Reference Control Architecture in the Presence of Measurement Noise and Actuator Dynamics

Gerardo De La Torre, Tansel Yucelen, and Eric Johnson

***Abstract*— In this paper the command governor-based model reference control architecture is developed and analyzed for uncertain dynamical systems in the presence measurement noise and actuator dynamics. Specifically, the command governor is a dynamical system that adjusts the trajectory of a given command in order to enable an uncertain system to be able to follow an ideal reference system capturing a desired closed-loop dynamical system behavior both in transient-time and steady-state. In this paper, we present modifications to the original command governor approach in order to increase its robustness properties against measurement noise and actuator dynamics. In particular, the modified architecture is shown to retain closed-loop system stability and predictable transient and steady-state performance. Illustrative numerical results are found to verify the theoretical findings.**

I. Introduction

There are been many control architectures formulated in order to control uncertain systems while achieving closed-loop stability and guaranteed system performance. Robust controllers rely on mathematical models that precisely capture uncertain parameter variations with lower and upper bounds. Once a set of all possible uncertainties is established a control law is created in order to maintain closed-loop stability over the entire set [1]. However, characterization of this set can be difficult in some applications [2], [3] and, as a result, overly conservative sets are often used in order to guarantee closed-loop system stability. On the other hand, adaptive controllers do not require a set of all possible uncertainties. Only knowledge of a parameterization of the system uncertainty given by an unknown ideal weight matrix and a known basis function is required to suppress the affects of system uncertainties and failures [4]–[7]. This framework has been shown to be effective for tracking the states (or the outputs) of a given ideal reference system capturing a desired closed-loop system behavior. Inherently, the adaptive control law makes the overall closed-loop system nonlinear. Therefore, it is difficult to compute relative gain and time-delay margins using linear system techniques and predict transient and steady-state performance [8], [9].

The command governor-based model reference control architecture was developed for the control of uncertain dynamical systems and has been shown to effectively suppress system uncertainty through augmentation of the uncertain dynamical system control input and the reference system control input [2], [10], [11]. Unlike traditional robust control frameworks, only parameterization of the system uncertainty given by unknown weights with known conservative bounds is needed in order to stabilize the uncertain dynamical system and achieve a guaranteed level of system performance. In particular, the controlled uncertain dynamical system approximates a given ideal reference system if the command governor design parameter is properly selected. Unlike robust control architectures, the proposed control architecture's performance is not dependent on the level of conservatism of the bounds of system uncertainty. This approach is advantageous over model reference adaptive control approaches since linearity of the uncertain dynamical system is preserved through linear control laws and, hence, the closed-loop performance is predictable for different command spectrums. In this paper, we present modifications to the original command governor approach in order to increase its robustness properties against measurement noise and actuator dynamics. In particular, the modified architecture is shown to retain closed-loop system stability and predictable transient and steady-state performance.

The paper is organized as follows. Section 2 outlines notation and mathematical preliminaries used throughout the paper and Section 3 summarizes the problem formulation and the command governor-based MRC architecture. Framework modifications in order to improve robustness with respect to high frequency system content and address the issue of actuator dynamics are presented in Sections 4 and 5, respectively. In Section 6 an illustrative example to demonstrate the performance of the proposed architecture is presented. Finally, conclusions are summarized in Section 7.

II. Notation and Mathematical Preliminaries

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbb{R}_+$ (resp., $\overline{\mathbb{R}}_+$) denotes the set of positive (resp., nonnegative-definite) real numbers, $\mathbb{S}^{n\times n}$ denotes the set of $n \times n$ symmetric real matrices, $\mathbb{D}^{n\times n}$ denotes the set of $n \times n$ real matrices with diagonal scalar entries, $0_{n\times m}$ denotes a $n \times m$ zero matrix, $(\cdot)^{\mathrm{T}}$ denotes transpose, $(\cdot)^{-1}$ denotes inverse, $(\cdot)^{+}$ denotes the Moore-Penrose generalized inverse, "$\equiv$" denotes equivalency, "$\approx$" denotes approximate equality, and "$\triangleq$" denotes equality by definition, I_n denotes a $n \times n$ identity matrix and wherever appropriate the subscript n is removed for ease of exposition. In addition, we write $\det(A)$ for the determinant of a Hermitian matrix A, s for the Laplace variable, $\mathcal{G}_{u\to y}(s, \chi)$ for the transfer function from

G. De La Torre and E. N. Johnson are with the School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, GA 30332, USA. E-mails: gdelato2@gatech.edu, eric.johnson@ae.gatech.edu.

T. Yucelen is with Mechanical and Aerospace Engineering Department, Missouri University of Science and Technology, Rolla, MO 65409, USA. E-mail: yucelen@mst.edu.

input $u(s)$ to output $y(s)$ with dependence on parameter χ, $\mathcal{L}(\cdot)$ for the Laplace transform, $\sigma_{\max}(A)$ for the maximum singular value of a Hermitian matrix A, A^{L} for the left inverse, $(A^{\mathrm{T}}A)^{+}A^{\mathrm{T}}$, of $A \in \mathbb{R}^{n\times m}$, $f(t) * g(t)$ for the convolution of functions $f(t)$ and $g(t)$, and $\|\cdot\|_{\mathcal{L}_1}$ and $\|\cdot\|_1$ for the $\mathcal{L}_1$ norm and the 1-norm of a matrix, respectively [12], [13].

III. Command Governor-Based Model Reference Control Overview

Consider a class of uncertain dynamical systems given by

$$\dot{x}(t) = Ax(t)+Bu(t)+D\phi(x,t),\ x(0)=x_0,\ t\in\overline{\mathbb{R}}_+, \quad (1)$$

where $x(t)\in\mathbb{R}^n$ is the state vector available for feedback, $u(t)\in\mathbb{R}^m$ is the control input restricted to the class of admissible controls consisting of measurable functions, $A\in\mathbb{R}^{n\times n}$ and $D\in\mathbb{R}^{n\times m}$ are the known system matrices such that the pair (A,D) is controllable and $\det(D^TD)\neq 0$, $B\in\mathbb{R}^{n\times m}$ is an unknown control input matrix, and $\phi:\mathbb{R}^n\times\bar{\mathbb{R}}_+\to\mathbb{R}^m$ represents system uncertainty.

Assumption 3.1. The set of possible uncertainty in (1) is given as

$$\phi(\cdot,\cdot)\in\Phi \triangleq \{\phi:\mathbb{R}^n\times\bar{\mathbb{R}}_+\to\mathbb{R}^m:\phi(0,\cdot)=0,\\ \phi^T(x,t)\phi(x,t)\le w^\star x^Tx,\ x\in\mathbb{R}^n, t\in\bar{\mathbb{R}}_+\}$$

where $w^\star\in\mathbb{R}_+$ is a known (conservative) bound [14]. Equivalently, the set of possible uncertainty can be given as

$$\phi(\cdot,\cdot)\in\Phi \triangleq \{\phi:D\times\bar{\mathbb{R}}_+\to\mathbb{R}^m:\phi(0,\cdot)=0,\\ \phi^T(x,t)\phi(x,t)\le w^\star x^Tx,\ x\in D, t\in\bar{\mathbb{R}}_+\}$$

where $D\subseteq\mathbb{R}^n$ is a known set of allowable system states.

Assumption 3.2. The unknown control input matrix in (1) is parameterized as

$$B = D\Lambda, \quad (2)$$

where $\Lambda\in\mathbb{R}_+^{m\times m}\cap\mathbb{D}^{m\times m}$ is an unknown control effectiveness matrix such that $\Lambda_{\mathrm{L}}^\star\le\|\Lambda\|_1\le\Lambda_{\mathrm{U}}^\star$ and $\Lambda_{\mathrm{L}}^\star,\Lambda_{\mathrm{U}}^\star\in\mathbb{R}_+$ are known (conservative) bounds. The nominal case is recovered when $\Lambda=\mathrm{I}_m$.

Next, consider the reference system given by

$$\dot{x}_{\mathrm{r}}(t)=A_{\mathrm{r}}x_{\mathrm{r}}(t)+B_{\mathrm{r}}c(t),\quad x_r(0)=x_{r0},\quad t\in\overline{\mathbb{R}}_+, \quad (3)$$

where $x_{\mathrm{r}}(t)\in\mathbb{R}^n$ is the reference state vector,

$$c(t) \triangleq c_{\mathrm{d}}(t)+c_{\mathrm{g}}(t), \quad (4)$$

$c_{\mathrm{d}}(t)\in\mathbb{R}^m$ is a bounded tracking command (or $c_{\mathrm{d}}(t)\equiv 0$ for stabilization), $c_{\mathrm{g}}(t)\in\mathbb{R}^m$ is the command governor signal to be defined later, $A_{\mathrm{r}}\in\mathbb{R}^{n\times n}$ is the Hurwitz reference system matrix, and $B_{\mathrm{r}}\in\mathbb{R}^{n\times m}$ is the reference command input matrix. We will also consider an *ideal* reference system given as

$$\dot{x}_{\mathrm{r}}(t)=A_{\mathrm{r}}x_{\mathrm{r}}(t)+B_{\mathrm{r}}c_{\mathrm{d}}(t),\quad x_r(0)=x_{r0},\quad t\in\overline{\mathbb{R}}_+ \quad (5)$$

that captures a desired closed-loop dynamical system response.

Remark 3.1. In the standard model reference (adaptive) control formulation [4]–[6] there is no distinction between the two reference systems (3) and (5) since, in that case, the command governor signal $c_g(t)$ in (4) is equivalently zero. The model reference formulation summarized here is different since it is possible for the uncertain system to track the *ideal* reference system (5) in both transient time and steady-state by using a control formulation that utilizes the reference system (3).

Let the feedback control law be given by

$$u(t)=K_1x(t)+K_2c(t), \quad (6)$$

where $K_1\in\mathbb{R}^{m\times n}$ and $K_2\in\mathbb{R}^{m\times m}$ are the feedback and the feedforward gains, respectively, such that $A_{\mathrm{r}}=A+DK_1$, $B_{\mathrm{r}}=DK_2$, and $\det(K_2)\neq 0$ hold.

Next, let the command governor signal be

$$c_{\mathrm{g}}(t)\triangleq G\eta(t), \quad (7)$$

where $G\in\mathbb{R}^{m\times n}$ is the matrix defined by

$$G\triangleq K_2^{-1}D^{\mathrm{L}}=K_2^{-1}(D^TD)^{-1}D^T, \quad (8)$$

and $\eta(t)\in\mathbb{R}^n$ is the command governor output generated by

$$\begin{aligned}\dot{\xi}(t) &= -\lambda\xi(t)+\lambda e(t),\quad \xi(0)=0,\quad t\in\overline{\mathbb{R}}_+, &(9)\\ \eta(t) &= \lambda\xi(t)+(A_{\mathrm{r}}-\lambda\mathrm{I}_n)e(t), &(10)\end{aligned}$$

where $\xi(t)\in\mathbb{R}^n$ is the command governor state vector and $\lambda\in\mathbb{R}_+$ is the command governor rate. Using (6) in (1) yields

$$\begin{aligned}\dot{x}(t) &= A_{\mathrm{r}}x(t)+B_{\mathrm{r}}[c_{\mathrm{d}}(t)+c_{\mathrm{g}}(t)]+D[\Lambda-\mathrm{I}]u(t)\\ &\quad +D\phi(x,t),\ x(0)=x_0,\ t\in\overline{\mathbb{R}}_+, \quad (11)\end{aligned}$$

and the system error dynamics are given by using $e(t)\triangleq x(t)-x_{\mathrm{r}}(t)$, (3) and (11) as

$$\begin{aligned}\dot{e}(t) &= A_{\mathrm{r}}e(t)+D[\Lambda-\mathrm{I}]u(t)\\ &\quad +D\phi(x,t),\ e(0)=e_0,\ t\in\overline{\mathbb{R}}_+, \quad (12)\end{aligned}$$

where $e_0\triangleq x_0-x_{\mathrm{r}0}$. The problem can be reformulated into a robust analysis framework to provide conditions for closed-loop system stability by defining the following augmented dynamical system using (3), (7) and (11) as

$$\begin{aligned}\dot{\theta}(t) &= \tilde{F}(\lambda)\theta(t)+\tilde{G}\delta(t)+\tilde{J}c_{\mathrm{d}}(t), &(13)\\ y(t) &= \tilde{H}(\lambda)\theta(t)=[x(t),\bar{u}(t)]^T, &(14)\\ \delta(t) &= [\Lambda-\mathrm{I}]u(t)+\phi(x,t), &(15)\end{aligned}$$

where $\theta(t)=[x(t),x_{\mathrm{r}}(t),\xi(t)]^T\in\mathbb{R}^{3n}$ is the augmented system state, $\tilde{F}(\lambda)\in\mathbb{R}^{3n\times 3n}$ is the augmented system matrix, $\tilde{G}\in\mathbb{R}^{3n\times m}$ is the augmented uncertainty input matrix, $\tilde{J}\in\mathbb{R}^{3n\times m}$ is the augmented tracking command input matrix, $y(t)\in\mathbb{R}^n$ is the system output, and $\tilde{H}\in\mathbb{R}^{2n\times 3n}$ is the augmented system output matrix.

The following results and remarks summarized the stability analysis of the command governor-based model reference control framework. Figure 1 illustrates the system as a set interconnected transfer function blocks.

Lemma 3.1. Let $\Lambda \equiv \mathrm{I}, \phi(\cdot,\cdot) \equiv 0$ and $c_d(\cdot) \equiv 0$, then the augmented dynamical system given by (13) is asymptotically stable.

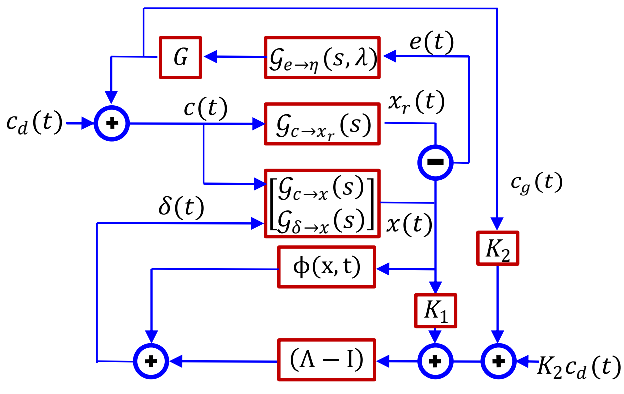

Fig. 1. Augmented System Represented as Interconnected Transfer Functions Blocks

Lemma 3.2. Consider a transfer function defined by

$$\mathcal{G}(s,\lambda) = \left(\frac{\lambda s}{s+\lambda}\mathrm{I}_n - A\right)(s\mathrm{I}_n - A)^{-1} \tag{16}$$

where A is Hurwitz and let $a(s)$ and $b(s)$ be bounded continuous signals. If $\mathcal{G}(s,\lambda)a(s) = b(s)$, then $a(t)+\epsilon(t,\lambda) = b(t)$ where $\epsilon(t,\lambda)$ is a bounded function and $\epsilon(t,\lambda) \approx 0$ and $b(t) \approx a(t)$ when λ is sufficiently large and $t \in \mathbb{R}_+$.

For the following results, Figure 1 is used to define the closed-loop transfer function from the system uncertainty $\delta(s)$ to the system state $x(s)$ as $\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda) = \mathcal{G}_{\delta\to x}(s) + \mathcal{G}_{c\to x}(s)G\mathcal{G}_{e\to\eta}(s,\lambda)\mathcal{G}_{\delta\to x}(s)$ and to define the closed-loop transfer function from the $\delta(s)$ to $\bar{u}(s)$ as $\mathcal{G}_{\delta\to\bar{u}}(s,\lambda) = K_2 G\mathcal{G}_{\delta\to\eta}(s,\lambda) + K_1\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda)$.

Lemma 3.3. Consider the uncertain dynamical system given by (13)-(15) subject to Assumption 3.1 and $\Lambda \equiv \mathrm{I}$. If $\|\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda^\star)\|_{\mathcal{L}_1} w^\star < 1$, then the system is BIBO stable when $\lambda = \lambda^\star$.

Lemma 3.4. Consider the uncertain dynamical system given by (13)-(15) subject to Assumption 3.1 and $\Lambda \equiv \mathrm{I}$. There always exist a $\lambda^\star$ such that $\|\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda^\star)\|_{\mathcal{L}_1} w^\star < 1$.

Remark 3.2. Closed-loop stability of the uncertain dynamical system is guaranteed by sufficiently increasing λ when $\Lambda \equiv \mathrm{I}$ and $\phi(\cdot,\cdot) \not\equiv 0$ for any $\phi(\cdot,\cdot) \in \Phi$. Since it can be shown that $\|\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda^\star)\|_{\mathcal{L}_1} \approx 0$ for sufficiently large λ.

Lemma 3.5. Consider the uncertain dynamical system given by (13)-(15) subject to Assumption 3.2 and $\phi(\cdot,\cdot) \equiv 0$. If $\|\mathcal{G}_{\delta\to\bar{u}}(s,\lambda^\star)\|_{\mathcal{L}_1}\|\Lambda - \mathrm{I}\|_1 < 1$ then the system is BIBO stable when $\lambda = \lambda^\star$.

Remark 3.3. Bounds for allowable Λ can be trivially computed since it can be shown for sufficiently large λ that $\mathcal{G}_{\delta\to\bar{u}}(s,\lambda) \approx -\mathrm{I}$. Thus, $\|\Lambda - \mathrm{I}\|_1 < 1$. Note, the case when $\Lambda = 0$ does not satisfy this requirement. This is expected since physically this corresponds to a case when all of the system's actuators are disabled.

Theorem 3.1. Consider the uncertain dynamical system given by (13)-(15) subject to Assumptions 3.1 and 3.2, $\Lambda \not\equiv \mathrm{I}$ and $\phi(x,t) \not\equiv 0$. If $\|\mathcal{G}_{\delta\to\bar{u}}(s,\lambda^\star)\|_{\mathcal{L}_1}\|\Lambda - \mathrm{I}\|_1 + \|\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda^\star)\|_{\mathcal{L}_1} w^\star < 1$ then the system is BIBO stable when $\lambda = \lambda^\star$.

Theorem 3.2. Consider the uncertain dynamical system given by (1) subject to Assumptions 3.1 and 3.2, the reference system given by (3), the feedback control law given by (6), and the command governor given by (7)-(10). For sufficiently large λ and $t \in \mathbb{R}_+$, (1) approximates

$$\dot{x}(t) = A_\mathrm{r}x(t) + B_\mathrm{r}c_\mathrm{d}(t), \tag{17}$$

$$z(t) = C_\mathrm{r}x(t), \tag{18}$$

where $A_\mathrm{r} = A + BK_1$ is Hurwitz, $B_\mathrm{r} = BK_2$, $C_\mathrm{r} = \mathrm{I}_n$.

Remark 3.4. It has been shown that for a sufficiently large command governor rate λ the system is stable for any bounded uncertainty and behaves like the ideal reference system (17) and (18).

IV. Measurement Noise

It was shown in Theorem 3.2 for a large command governor rate, λ, the uncertain dynamical system (1) approximates the *ideal* reference system (17). Still, a large command governor rate may lead to measurement noise amplification. This section presents a modification to the command governor-based model reference control framework such that its output $\eta(t)$ is less sensitive to the high-frequency content such as measurement noise when a large command governor rate is chosen. For this propose, let the modified command $c(t)$ be given as

$$c(t) = c_\mathrm{d}(t) + G\eta_\mathrm{f}(t), \tag{19}$$

where $G\eta_\mathrm{f}(t)$ is the modified command governor signal with $G \in \mathbb{R}^{m\times n}$ being the matrix defined by (8) and $\eta_\mathrm{f}(t) \in \mathbb{R}^n$ being the modified command governor output generated by

$$\dot{\xi}_\mathrm{f}(t) = -\lambda\xi_\mathrm{f}(t) + \lambda e_\mathrm{f}(t),\ \xi_\mathrm{f}(0) = 0,\ t \in \overline{\mathbb{R}}_+, \tag{20}$$

$$\eta_\mathrm{f}(t) = \lambda\xi_\mathrm{f}(t) + (A_\mathrm{r} - \lambda\mathrm{I}_n)e_\mathrm{f}(t), \tag{21}$$

where $\xi_\mathrm{f}(t)\in\mathbb{R}^n$ is the command governor state vector, $\lambda \in \mathbb{R}_+$ is the command governor rate and $e_\mathrm{f}(t)$ is generated by

$$\dot{e}_\mathrm{f}(t) = -\beta e_\mathrm{f}(t) + \beta e(t), \quad e_\mathrm{f}(0) = e(0), \quad t \in \bar{\mathbb{R}}_+, \tag{22}$$

where $\beta \in \mathbb{R}_+$.

Remark 4.1. The filter described by (22) is a simple example of a possible low-pass filter. More elaborate filter architectures can be selected to serve the same purpose. That is, the filter selection is not unique. Nevertheless, as shown later, filter selection changes the set of allowable Λ and $\phi(\cdot,\cdot)$ and overall transient time and steady-state system performance. The following analysis assumes the filter architecture as presented in (22). However, other architectures can easily be analyzed similarly.

The augmented system state vector can be modified to included the modified command governor state and error as

$$\theta_\mathrm{m}(t) = [x(t), x_\mathrm{r}(t), e_\mathrm{f}(t), \xi_\mathrm{f}(t)]^T \in \mathbb{R}^{4n}. \tag{23}$$

The system matrices are now defined as $\tilde{G}_\mathrm{m} = [D^T, 0_{m\times 3n}]^T$, $\tilde{J}_\mathrm{m} = [B_\mathrm{r}^T, B_\mathrm{r}^T, 0_{m\times 2n}]^T$,

$$\tilde{H}_\mathrm{m}(\lambda) = \begin{bmatrix} \mathrm{I}_n & 0_{n\times n} & 0_{n\times n} & 0_{n\times n} \\ K_1 & 0_{n\times n} & D^L\bar{A}_\mathrm{r} & \lambda D^L \end{bmatrix}, \tag{24}$$

$$\tilde{F}_{\mathrm{m}}(\lambda) = \begin{bmatrix} A_{\mathrm{r}} & 0_{n\times n} & B_{\mathrm{r}}G\bar{A}_{\mathrm{r}} & \lambda B_{\mathrm{r}}G \\ 0_{n\times n} & A_{\mathrm{r}} & B_{\mathrm{r}}G\bar{A}_{\mathrm{r}} & \lambda B_{\mathrm{r}}G \\ \beta \mathrm{I}_n & -\beta \mathrm{I}_n & -\beta \mathrm{I}_n & 0_{n\times n} \\ 0_{n\times n} & 0_{n\times n} & \lambda \mathrm{I}_n & -\lambda \mathrm{I}_n \end{bmatrix}, \quad (25)$$

and $\bar{A}_{\mathrm{r}} = A_{\mathrm{r}} - \lambda \mathrm{I}_n$. The following result extends the result in Lemma 3.1 to included the newly added filter.

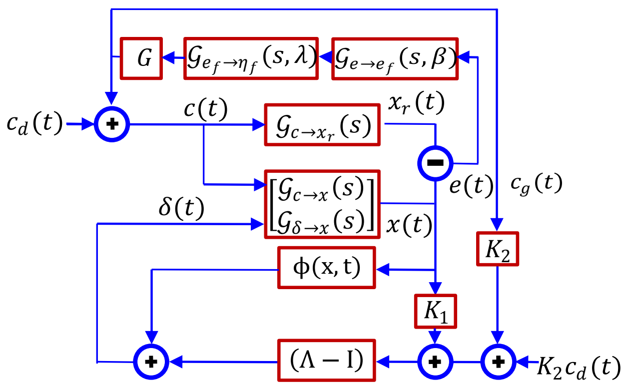

Fig. 2. Modified Augmented System Represented as Transfer Functions

Lemma 4.1. Let $\Lambda \equiv \mathrm{I}, \phi(\cdot,\cdot) \equiv 0$ and $c_d(\cdot) \equiv 0$, then the modified augmented dynamical system given by (13), $\theta(t) = \theta_{\mathrm{m}}(t)$, $\tilde{F}(\lambda) = \tilde{F}_{\mathrm{m}}(\lambda), \tilde{G} = \tilde{G}_{\mathrm{m}}$ and $\tilde{J} = \tilde{J}_{\mathrm{m}}$ is asymptotically stable.

For brevity the proof is omitted.

The modified system can be represented by interconnected transfer function blocks as shown in Figure 2. The affect the modification has on the set of allowable Λ and $\phi(\cdot,\cdot)$ can be seen by solving for $x(t)$ from the closed-loop transfer function $\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda)$ as

$$\begin{aligned} x(t) &= \mathcal{L}^{-1}[\mathcal{G}_{\delta\to x}(s)\delta(s) \\ &\quad + \mathcal{G}_{c\to x}(s)G\frac{-\beta}{s+\beta}\bar{\mathcal{A}}_r(s,\lambda)D\delta(s)] \end{aligned} \quad (26)$$

where $\bar{\mathcal{A}}_r(s,\lambda) = (\frac{\lambda s}{s+\lambda}\mathrm{I}_n - A_{\mathrm{r}})(s\mathrm{I}_n - A_{\mathrm{r}})^{-1}$. For sufficiently large λ,

$$\begin{aligned} x(t) &\approx \mathcal{L}^{-1}[(s\mathrm{I}_n - A_{\mathrm{r}})^{-1}D\delta(s) \\ &\quad -(s\mathrm{I}_n - A_{\mathrm{r}})^{-1}D\frac{-1}{\frac{s}{\beta}+1}\delta(s)]. \end{aligned} \quad (27)$$

For illustrative purposes assume the filter given by (22) is an ideal filter such that

$$\begin{aligned} \delta(j\omega) &= \frac{-1}{\frac{j\omega}{\beta}+1}\delta(j\omega), \quad \omega \le \beta \\ 0 &= \frac{-1}{\frac{j\omega}{\beta}+1}\delta(j\omega), \quad \omega > \beta. \end{aligned}$$

Thus,

$$x(t) \approx \mathcal{L}^{-1}[(j\omega\mathrm{I}_n - A_{\mathrm{r}})^{-1}D\delta(j\omega)], \quad \omega > \beta. \quad (28)$$

In general, $\|\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda)\|_{\mathcal{L}_1} \not\approx 0$ for sufficiently large λ and there does not exist a $\lambda^\star$ such that $\|\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda^\star)\|_{\mathcal{L}_1}\|w^\star <$ 1 for all $w^\star$. Thus, the set of allowable $\phi(\cdot,\cdot)$ is limited. The modified closed-loop transfer function from the system uncertainty $\delta(s)$ to the $\bar{u}(s)$ is given as

$$\mathcal{G}_{\delta\to\bar{u}}(s,\lambda) = K_2G\mathcal{G}_{\delta\to\eta_{\mathrm{f}}}(s,\lambda) + K_1\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda). \quad (29)$$

For sufficiently large λ,

$$\begin{aligned} \mathcal{G}_{\delta\to\bar{u}}(s,\lambda) &= \frac{-1}{\frac{s}{\beta}+1} + K_1[(s\mathrm{I}_n - A_{\mathrm{r}})^{-1}D \\ &\quad -(s\mathrm{I}_n - A_{\mathrm{r}})^{-1}D\frac{-1}{\frac{s}{\beta}+1}]. \end{aligned} \quad (30)$$

Thus, for sufficiently large λ, $\|\mathcal{G}_{\delta\to\bar{u}}(s,\lambda)\|_{\mathcal{L}_1} \ge 1$. The bounds of allowable Λ given in remark 3.3 for the case when $\Lambda \ne \mathrm{I}$ and $\phi(\cdot,\cdot) = 0$ may no longer be valid

Remark 4.2. Note that, for sufficiently large λ, $\|\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda)\|_{\mathcal{L}_1}$ and $\|\mathcal{G}_{\delta\to\bar{u}}(s,\lambda)\|_{\mathcal{L}_1}$ are determined by β, the reference system and the system's input matrix and not by the system's uncertainty. Thus, the set of allowable Λ and $\phi(\cdot,\cdot)$ can always be determined.

Theorem 4.1. Consider the modified uncertain dynamical system given by (13)-(15), $\theta(t) = \theta_{\mathrm{m}}(t)$, $\tilde{F} = \tilde{F}_{\mathrm{m}}, \tilde{G} = \tilde{G}_{\mathrm{m}}$, $\tilde{H} = \tilde{H}_{\mathrm{m}}$ and $\tilde{J} = \tilde{J}_{\mathrm{m}}$ subject to Assumptions 3.1 and 3.2, $\Lambda \not\equiv \mathrm{I}$ and $\phi(x,t) \not\equiv 0$. If $\|\mathcal{G}_{\delta\to\bar{u}}(s,\lambda^\star)\|_{\mathcal{L}_1}\|\Lambda - \mathrm{I}\|_1 + \|\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda^\star)\|_{\mathcal{L}_1}w^\star < 1$ then the system is BIBO stable when $\lambda = \lambda^\star$.

Proof. This is a direct consequence of Theorem 3.1. ■

Since the filter deteriorates the command governor ability to suppress the system uncertainty signal, $\delta(t)$, the uncertain dynamical system may not be able to approximate the *ideal* reference system as previously shown in Theorem 3.2. Still, as shown in the next result, the uncertain dynamical system approximates the ideal reference system augmented with the difference between the filtered and unfiltered command governor output.

Lemma 4.2. Consider the modified uncertain dynamical system given by (1) subject to Assumptions 3.1 and 3.2, the reference system given by (3), the feedback control law given by (6) and (19), and the command governor given by (20)-(21). For sufficiently large λ and $t \in \mathbb{R}_+$, (1) approximates

$$\dot{x}(t) = A_{\mathrm{r}}x(t) + B_{\mathrm{r}}c_{\mathrm{d}}(t) + \eta_{\mathrm{f}} - \eta \quad (31)$$
$$z(t) = C_{\mathrm{r}}x(t), \quad (32)$$

where $A_{\mathrm{r}} = A + BK_1$ is Hurwitz, $B_{\mathrm{r}} = BK_2$, $C_{\mathrm{r}} = \mathrm{I}_n$.

For brevity the proof is omitted.

V. Actuator Dynamics

While previously deriving stability conditions we have neglected the affects of actuator dynamics on the system. The proposed command governor architecture suppress the affect of system uncertainty (Λ and $\phi(\cdot,\cdot)$) by augmenting the input control signal with a command governor signal, $c_{\mathrm{g}}(t)$. As a result, an actuator's input control signal may contain high frequency content outside of the operating bandwidth of the actuator. Actuator dynamics may inhibit the command governor ability to suppress the affect of system uncertainty due to actuator response time and bandwidth. In this section, we analyze the affect of actuator dynamics on the proposed model reference control architecture and discuss a modification to the framework to address this issue.

Consider a class of uncertain dynamical systems given by

$$\dot{x}(t) = Ax(t) + D[u_{\rm a}(t) + \phi(x,t)],\ x(0) = x_0,\ t \in \overline{\mathbb{R}}_+, \quad (33)$$

where $x(t) \in \mathbb{R}^n$ is the state vector available for feedback, $u_{\rm a}(t) \in \mathbb{R}^m$ represents the effective control input to the system, $A \in \mathbb{R}^{n\times n}$ and $D \in \mathbb{R}^{n\times m}$ are known system matrices such that the pair (A, D) is controllable and $\det(D^T D) \neq 0$, and $\phi : \mathbb{R}^n \times \overline{\mathbb{R}}_+ \to \mathbb{R}^m$ is the system uncertainty. Assumption 3.1 is assumed for this system as well. Furthermore, consider a class of dynamical actuators represented by

$$\dot{u}_{\rm a}(t) = \Lambda[A_{\rm a}u_{\rm a}(t) + B_{\rm a}u(t)],\ u_{\rm a}(0) = u_{{\rm a}_0},\ t \in \overline{\mathbb{R}}_+, \quad (34)$$

where $u(t) \in \mathbb{R}^m$ is the control input restricted to the class of admissible controls consisting of measurable functions and given by (6), $A_{\rm a} \in \mathbb{R}^{m\times m}$ and $B_{\rm a} \in \mathbb{R}^{m\times m}$ are the system matrices representing known actuator dynamics and $\Lambda \in \mathbb{R}_+^{m\times m} \cap \mathbb{D}^{m\times m}$ is an unknown control effectiveness matrix such that $\Lambda_{\rm L}^\star \leq \|\Lambda\|_1 \leq \Lambda_{\rm U}^\star$ and $\Lambda_{\rm L}^\star, \Lambda_{\rm U}^\star \in \mathbb{R}$ are known (conservative) bounds as discussed in Assumption 3.2. Using the feedback control input (6), the uncertain dynamical system can be written as

$$\begin{aligned}\dot{x}(t) &= A_{\rm r}x(t) + B_{\rm r}[c_{\rm d}(t) + c_{\rm g}(t)] \\ &\quad + D[u_{\rm a}(t) - u(t) + \phi(x,t)], \\ &= A_{\rm r}x(t) + B_{\rm r}[c_{\rm d}(t) + c_{\rm g}(t)] + D\delta(t), \quad (35)\end{aligned}$$

and the system error dynamics are given as

$$\dot{e}(t) = A_{\rm r}e(t) + D\delta(t),\ e(0) = e_0,\ t \in \overline{\mathbb{R}}_+. \quad (36)$$

Now consider a filter on the input of the system (6) such that

$$\mathcal{G}_{u\to u_{\rm f}}(s)u(s) = u_{\rm f}(s). \quad (37)$$

and the following condition holds

$$\|[\mathcal{G}_{u_{\rm f}\to u_{\rm a_f}}(s) - {\rm I}]\mathcal{G}_{u\to u_{\rm f}}(s)\|_{\mathcal{L}_1} < 1, \quad (38)$$

where $\mathcal{G}_{u_{\rm f}\to u_{\rm a_f}}(s)$ represents the actuator dynamics given as

$$\dot{u}_{\rm a_f}(t) = \Lambda[A_{\rm a}u_{\rm a_f}(t) + B_{\rm a}u_{\rm f}(t)], \quad (39)$$

$u_{\rm f} \in \mathbb{R}^m$ is the filtered input, $u_{\rm a_f}(t)$ filtered effective control input and $u(t)$ is the control input given by (6). Intuitively, the filter should be selected by considering the bandwidth of the actuator. By limiting the bandwidth of the control input it is ensure the difference between the filtered computed control and filtered effective control input is limited. Notice that the set of allowable Λ will be dependent on the input filter.

The class of uncertain dynamical systems (33) with input filter (37) is given as

$$\dot{x}(t) = Ax(t) + D[u_{\rm a_f}(t) + \phi(x,t)] \quad (40)$$

and the reference system (3) with an input filter (37) is given as

$$\dot{x}_{\rm r}(t) = Ax_{\rm r}(t) + Du_{\rm f}(t). \quad (41)$$

Finally, the error dynamics between the reference system and the uncertain system are

$$\dot{e}(t) = Ae(t) + D[u_{\rm a_f}(t) - u_{\rm f}(t) + \phi(x,t)]. \quad (42)$$

Conditions for closed-loop stability can be derived following a similar procedure as done Sections 3 and 4. However, as seen before, results will diverge from results in Section 3 since the input filter inhibits the command governor ability to suppress system uncertainty. The allowable set of Λ and $\phi(\cdot,\cdot)$ will be dependent on the selection of the input filter.

Lemma 5.1. Consider the modified linear uncertain dynamical system given by (40) subject to Assumption 3.1, the reference system given by (41), the feedback control law given by (6) and (37), the command governor given by (9)-(10), actuator dynamics (39) and the input filter condition (38). For sufficiently large λ, (40) approximates

$$\begin{aligned}\dot{x}(t) &= A_{\rm r}x(t) + B_{\rm r}c_{\rm d}(t) + D[u_{\rm f}(t) - u(t)] \quad (43)\\ z(t) &= C_{\rm r}x(t), \quad (44)\end{aligned}$$

where $A_{\rm r} = A + BK_1$ is Hurwitz, $B_{\rm r} = BK_2$, $C_{\rm r} = {\rm I}_n$.

For brevity the proof is omitted.

VI. Illustrative Numerical Examples

Consider a nonlinear uncertain dynamical system given as

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [\Lambda u(t) + \phi(x,t)],$$

where $x(t) = [x_1(t)\ x_2(t)]^T, x_1(0) = 0, x_2(0) = 0$, and the unknown system uncertainty is given as $\Lambda = 0.9$ and $\phi(x,t) = 0.2(x_1^2 + x_2^2) + 0.3x_1x_2$. The dynamical system is known to operate in $D = \{x \in \mathbb{R}^2 : x_1 \in (-8, 8), x_2 \in (-1.5, 1.5)\}$. The ideal reference system is selected such that the feedback control law (6) is given by $K_1 = [-0.2\ -0.8]$ and $K_2 = 1.1$. Suppose for this system conservative estimates of system uncertainty are given as $w^\star = 3$ (actual bound is approximately 2.08), $\Lambda_{\rm L}^\star = 0.6$, and $\Lambda_{\rm U}^\star = 1.4$. In addition, we consider actuator dynamics given by

$$\dot{u}_{\rm a}(t) = \Lambda[-5u_{\rm a}(t) + 5u(t)], \quad u_{\rm a}(0) = 0, \quad t \in \overline{\mathbb{R}}_+. \quad (45)$$

For this study $\bar{e}(t)$ refers to the error vector between the nonlinear uncertain dynamical system and the *ideal* reference system (17).

Figure 3 shows $\|\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda)\|_{\mathcal{L}_1}$ and $\|\mathcal{G}_{\delta\to u}(s,\lambda^\star)\|_{\mathcal{L}_1}$ as functions of λ. As shown in Section 4, $\|\mathcal{G}_{\delta\to x}(s,\lambda)\|_{\mathcal{L}_1} \to 0$ and $\|\mathcal{G}_{\delta\to u}(s,\lambda^\star)\|_{\mathcal{L}_1} \to 1$ as $\lambda \to \infty$. Figure 4 shows the responses to a unity step of the ideal reference system and the nonlinear uncertain dynamical system with no command governor architecture implemented. The nonlinear uncertain dynamical system is unstable.

When $\lambda = 15$ an input filter $\mathcal{G}_{u\to u_{\rm f}}(s)u(s) = \frac{25}{s+25}$ was used. It was calculated that $\|[\mathcal{G}_{u_{\rm f}\to u_{\rm a_f}}(s) - {\rm I}]\mathcal{G}_{u\to u_{\rm f}}(s)\|_{\mathcal{L}_1} \approx 0.85$. Thus, $\|\mathcal{G}_{\delta\to\bar{u}}(s,\lambda^\star)\|_{\mathcal{L}_1}\|[\mathcal{G}_{u_{\rm f}\to u_{\rm a_f}}(s) - {\rm I}]\mathcal{G}_{u\to u_{\rm f}}(s)\|_{\mathcal{L}_1} + \|\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda^\star)\|_{\mathcal{L}_1}w^\star \approx 1.11 \times 0.85 + 0.21 \times 3 = 1.57$. Figure 5 shows the system response. Even though stability conditions were not met the uncertain dynamical system was able to track the ideal reference system well. Next the

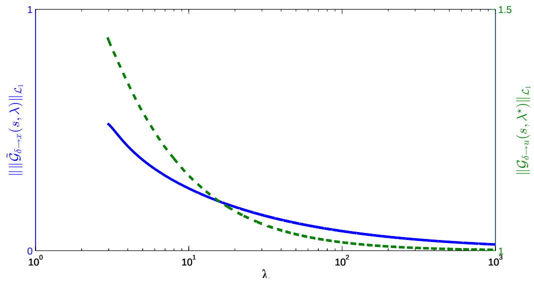

Fig. 3. $\|\tilde{\mathcal{G}}_{\delta\to x}(s,\lambda)\|_{\mathcal{L}_1}$(solid line) and $\|\mathcal{G}_{\delta\to u}(s,\lambda^\star)\|_{\mathcal{L}_1}$ (dotted line) as functions of λ

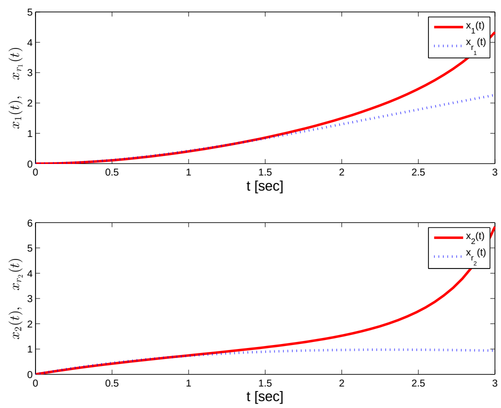

Fig. 4. Ideal reference system and nonlinear uncertain dynamical system responses for a unity step command.

available input bandwidth are further restricted by using an input filter of $\mathcal{G}_{u\to u_\mathrm{f}}(s)u(s) = \frac{5}{s+5}$. Figure 6 shows the system response. By comparing Figure 5 and Figure 6 it is seen tracking error increases when the available input bandwidth decreases.

VII. Conclusion

We proposed modifications to the original command governor approach in order to increase its robustness properties against measurement noise and actuator dynamics. We theoretically showed that our modified architecture retains closed-loop system stability and achieves predictable performance both in transient-time and steady-state by approximating an ideal reference system augmented with a mismatch term when the command governor rate is chosen judiciously.

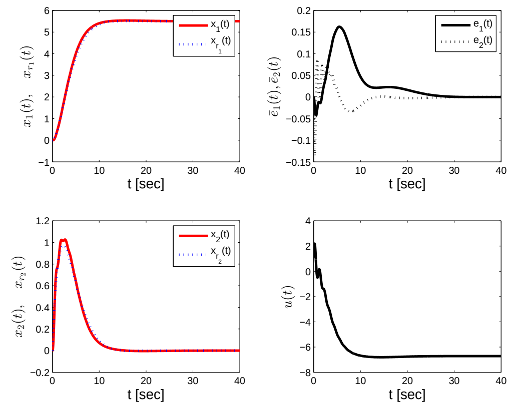

Fig. 5. System response and tracking error with command governor rate $\lambda = 15$ and input filter $\mathcal{G}_{u\to u_\mathrm{f}}(s)u(s) = \frac{25}{s+25}$ for a unity step command.

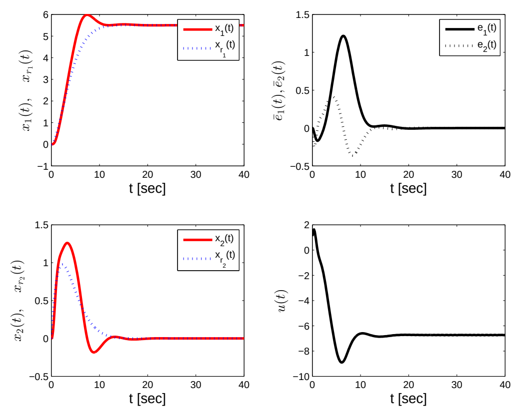

Fig. 6. System response and tracking error with command governor rate $\lambda = 15$ and input filter $\mathcal{G}_{u\to u_\mathrm{f}}(s)u(s) = \frac{5}{s+5}$ for a unity step command.

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