Decentralized Adaptive Control of Uncertain Systems with Gain Scheduled Reference Models

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Abstract—Control theoretic concepts for decentralized adaptive control of uncertain systems with gain scheduled reference model is developed. For each subsystem a single Lyapunov matrix is computed, using convex optimization tools, for multiple linearizations near equilibrium and non-equilibrium points of the nonlinear closed-loop gain scheduled reference subsystem. This approach guarantees stability of the closed-loop gain scheduled reference model. Then, decentralized adaptive state feedback control architecture is developed and its stability is proved. Specifically, the resulting closed-loop system is shown to have bounded solutions with bounded tracking error for all the subsystems. Simulation results for two different models of a turboshaft engine, including the nominal engine model and an engine model with a new core subsystem, illustrate the possibility of stable decentralized adaptive control of gas turbine engines with proper tracking performance.

I. INTRODUCTION

During the past decades there has been a growing interest in decentralized adaptive control [1], [2], [3], [4], [5]. The problem deals with a system composed of $N$ subsystems $S_k$, each of whose inputs is chosen by $N$ controllers $C_k$, where $k = 1, 2, ..., N$. The parameters of the subsystems are assumed to be unknown, and the controllers have to generate their inputs adaptively, using all information available to them, to achieve some desired objectives.

Control theoretic concepts for gain scheduled model reference adaptive control of gas turbine engines have been developed in [6], [7]. Since controlling the systems which operate in large operating envelopes, such as gas turbine engines, is not practical close to just one operating point, there is a need to use linear parameter dependent models that cover the entire operating envelope of the system (i.e. control the system for multiple operating points). The contribution of this paper is the development of a decentralized adaptive control approach for systems with gain scheduled reference models; the controller can be used to control the dynamical systems with multiple subsystems over large operating envelopes for a continuum of equilibria. The decentralized adaptive control design developed here is based on the results from [6], [7]. The developed decentralized controller, then, is applied to a high fidelity physics-based model of a JetCat SPT5 turboshaft engine with two subsystems. Using this architecture, we can match different engine cores to different props, and the whole propulsion system could work without anymore performance tuning. Simulation results show that the gas turbine engine with two subsystems (i.e., engine core and engine propeller) can be controlled for large throttle commands in a stable manner and with proper tracking performance.

The rest of this paper is organized as follows. In section II, linear parameter dependent model is developed and the decentralized version is also presented. In section III, the decentralized adaptive control for systems with gain scheduled reference systems is presented. Then, uniform ultimate boundedness of the error signals for all the subsystems of the decentralized system is proven. In section IV, simulation results are presented. The simulations study the efficiency of the developed decentralized adaptive control architecture for controlling the system with a new engine core subsystem along with the nominal engine prop subsystem. Section V, concludes the paper.

II. LINEAR PARAMETER DEPENDENT REFERENCE MODEL

The contents of this section is mostly adopted from the previous work by the authors [6], [8]. For a brief overview, consider the nonlinear dynamical system

$$
\begin{align*}
\dot{x}(t) &= f(x(t), u(t)), \\
y(t) &= g(x(t), u(t)),
\end{align*}
$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $y(t) \in \mathbb{R}^m$ is the output vector, $f(.)$ is an $n$-dimensional differentiable nonlinear vector function which represents the plant dynamics, and $g(.)$ is an $m$-dimensional differentiable nonlinear vector function which generates the plant outputs. We intend to design a feedback control such that $y(t)$ properly tracks a reference signal $r(t)$ as $t$ goes to infinity, where $r(t) \in D_r \subset \mathbb{R}^m$, and $D_r$ is a compact set. Assume that for each $r \in D_r$, there is a unique pair $(x^e_r, u_e)$ that depends continuously on $r$ and satisfies the equations

$$
0 = f(x^e_r, u_e), \quad r = g(x^e_r, u_e),
$$

where $x^e_r$ is the desired equilibrium point and $u_e$ is the steady-state control that is needed to maintain equilibrium at $x^e_r$. It is often useful to parameterize the family of system equilibria as follows:

Definition 1: The functions $x^e_r(\alpha(t)), u_e(\alpha(t))$, and $r_e(\alpha(t))$ define an equilibrium family for the plant (1) on the set $\Omega$ if

$$
\begin{align*}
f(x^e_r(\alpha(t)), u_e(\alpha(t))) &= 0, \\
g(x^e_r(\alpha(t)), u_e(\alpha(t))) &= r_e(\alpha(t)), \quad \alpha \in \Omega.
\end{align*}
$$

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The family of plant linear models, for all \( \alpha \in Q \) can be written as
\[
\delta x^P(t) = A^P(\alpha(t)) \delta x^P(t) + B^P(\alpha(t)) \delta u(t),
\]
\[
\delta y(t) = C^P(\alpha(t)) \delta x^P(t) + D^P(\alpha(t)) \delta u(t),
\]
where \( \delta x^P(t) = x^P(t) - x^P_e(\alpha(t)) \), \( \delta y(t) = y(t) - y_e(\alpha(t)) \), and \( \delta u(t) = u(t) - u_e(\alpha(t)) \). \( A^P(\alpha(t)), B^P(\alpha(t)), C^P(\alpha(t)), \) and \( D^P(\alpha(t)) \) are the parameterized plant linearization family matrices and \( x^P_e(\alpha(t)) = 0, y_e(\alpha(t)) = 0 \), \( \forall \alpha \). The plant (1) with the filtered inputs, and its general controller can be written as
\[
\dot{z}^P(t) = \begin{bmatrix} f^P(x^P(t), u(t)) \\ g^P(x^P(t), u(t), r(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ g(x^P(t), r(t)) \end{bmatrix},
\]
\[ v(t) = \begin{bmatrix} f(x(t), r(t)) \\ g(x(t), r(t)) \end{bmatrix},
\]
and the closed-loop nonlinear system is
\[
\dot{x}(t) = F(x(t), r(t)),
\]
where \( x(t) \in D_x \subset \mathbb{R}^{n+2m}, \) and \( r(t) \in D_r \subset \mathbb{R}^m \). The augmented linear family of systems for the augmented plant is
\[
\begin{bmatrix} \dot{x}^P(t) \\ \dot{u}(t) \end{bmatrix} = \begin{bmatrix} A^P(\alpha(t)) & B^P(\alpha(t)) \\ 0 & -\eta_e I \end{bmatrix} \begin{bmatrix} x^P(t) \\ u(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \eta_e x \end{bmatrix},
\]
\[
\begin{bmatrix} \delta x^P(t) \\ \delta u(t) \end{bmatrix} = \begin{bmatrix} A_{aug}(\alpha(t)) \\ C_{aug}(\alpha(t)) \end{bmatrix} \begin{bmatrix} \delta x^P(t) \\ \delta u(t) \end{bmatrix},
\]
and the controller is defined to be
\[
\begin{bmatrix} \dot{x}^c(t) \\ v(t) \end{bmatrix} = \begin{bmatrix} -\epsilon_e I & I \\ K^T(\alpha(t)) & 0 \end{bmatrix} \begin{bmatrix} x^c(t) \\ \delta y(t) \end{bmatrix}.
\]
For the case where we have \( \delta y(t) = \delta x^P(t) \), (i.e. \( C^P(\alpha(t)) = I, D^P(\alpha(t)) = 0 \)) the linearized closed-loop system (7) with controller (8) becomes
\[
\begin{bmatrix} \delta x^P(t) \\ \delta u(t) \end{bmatrix} = \begin{bmatrix} A^P(\alpha(t)) & B^P(\alpha(t)) \\ 0 & -\eta_e I \end{bmatrix} \begin{bmatrix} \delta x^P(t) \\ \delta u(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \eta_e v \end{bmatrix}.
\]

A. Stability Analysis of the Reference Model

**Assumption 1:** The matrix \( A_m \) is bounded by \( ||A_m(\alpha)|| \leq k_A \), for all \( t \geq 0 \), where \( k_A \) is a constant.

**Remark 1:** The feasibility of this assumption can be investigated by extensive numerical simulation studies of the physical system that is being investigated, using a high fidelity dynamic model. For systems such as gas turbine engines, it already has been investigated in \([7],[6],[8]\).

**Theorem 1:** \([7],[6]\) Consider the closed-loop system (6), and assume there is a family of equilibrium points \( (x_e(t), r_e(t)) \) such that \( F(x_e(t), r_e(t)) = 0 \). Define \( A_{nl} = \frac{\partial F}{\partial x} \in S \), \( \forall x \in D_x \), where \( S := \{ A_{nl}, \forall x \in D_x \} \) is the set of linearizations of system (6). Assume there exist symmetric positive definite matrices \( P \) and \( Q \), such that
\[
PA_{nl}^r + P^T A_{nl} \leq -Q, \ \forall A_{cl} \in S,
\]
then the system (6) is stable. In other words, assuming the initial state is sufficiently close to some equilibrium, then the closed-loop system remains in a neighborhood of the equilibrium manifold for all \( t \geq 0 \).

**Remark 2:** In practice we can not obtain \( S \), instead, we can linearize system (6) for a large number of points \( x_i, i = 1, \ldots, L \), which we claim is sufficient to cover the set of actual operating conditions, to show the stability of the closed-loop system. Define \( S := \text{Co}\{A_{nl1}, \ldots, A_{nll}\} \) as a matrix polytope described by its vertices, where \( A_{nl} = \frac{\partial F}{\partial x} \in S \), for all \( i \in [1,2,\ldots,L] \). Note that \( A_{nl} \) can be obtained by linearizing the nonlinear system (6) at non-equilibrium points (transient condition), and also at equilibrium points (steady state condition), which in this paper, are represented by \( A_m(\alpha_i) \). Then using convex optimization tools \([9],[10]\), for some matrix \( Q = Q^T \), we compute a single symmetric positive definite matrix \( P \), such that
\[
P A_{nl}^r + P^T A_{nl} \leq -Q, \ \forall i \in [1,2,\ldots,L].
\]
With assumption 1 satisfied, and the claim that \( A_m(\alpha) \in S \), for all \( \alpha \in \Omega \), then system (9) is also stable. In the next section, we will show how to verify the above claim.

**Remark 3:** For the purpose of stability analysis, there is a need for multiple linearizations of the closed-loop system to construct a feasible set \( S \). The minimum number of required linearizations, \( L \), depends on the physical system; and it changes for different dynamical systems. This knowledge usually can be obtained through an extensive numerical simulation study of the dynamical system, using a high fidelity model \([11],[12]\).
Lemma 1: If matrices $P$ and $Q$ exist, such that LMI (11) is satisfied, and $A_m(\alpha(t)) \in S$, for all $\alpha \in \Omega$, then system (9) is stable.

$$PA_m(\alpha(t)) + A_m^T(\alpha(t))P \leq -Q, \forall \alpha \in \Omega.$$ (12)

Remark 4: The existence of a single matrix $P$ which guarantees the stability of a closed-loop system over some operating envelope has already been shown for dynamical systems such as gas turbine engines [7], [6], [8] and high performance aircraft [13]. The numerical verification of the assumption in Lemma 1, that is the linearized plant lives in the convex hull of the linearization matrix samples, for gas turbine engines can be found in [7], [6].

B. Decentralized Linear Parameter Dependent Modeling

Here, a decentralized model of the plant is described. Each one of the subsystems is modeled as a single input, single output (SISO) sub-plant. Each subsystem with its filtered input and its controller can be defined as

$$\begin{bmatrix} \dot{x}_k(t) \\ \dot{u}_k(t) \\ \dot{y}_k(t) \end{bmatrix} = \begin{bmatrix} f_k(x_k(t), u(t)) \\ -\eta_k \delta u(t) \\ g_k(x_k(t), u(t), r_k(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ \eta_c \\ 0 \end{bmatrix} v_k(t),$$

and the closed-loop nonlinear subsystem can be written as

$$\dot{x}_k(t) = F_k(x_k(t), x_q(t), r_k(t)), \tag{14}$$

where $x_k(t) \in D_{x_k} \subset \mathbb{R}^{n_k+2}$, and $r_k(t) \in D_{r_k} \subset \mathbb{R}$, and $x_q(t) \in D_{x_q}$ includes all the states from the other subsystems interconnecting with the $k$th subsystem. Now, similar to controller (8), for all $\alpha \in \Omega$, the parameter dependent controller for each subsystem is defined as

$$\begin{bmatrix} \dot{x}_k^*(t) \\ \dot{u}_k(t) \end{bmatrix} = \begin{bmatrix} -\epsilon_c & 1 & -1 \\ k_{i,k}(\alpha(t)) & 0 & 0 \end{bmatrix} \begin{bmatrix} x_k^*(t) \\ \delta y_k(t) \\ \delta r_k(t) \end{bmatrix}. \tag{15}$$

The Linear family of systems for the augmented subsystem (13) becomes

$$\begin{bmatrix} \dot{x}_k(t) \\ \dot{u}_k(t) \end{bmatrix} = \begin{bmatrix} A_k(\alpha(t)) & b_k(\alpha(t)) & b_r \delta r_k(t) \\ \delta y_k(t) \\ \delta r_k(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_k(t),$$

with the state feedback controller

$$v_k(t) = K_k^T(\alpha(t))\delta x_k(t), \forall \alpha \in \Omega, \tag{16}$$

where $\delta x_k(0) = \delta x_{k0}$, and $\delta x_k(t) \in \mathbb{R}^{n_k+2}$ is the $k$th subsystem state vector, $v_k(t) \in \mathbb{R}$ is the $k$th subsystem control input, and $K_k^T(\alpha(t)) \in \mathbb{R}^{n_k+2}$ is the vector of parameter dependent control gains for subsystem $k$, and $\delta r_k(t) \in \mathbb{R}$ is the $k$th subsystem reference signal. $h_k(\delta x_q(t), \alpha(t))$ is the interconnection of all other subsystems on the $k$th subsystem. Subscript $k$ represents the $k$th subsystem, where $k \in \{1, ..., N\}$; in turboshaft engine control example $k \in \{Co, Pr\}$. To design a reference model for each subsystem, we ignore the effects of the interconnection terms from other subsystems and for a desired performance, we find out the specific controller $K_k(\alpha(t)) = [0, 0, k_{i,k}(\alpha)]^T$; as a result we obtain the following closed-loop system

$$\begin{bmatrix} \delta x_k^*(t) \\ \delta u_k(t) \\ \delta y_k(t) \end{bmatrix} + \begin{bmatrix} A_k(\alpha(t)) & b_k(\alpha(t)) & b_r \delta r_k(t) \\ 0 & -\eta_c & \eta_c k_{i,k}(\alpha(t)) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} \delta r_k(t), \forall \alpha \in \Omega.$$

The stability of reference model for each subsystem is guaranteed by Lemma 1.

Remark 5: Using pre-designed linear controllers available for important operating points of the system, $K_i^T(\alpha(t))$ can be obtained based on a stability preserving interpolation approach described in [14] with respect to the scheduling parameter $\alpha$ in a smooth, continuous way. An approach by which the interpolated controller stabilizes the linearized plant for all $\alpha \in \Omega$. Another approach to compute $K_i^T(\alpha(t))$ by polynomial approximation as a function of $\alpha$.

III. DECENTRALIZED ADAPTIVE CONTROL

A. Projection Operator

The definitions and lemmas presented here are mainly adopted from [15], [16], [17], and the proofs can be found in those references.

Definition 2: Consider a convex compact set with a smooth boundary $\Omega_c = \{ \theta \in \mathbb{R}^n | f(\theta) \leq c \}$, $0 \leq c \leq 1$, where $f: \mathbb{R}^n \to \mathbb{R}$ is a smooth convex function defined as $f(\theta) = \frac{\theta^T \theta}{\epsilon_\theta + \theta_{max}^2}$, $\theta_{max}$ is the norm bound imposed on the parameter vector $\theta$, and $\epsilon_\theta$ denotes the convergence tolerance of our choice. Let the true value of the parameter $\theta$, denoted by $\theta^*$, belong to $\Omega_0$, i.e. $\theta^* \in \Omega_0$, the projection operator for two vectors, $\theta, y \in \mathbb{R}^n$ is defined as

$$\text{Proj}_\theta(y) = \begin{cases} y \quad & \frac{\partial f}{\partial \theta} \neq 0 \land \frac{\partial f}{\partial \theta}^T y > 0 \\ y - \frac{\partial f (\theta)^T}{\partial \theta} \frac{\partial f (\theta)}{\partial \theta} y & \text{otherwise} \end{cases} \tag{19}$$

where $\nabla f(\theta) = \left( \frac{\partial f(\theta)}{\partial \theta_1}, ..., \frac{\partial f(\theta)}{\partial \theta_n} \right) $ is the gradient of $f$ evaluated at $\theta$ and it is computed as $\nabla f(\theta) = \frac{2\theta}{\epsilon_\theta + \theta_{max}^2}$.

Definition 3: [18], [16] A variant of the projection algorithm, $\Gamma$-projection, updates the parameter along a symmetric positive definite gain $\Gamma$ as defined below

$$\text{Proj}_\Gamma(\theta, y) = \begin{cases} y - \frac{\nabla f(\theta)}{\nabla f(\theta)^T \nabla f(\theta)} \nabla f(\theta), & \text{if } f(\theta) > 0 \land \nabla f(\theta)^T y > 0 \\ \frac{1}{\epsilon_\theta} y, & \text{otherwise} \end{cases} \tag{20}$$

Lemma 2: Given $\theta^* \in \Omega_0$, then (\( \theta - \theta^* \))$^T (\Gamma^{-1} \text{Proj}_\Gamma(\theta, y) - y) \leq 0$. 


B. Control Design and Stability Analysis

Consider a system $S$ consists of $N$ subsystems $S_1, S_2, \ldots, S_N$ that are interconnected. Each of the subsystems is modeled as a single input, single output (SISO) linear parameter dependent model. For convenience, we shall assume that each subsystem $S_k$ has a controller $C_k$ which computes the control input $u_k$ to $S_k$. The subsystems $S_k$ are described by the equations

$$S_k: \dot{\delta x}_k(t) = A_k(\alpha(t))\delta x_k(t) + b_k v_k(t) + b_{rk} \delta r_k(t)$$

$$\delta y_k(t) = C_k\delta x_k(t),$$

where $\delta x_k(0) = \delta x_{0k}$, and $\delta x_k(t) \in \mathbb{R}^{n_k}$ is the $k$th subsystem state vector, $v_k(t) \in \mathbb{R}$ is the $k$th subsystem control input, and $\delta r_k(t) \in \mathbb{R}$ is the $k$th subsystem reference signal. $h_k(\delta x_k(t), \alpha(t))$ is the interconnection of all other subsystems on the $k$th subsystem. Note that $\delta x_k(t) = [\delta x_{1k}^T(t), \ldots, \delta x_{nk}^T(t)]^T$. Subscript $k$ represents the $k$th subsystem, where $k \in \{1, \ldots, N\}$.

Assumption 2: For the interconnection term $h_k(\delta x_k(t), \alpha(t))$, there exist positive constants $c_{vk} \in \mathbb{R}$, for each subsystem $q \neq k$, such that, it is satisfying

$$\|h_k(\delta x_k(t), \alpha(t))\| \leq \sum_{q=1}^{N} \sum_{q \neq k} c_{vk} \|\delta x_q(t)\|, \text{ for all } \alpha \in \Omega.$$

Remark 6: This assumption is a result of Assumption 1, which is about the boundedness of $A_m(\alpha(t))$.

Remark 7: The feasibility of this assumption has already been verified in [7] by numerical simulation studies, for gas turbine engine applications which we consider as the main application of this paper. For other systems, modeling and numerical studies are also needed for such verification.

The linear parameter varying reference model for the $k$th subsystem, for all $\alpha \in \Omega$ is expressed as

$$\dot{\delta x}_m,k(t) = A_{m,k}(\alpha(t))\delta x_{m,k}(t) + b_{rm} \delta r_k(t),$$

where $\delta r_k(t) \in \mathbb{R}$ is a bounded continuous reference input signal. The parameter matrix $A_{m,k} \in \mathbb{R}^{n_k \times n_k}$ is chosen with $A_{m,k}$ being Hurwitz. The boundedness of all the reference trajectories is required in a decentralized tracking control problem, which has been showed in the previous section. Note that $\delta r_k(t) \in \mathbb{R}$ is the command signal such that $\|\delta r_k(t)\| \leq r_{max,k}$.

The decentralized adaptive control of a linear parameter dependent systems can be stated as follows: Given $N$ subsystems described by (21), and $N$ reference models described by (22), and assuming that controller $C_k$ of $S_k$ can generate an input $v_k(t)$ such that all the signals in the system are bounded and $\lim_{t \to \infty} \|\delta x_k(t) - \delta x_{m,k}(t)\| = 0$. Since the effect of the interactions of subsystems on each other is bounded, we can use the following adaptive state feedback controller for each subsystem

$$C_k: v_k(t) = \hat{K}_k^T(t)\delta x_k(t),$$

with $\hat{K}_k(t) \in \mathbb{R}^{n_k}$ is the time-varying estimate of the nominal controller parameters $K_k^*(t)$.

Assumption 3: For each subsystem $S_k$, there exists an ideal gain matrix $K_k^* (\alpha(t)) = [0, 0, K_{k1}^* (\alpha(t))]$, that results in perfect matching between the reference model (22) and the plant (21) such that for all $\alpha \in \Omega$

$$A_{m,k}(\alpha(t)) = A_k(\alpha(t)) + b_k K_k^T(\alpha(t)).$$

Remark 8: The feasibility of this assumption has already been verified in [7], for gas turbine engine applications which we consider as the main application of this paper. For other systems, modeling and numerical studies are needed for such verification.

Assumption 4: Let $K_k^*(\alpha(t)) \in \theta_k$ for all $\alpha \in \Omega$, where $\theta_k$ is a known convex compact set. We also assume that $K_k^*(\alpha(t))$ is continuously differentiable, and the derivative is uniformly bounded, $\|K_k^*(\alpha(t))\| \leq \bar{d}_k < \infty$ for all $\alpha \in \Omega$.

Remark 9: $\alpha(t)$ is defined to be $\alpha(t) = \|y(t)\| = ||x^p(t)||$: since it is a function of endogenous variables (i.e., the plant states), its boundedness is guaranteed by boundedness of the plant states. As a result, its derivative $\dot{\alpha}(t) = \frac{x^p(t)^T x^p(t)}{||x^p(t)||^2}$ is also bounded. More details can be found in [19], [11], [7], [6], [20].

Remark 10: Compact set $\theta_k$ can be obtained by extensive numerical simulation studies of the system that the controller is being designed for. Smoothness, continuity, and differentiability of $K_k(\alpha(t))$, and also uniform boundedness of $K_k(\alpha(t))$, can be guaranteed, by using proper design and computation process for $K_k(\alpha(t))$ (see Remark 5).

With adaptive controller (23), the closed-loop form of subsystem $S_k$ becomes

$$\dot{\delta x}_k(t) = A_{m,k}(\alpha(t))\delta x_k(t) + b_k \hat{K}_k^T(t)\delta x_k(t) + b_{rk} \delta r_k(t) + h_k(\delta x_k(t), \alpha(t)),$$

where $\hat{K}_k(t) = K_k(t) - K_k^*(t)$. The error equation in terms of state tracking error $\dot{\delta}x_k(t) = \delta x_k(t) - \delta x_{m,k}(t)$ and controller parameters is

$$\dot{\delta}x_k(t) = A_{m,k}(\alpha(t))\delta x_k(t) + b_k \hat{K}_k^T(t)\delta x_k(t) + h_k(\delta x_k(t), \alpha(t)).$$

Based on the error model (26), adaptive laws are presented using the Lyapunov design method. Here we consider the case that for each subsystem a single quadratic Lyapunov function exists for the error model (26). If for the Hurwitz matrices $A_{m,k}(\alpha(t))$, for each subsystem, for all $\alpha \in \Omega$, there exist a single Lyapunov matrix $P_k = P_k^T > 0$, and a positive definite matrix $Q_k$ such that

$$P_k A_{m,k}(\alpha(t)) + A_{m,k}(\alpha(t))^T P_k \leq -Q_k, \forall \alpha \in \Omega$$

we use the following adaptive law:

$$\dot{\hat{K}}_k(t) = \text{Proj}_T \left(\hat{K}_k(t) - \dot{\delta}x_k(t)c_k^T(t) P_k b_k\right),$$

where $\Gamma_k = \Gamma_k^T$. A visualization of the decentralized gain scheduled model reference adaptive control architecture is given in Figure 1.

**Theorem 2:** Consider the system $S$ consisting of $N$ interconnected subsystems $S_k$ described by (21) subject to Assumption 2. Consider, in addition for subsystems $S_k$, the
adaptable control laws $C_k$ defined in (23), with adaptable laws defined in (28) subject to Assumptions 3 and 4. Then the error signals $e_k(t)$ are uniformly ultimately bounded (UUB) for all $k = 1, 2, ..., N$.

**Proof:** See [21] for detailed proof.

### IV. TURBOSHAFT ENGINE EXAMPLE

We apply the developed decentralized controller to a high fidelity physics-based model of JetCat SPT5 turboshaft engine driving a variable pitch propeller developed in [22]. To show the stability of the closed-loop reference model for each subsystem, 40 different (30 equilibrium, and 10 non-equilibrium) linearizations are used, to solve inequality (11); the inequality is solved in MATLAB with using YALMIP [9] and SeDuMi [10] packages. The numerical values for $Q_{Co}$, $Q_{Pr}$, and the matrices $P_{Co}$ and $P_{Pr}$ for the subsystems are

$$P_{Co} = \begin{bmatrix}
4.9034 & 0.9895 & -0.6234 \\
0.9895 & 1.7176 & -0.1078 \\
-0.6234 & -0.1078 & 3.4583
\end{bmatrix},$$

$$P_{Pr} = \begin{bmatrix}
1.9015 & 0.0513 & 0.1912 \\
0.0513 & 0.3882 & -0.0553 \\
0.1912 & -0.0553 & 1.0811
\end{bmatrix},$$

where the condition numbers are $\kappa(P_{Co}) = 3.6384$ and $\kappa(P_{Pr}) = 5.1066$. $Q_{Co} = 0.1 \times I_3$ and $Q_{Pr} = 0.1 \times I_3$. These simulations include the control of the nominal model (NomEng), and also control of the engine with a new core (NewCore). These decentralized adaptive control case studies, simulate the engine acceleration from the idle thrust to the cruise condition and then its deceleration back to the idle condition in a stable manner, with proper tracking performance. The initial conditions for each subsystems, and the numerical values for the corresponding adaptive controllers are $x_{Co}(0) = x_{m,Co}(0) = [0.295, 0.145, 0]^T$, $x_{Pr}(0) = x_{m,Pr}(0) = [0.161, 16, 0]^T$, $K_{Co}(0) = [0, 0, -0.49]^T$, $K_{Pr}(0) = [0, 0, -0.49]^T$, $\Gamma_{Co} = \text{diag}(40, 40, 40)$, $\Gamma_{Pr} = \text{diag}(30, 30, 30)$, $K^{*}_{Co} \in \theta_{K_{Co}} = \{[-2, 0], [-2, 0], [-2, 0]\}$, and $K^{*}_{Pr} \in \theta_{K_{Pr}} = \{[-2, 0], [-2, 0], [-2, 0]\}$. To simulate a new engine core, we assumed the high pressure spool inertia is $I_{hps,new} = 0.8I_{hps,nom}$, where $I_{hps,nom} = 4 \times 10^{-5} (kg.m^2)$. Simulation results for this scenario are shown in Figures 2 to 7. Figures 2 and 3, show the output of the core subsystem ($x^p_{Co}(t)$) and prop subsystem ($x^p_{Pr}(t)$) tracking their reference signals. Figure 4, shows the evolution of the control inputs to the augmented engine core ($u_{Co}(t)$), and prop ($u_{Pr}(t)$) subsystems, each element is corresponding to one of the control inputs to the original subsystem.
adaptive laws designed for each subsystem. Figure 7, shows the evolution of the infinity norm of the errors $|e_{Co}(t)|$ and $|e_{Pr}(t)|$. The smallness of the errors suggest that the subsystems closely track the desired reference trajectories. It also verifies the Assumption 2, which is on the boundedness of the coupling effects of the subsystems on each other, for the gas turbine engine control example.

V. CONCLUSIONS

Gain scheduled reference models were developed for each subsystem of the decentralized architecture. Using convex optimization tools, a single quadratic Lyapunov function was computed for each subsystem, which guaranteed the stability of the gain scheduled gas turbine engine core and prop reference models. Rigorous stability analysis was done by proving the uniform ultimate boundedness of the error signals for all the subsystems. Sufficient conditions for uniform ultimate boundedness of the entire system were derived. Through the simulation based on a physics-based nonlinear model of a JetCat SPT5 turboshaft engine with a new core, it was demonstrated the proposed decentralized adaptive controllers track their reference models in each subsystem for the entire flight envelope of the engine.

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