to avoid the phenomena of parameter drift. An example of such a modification is the σ-modification [8].

Remark 3.4. Since $\lambda = \infty$ is not physically possible, an adaptive control law is necessary to ensure stability. Consequently, there is a trade-off between the command governor gain λ and the adaptive learning rates Γ_σ and Γ_u. If the upper bound of the uncertainty is known, however, the command governor alone can be used [5].

4 Constraint Enforcement

In the spirit of the command governor, presented above, an additional command is added to both, the uncertain dynamical system (1) through the control laws (5) and (6), and the reference model (4). This additional constraint command $c_c(t) \in \Re^m$ is designed in order to enforce constraints on the dynamical system by reducing the overall command. Then, the overall command becomes

$$c(t) = c_D(t) + K_c^{-1} \left[D^T D\right]^{-1} D^T g(t) + c_c(t). \tag{21}$$

The overall control structure is displayed in Fig. 1. Obviously, the controller and the reference model are driven by the overall command assembled in (21).

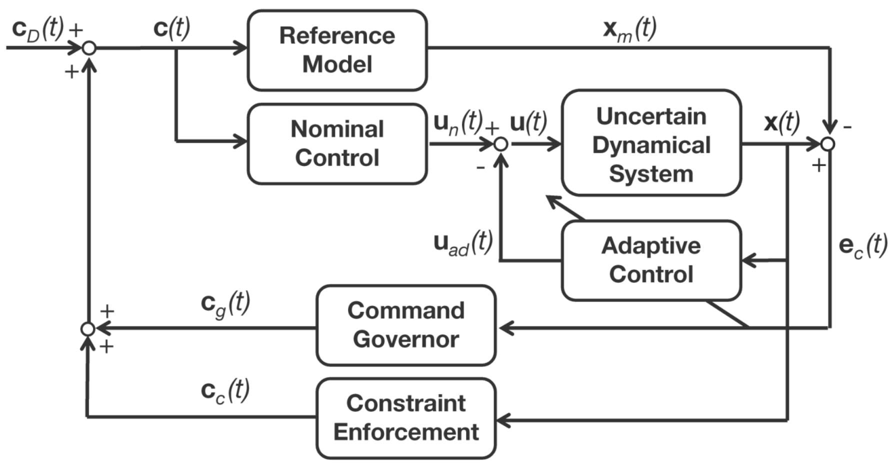

Fig. 1 Block diagram of overall control structure

The constraint command $c_c(t)$ is designed based on a gradient-based minimization of a cost function,

$$J = \frac{1}{2} x^T(t) D Q D^T x(t), \tag{22}$$

where $Q \in \Re^{m \times m}$ is a weight matrix. This cost function can be minimized by application of the negative gradient,

$$\dot{x}_{\text{lim}}(t) = -\frac{\partial J}{\partial x(t)} = -D Q D^T x(t). \tag{23}$$

Here, $\dot{x}_{\text{lim}}(t)$ is the additional change of the states due to constraint enforcement. However, (23) would basically try to enforce the constraint $x(t) = 0$ and consequently pull back the states $x(t)$ to the origin. Therefore, the constraint enforcement term should just be active if the states are actually about to violate the constraint.

Therefore, (23) is modified:

$$\dot{x}_{\lim}(t) = -DQD^T\mu(t)x(t), \tag{24}$$

where $\mu(t) \in \Re^{n\times n}$ is a diagonal, positive semidefinite matrix with scalar entries $\mu_i(t)$ on the diagonal. In the spirit of constraint enforcement in optimal control [4], each $\mu_i(t) = 0$, if its corresponding state $x_i(t)$ is within the set $X_{i,0} \triangleq \{x_i(t) \in \Re, x_{i,\min} < x_i(t) < x_{i,\max}\}$ and $\mu_i(t) = \vartheta_i$, if $x_i(t) \in X_{i,\vartheta} \triangleq \{x_i(t) \in \Re, x_i(t) = x_{i,\min} \cup x_i(t) = x_{i,\max}\}$, where $\vartheta_i \in \Re_+$ is a gain chosen by the designer. As a result, the gradient based method proposed drives the states $x_i(t)$ to its origin in order to try to enforce the constraint $\mu_i(t)x_i(t) = 0$. However, the actual objective here is *not* to keep the states at zero, but to keep the states within the set defined by $X_i \triangleq X_{i,0} \cup X_{i,\vartheta} = \{x_i(t) \in \Re, x_{i,\min} \le x_i(t) \le x_{i,\max}\}$. This point is elucidated below.

It is obvious that the term (24) is just relevant for $\mu(t) \neq 0$ and therefore, for at least one $x_i(t) \in X_{i,\vartheta}$ or $x_i(t) = x_{i,\lim}$, where $x_{i,\lim}$ represents either $x_{i,\min}$ or $x_{i,\max}$ depending on $x_i(t)$. For example, for $x_{i,\min} = -x_{i,\max}$, the state limits can be determined by $x_{i,\lim} = x_{i,\max}\mathrm{sign}(x_i(t))$. Then, it is sufficient to use the $x_{i,\lim}$ instead of $x_i(t)$, and (24) becomes $\dot{x}_{\lim}(t) = -DQD^T\mu(t)x_{\lim}$, which can be included into the overall command $c(t)$ as

$$c_c(t) = -K_c^{-1}QD^T\mu(t)x_{\lim}. \tag{25}$$

(25) is applied on both, reference model and system. Consequently, the error dynamics (13) do not change. As a result, the command governor of section 3 remains unchanged, as well. Furthermore, it should be noted that the constraint command $c_c(t)$ is uniformly, ultimately bounded, since $\mu(t) \le \vartheta$.

Finally, the transition from $\mu_i(t) = 0$ to $\mu_i(t) = \vartheta_i$ is desired to be continuous. Therefore, two definitions for $\mu_i(t)$ are proposed as follows. First, consider

$$\mu_i(t) \triangleq \begin{cases} 0, \ |x_i(t)| \le c_{D,i,\max} + \varepsilon_i \\ \vartheta_i, \ |x_i(t)| = |x_{i,\lim}| \\ \vartheta_i \frac{|x_i(t)| - (c_{D,i,\max} + \varepsilon_i)}{|x_{i,\lim}| - (c_{D,i,\max} + \varepsilon_i)}, \ c_{D,i,\max} + \varepsilon_i < |x_i(t)| < |x_{i,\lim}| \end{cases} \tag{26}$$

where $c_{D,i,\max} \in \Re_+$ represents the norm of either the maximum or minimum command allowed, depending on $x_i(t)$ and $\varepsilon_i \in \Re_+$ is a small positive constant that determines the magnitude of the transition zone. It is obvious that this definition of $\mu_i(t)$ is just feasible, if the maximum (minimum) command $c_{D,i,\max}$ is smaller (larger) than the state limit $x_{i,\lim}$, which represents $X_{i,\vartheta}$. However, in real-world applications, there will be always a margin between the maximum (or, minimum) command possible and the set $X_{i,\vartheta}$ and therefore, this is a reasonable assumption.

Smooth, uniformly continuous, transition from $\mu_i(t) = 0$ to $\mu_i(t) = \vartheta_i$ can also be achieved by

$$\mu_i(t) = \vartheta_i \tanh\left(\rho_i \left| \frac{x_i(t)}{|x_{i,\lim}| - |x_i(t)|} \right| \right), \tag{27}$$

where $\rho_i \in \Re_+$ is a small constant. However, except of at the origin $x_i(t) = 0$, the parameter $\mu_i(t)$ is different from zero and therefore, a steady-state error will occur.

Remark 4.1. If the actual desired reference model $\dot{x}_m(t) = A_m x_m(t) + B_m c_D(t)$ stays within X_0 for all times, the command governor already approximately enforces the constraint since (20) holds. However, the augmentation of the system by $c_c(t)$ gives the possibility to increase the command $c_{D,\max}$ to a magnitude that allows an overshoot of the desired reference model.

Remark 4.2. Due to the structure of (25), just states can be limited that are within the input range of the input matrix D. For states that are not in the input range of D, the tangency condition [4] has to hold. That is, all time derivatives of the state have to be feasible with the constraint at hand. An example of a possible application of the tangency condition is given in section 6. Furthermore, another workaround is presented, that can be used in certain cases.

Remark 4.3. For clarity, note that the reference model is bounded due to the Hurwitz matrix A_m and the boundedness of $c_D(t)$, $c_g(t)$ and $c_c(t)$. These variables can be chosen in a feasible way by the designer. Furthermore, note that the ideal reference model without modifications is approximately tracked if the constraint enforcement term is not active.

5 Stability Proof

Theorem 1. *Consider the uncertain dynamical system (1) subject to the assumptions (2) and (3), the reference model (4), the control law given by (5), (6) and (8), as well as the adaptive weight update laws (9) and (10), where the command is defined by (21). Furthermore, consider the command governor, consisting of (15) and (16), and the constraint enforcement command, given by (25). Then, the control error $e_c(t)$, the command governor states $\xi(t)$ and the weight estimation errors $\tilde{W}_\sigma(t)$ and $\tilde{W}_u(t)$ of the dynamical closed-loop system are Lyapunov stable for all $(e_{c,0}, \tilde{W}_{u,0}, \tilde{W}_{\sigma,0}, 0) \in \Re^n \times \Re^{m\times m} \times \Re^{s\times m} \times \Re^n$ and $t > 0$. Moreover,* $\lim_{t\to\infty} e_c(t) = 0$, $\lim_{t\to\infty} \xi(t) = 0$, $\lim_{t\to\infty} g(t) = 0$, *and* $\lim_{t\to\infty}(c(t) - c_D(t)) = c_c(t)$. *Besides, the closed-loop uncertain dynamical system approximates*

$$\dot{x}(t) \approx A_m x(t) + B_m c_D(t) + B_m c_c(t), \tag{28}$$

in transient time. Furthermore, if $|x(t)| \leq c_{D,\max} + \varepsilon$, then $\mu(t) = 0$ and $c_c(t) = 0$. Consequently, $\lim_{t\to\infty} c_c(t) = 0$ *and the desired reference model (20) is approximately tracked in transient time.*

Proof. Consider the Lyapunov function candidate $V(e_c(t), \tilde{W}_u(t), \tilde{W}_\sigma(t), \xi(t))$

$$V(\cdot) = e_c^T(t) P e_c(t) + \mathrm{tr}\left[\tilde{W}_u^T(t)\Gamma_u^{-1}\tilde{W}_u(t)\right] + \mathrm{tr}\left[\tilde{W}_\sigma^T(t)\Gamma_\sigma^{-1}\tilde{W}_\sigma(t)\right] + \alpha\xi^T(t)\xi(t). \tag{29}$$

This term is positive definite outside the equilibrium and radially unbounded. Differentiating along its trajectories (13), (9), (10), and (15), and application of the

Lyapunov Equation (11), results in

$$\dot{V}(\cdot) = -e_c^T(t)Re_c(t) - 2\alpha\lambda\xi^T(t)\xi(t) + 2\alpha\lambda\xi^T(t)e_c(t). \tag{30}$$

Now, by choosing the matrix R, the command governor gain λ and $\alpha > 0$ in an appropriate way, $R = R_0 + \alpha\lambda I_n$ with $R_0 \in \Re_+^{n\times n}$ being a positive definite matrix, it is shown that

$$\dot{V}(\cdot) = -e_c^T(t)R_0e_c(t) - \alpha\lambda\left(2\xi^T(t)\xi(t) - 2\xi^T(t)e_c(t) + e_c^T(t)e_c(t)\right). \tag{31}$$

With $\xi^T(t)\xi(t) - 2\xi^T(t)e_c(t) + e_c^T(t)e_c(t) = \|\xi(t) - e_c(t)\|_2^2$, it can be followed that

$$\dot{V}(\cdot) \le -e_c^T(t)R_0e_c(t) - \alpha\lambda\xi^T(t)\xi(t). \tag{32}$$

Thus, the system is Lyapunov stable for all $(e_{c,0}, \tilde{W}_{u,0}, \tilde{W}_{\sigma,0}, 0) \in \Re^n \times \Re^{m\times m} \times \Re^{s\times m} \times \Re^n$ and $t > 0$. Stability implies that the control error $e_c(t)$ and the command governor states $\xi(t)$ are bounded and, as a consequence of the stable and bounded reference model, the states $x(t)$ are also bounded. Therefore, $\sigma(x(t))$ and $u_n(t)$ are bounded and it follows from (9), (10), and (13) that $\dot{e}_c(t)$ is bounded, too. Furthermore, from (15), we know that $\dot{\xi}(t)$ is bounded as well. Consequently, $\ddot{V}(\cdot)$ is bounded and therefore, $\dot{V}(\cdot)$ is uniformly continuous and $V(\cdot)$ is lower bounded by 0 and strictly non-increasing for all $t > 0$. Hence, by application of Barbalat's Lemma [9], it follows that $\lim_{t\to\infty}\dot{V}(e_c(t), \tilde{W}_u(t), \tilde{W}_\sigma(t), \xi(t)) = 0$, and as a result, $\lim_{t\to\infty} e_c(t) = 0$, $\lim_{t\to\infty}\xi(t) = 0$. With (16), it can be directly followed that $\lim_{t\to\infty} g(t) = 0$, and, based on (21), $\lim_{t\to\infty}(c(t) - c_D(t)) = c_c(t)$. Furthermore, similar to (20), it can be followed that $\dot{x}(t) \approx A_m x(t) + B_m c_D(t) + B_m c_c(t)$. From (25) and (26), we furthermore know that $c_c(t) = 0$ for $|x(t)| \le c_{D,\max} + \varepsilon$ and in this case, the closed-loop uncertain dynamical system approximates the desired reference model (20) in transient time. This completes the proof. □

Remark 5.1. If the definition (27) is used, $\mu(t)$ will usually not become zero due to the definition of the hyperbolic tangent. Consequently, there will be always a small steady-state error as $c_c(t) \neq 0$.

6 Examples

Consider the nonlinear uncertain system

$$\begin{pmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{pmatrix} = \begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}\Lambda(u(t) + \Delta(x(t))), \tag{33}$$

which represents the wing rock dynamics of an aircraft. $x_1(t)$ is the bank angle of an aircraft in radians and $x_2(t)$ is the roll rate in radians per second. The uncertainty is given by $\Delta(x(t)) = 0.1414x_1(t) + 0.5504x_2(t) - 0.0624|x_1(t)|x_2(t) + 0.0095|x_2(t)|x_2(t) + 0.0215x_1^3(t)$ [17] and is derived from the aerodynamic coefficients of an aircraft. In order to show the performance of the overall system, the input uncertainty is arbitrarily chosen to be $\Lambda = 0.2$, which is equivalent to a 80% loss of control effectiveness. Furthermore, the nominal control consists of $K_x = \left(\omega^2 \; 2\omega\zeta\right)$ and $K_c = \omega^2$, where $\omega = 0.4 rad/s$ is the natural frequency of the desired reference model and $\zeta = 0.707$ is the corresponding damping ratio. The designer parameters are $R = I_2$, $\lambda = 25$, $\Gamma_\sigma = 0.1$, $\Gamma_u = 0.1$ and $\kappa = 1$. For demonstration purposes doublets of the bank angle are commanded and the limits of the roll rate are reduced. In order to enforce these constraints, definition (27) with $\rho_2 = 10^{-3}$ is used for $\mu_2(t)$ in $c_c(t) = -K_c^{-1} q \mu_2(t) x_{2,\text{lim}}$, where $q = 25$.

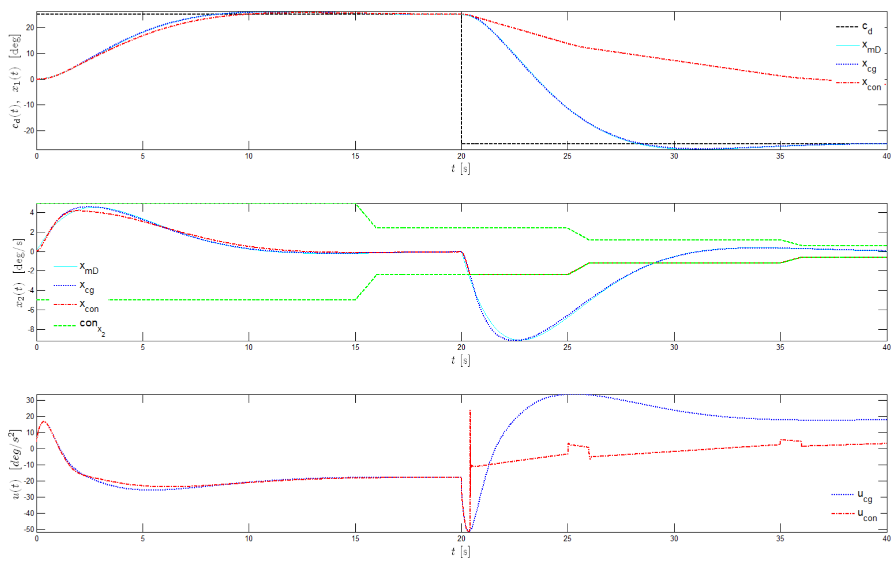

Fig. 2 Wing Rock Dynamics: Performance of constraint enforcement term

In Fig. 3 the limits of the roll rate are continuously reduced from $5°/s$ to $0.3°/s$. These limits are arbitrarily set in order to show the performance of the limiting term. The cyan solid lines represents the desired reference model (denoted as ”mD”), which is the model we want to track. The actual plant without constraint enforcement is represented by the blue dotted lines and the behavior of the plant subject to the constraint enforcement is shown by the red dash-dot lines. Obviously, the constrained plant reacts much slower due to the limits, but it is important to note that the constraints on the roll rate are enforced for the whole time.

As mentioned before, due to the structure of the constraint enforcement command (25) just states within the input range of the input matrix D can be limited. However, in a problem-dependent way, workarounds are possible. Two examples are given in this section. First, in spirit of the tangency condition [4], the bank angle is limited by changing the limits of the roll rate.

$$x_{2,\max} = \begin{cases} x_{2,\max,\max},\ x_1(t) \le (1-\delta)x_{1,\max} \\ \frac{x_{2,\max,\max}}{\delta^2 x_{1,\max}^2}(x_{1,\max} - x_1(t))^2,\ (1-\delta)x_{1,\max} \le x_1(t) \le x_{1,\max} \\ 0,\ x_1(t) \ge x_{1,\max} \end{cases}, \tag{34}$$

where $\delta = 0.15$ determines the transition zone and $x_{2,\max,\max} = 5^\circ/s$ is the maximum roll rate allowed. For the lower limit, (34) can be chosen similar, where $x_{2,\min,\min} = -5^\circ/s$ is the minimum roll rate allowed and $x_{1,\max} = 25^\circ$ and $x_{1,\min} = -25^\circ$ are the limits for the bank angle.

The constraint command is still given by $c_c(t) = -K_c^{-1} q \mu_2(t) x_{2,\lim}$ with the same parameters as above and $x_{2,\lim} = x_{2,\max}$ for $x_2(t) \ge 0$ or $x_{2,\lim} = x_{2,\min}$ for $x_2(t) < 0$, respectively.

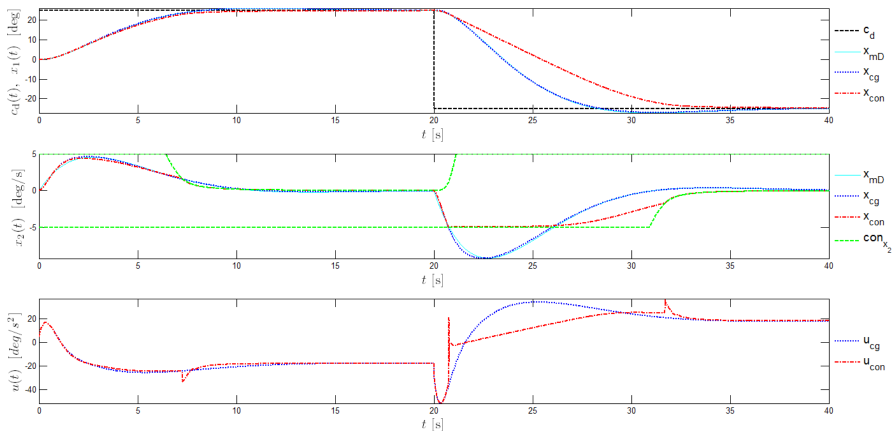

Fig. 3 Wing Rock Dynamics: Comparison of performance with and without constraint enforcment

Fig. 4 Wing Rock Dynamics: Detailed view of Fig. 3. Obviously, the constraint on the roll rate is met by the system, whereas the system without constraint enforcement approximately tracks the desired reference model

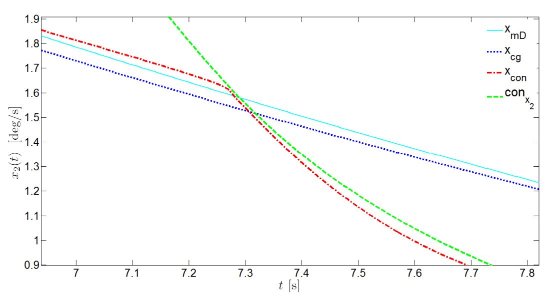

Fig. 3 shows the comparison of the system with and without constraint enforcement. Obviously, the desired reference model is approximately tracked in transient time by application of the command governor. Adding the constraint enforcement term, meets arbitrarily chosen constraints on the bank angle. Fig. 4 shows the influence of the constraint enforcement term in more detail.

Furthermore, consider the system

$$\begin{pmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{pmatrix} = \begin{bmatrix}a_{11} & 1\\ a_{21} & a_{22}\end{bmatrix}\begin{pmatrix}x_1(t)\\ x_2(t)\end{pmatrix} + \begin{pmatrix}0\\ 1\end{pmatrix}\Lambda(u(t)+\Delta(x(t))), \tag{35}$$

which represents the short-period longitudinal mode of an aircraft. $x_1(t)$ is the angle of attack of an aircraft in radians and $x_2(t)$ is the pitch rate in radians per second. In this example, taken from [12], the uncertainty is given by $\Lambda = 0.5$ and $\Delta(x(t)) = 0.4111x_1(t) + 0.8619x_2(t) + 1/2\exp\left(\frac{-(x_1(t)-\bar{x}_1)^2}{2\sigma^2}\right)$, where $\bar{x}_1 = 2/180\pi$ and $\sigma = 0.0233$. This set of uncertainties is equivalent to 50% increase in the static instability, 80% decrease in the pitch damping and a control effectiveness of 50% [12]. Additionally, the nominal control is given by $u_n(t) = -(2\omega\zeta a_{11} + \omega^2 + a_{11}^2 + a_{21})x_1(t) - (a_{11} + a_{22} + 2\omega\zeta)x_2(t) + \omega^2 x_{1,cmd}(t)$. The parameters are given by $a_{11} = -1.0189$, $a_{21} = -0.8223$, $a_{22} = -1.0774$, the natural frequency is $\omega = 4rad/s$ at a damping ratio of $\zeta = 0.6$. The adaptive control consists of the learning rates $\Gamma_\sigma = 30$ and $\Gamma_u = 50$ along with eleven evenly spaced Gaussian functions of the width σ with the centers placed between $10/180\pi$ and $-10/180\pi$ for $\sigma(x(t))$. The command governor gain is chosen as $\lambda = 30$.

The objective of the constraint enforcement is to limit the angle of attack to $|x_1(t)| \leq 5°$. The constraint enforcement term (25) is used, where Q reduces to a scalar and is chosen 1. Furthermore, since the angle of attack is not in the input range of D, the structure of the parameter matrix is changed to $\mu(t) = [\mu_1(t), 0; \mu_1(t), 0;]$ with definition (26). Additionally, $x_{1,\lim}(t) = 5°\mathrm{sign}(x_1(t))$, $x_{1,cmd,\max} = 4.8°$, $\varepsilon_1 = 0$ and $\vartheta_1 = 10$.

In Fig. 6 the performance of the constraint enforcing term is displayed. Here, the actual command is 4% less than the constraint that has to be met, which implies an overshoot of the desired reference model. However, the constraint is still met for all times. If a higher input is acceptable, even higher commands (over 4.9°) are possible. The combination of command governor and adaption ensures nearly perfect tracking of the desired reference model in transient time and therefore, no constraint term is needed if the desired reference model stays within the boundaries (this is the case for a command of 4.5°). That is, the command governor based adaptive control scheme presented above is enforcing the constraint by good tracking of a reasonable chosen reference model. The constraint enforcement term is an additional tool for keeping the system states within desired boundaries.

Finally, note that the approach proposed here is able to achieve steady-state angle of attacks of 4.9° with the constraints being met for all times. This is a significant improvement in comparison to the state limiter suggested by [12] (4.3°). In contrast to this approach, it is furthermore possible to limit systems of higher order without turning the adaption off.

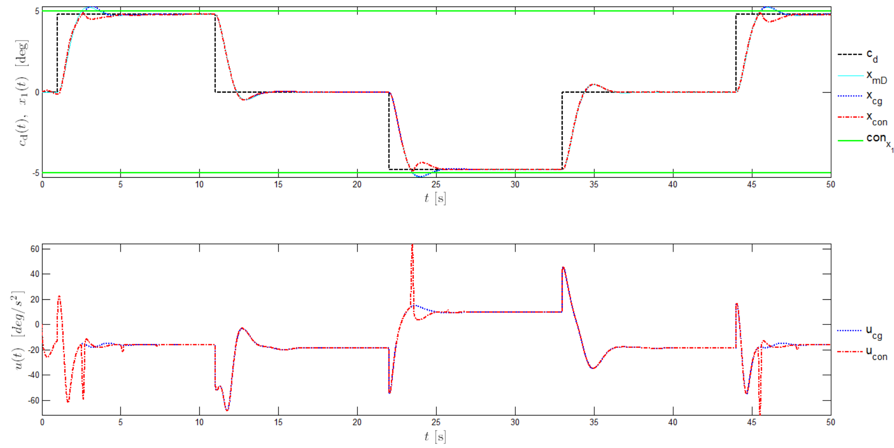

Fig. 5 angle of attack command 4.8°, comparison with and without constraint enforcement

7 Conclusion

We present a new method in order to enforce state constraints on dynamical systems in presence of uncertainties. Specifically, we first discuss the command governor structure [19] and show, that the uncertain dynamical system approximates the desired reference model. Secondly, we motivate the use of a novel augmentation of the command applied to an closed-loop reference model and the uncertain dynamical system, and show stability of the control system consisting of command governor, adaptive control and a constraint enforcement command. We furthermore discuss the fact that just constraints in the input range of D can be met. Finally, we show examples that discuss workarounds for this problem in certain cases and illustrate the compatibility of theory and numerical results. An improvement in comparison to the limiter term from [12] is shown. Future research will contain application of the constraint enforcement on an unmanned helicopter.

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