Frequency-Limited Adaptive Control of a Generic Transport Model in the Presence of System Uncertainty and Losses in Control Effectiveness

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Abstract

While adaptive control has been used in numerous applications to achieve system performance without excessive reliance on dynamical system models, the necessity of high-gain learning rates for safety-critical systems to achieve fast adaptation can be a serious limitation of most adaptive controllers. In order to address this problem, two novel adaptive control approaches developed recently aim to limit the bandwidth of the closed-loop adaptive control system in order to suppress the high-frequency content contained in the system error dynamics and adaptation algorithms. Specifically, this key feature of these frameworks allows for robust and fast adaptation by utilizing high-gain learning rates without inducing high-frequency oscillations in system states and the adaptive control signal. In this paper, we apply these two methods to a high-fidelity scaled transport aircraft model, namely the generic transport model, developed at NASA Langley Research Center in the presence of unexpected structural damage and uncertainty in control surface effectiveness.

I. Introduction

Often when designing automatic controllers for complex systems idealized assumptions, linearization, model order reduction, exogenous disturbances, and unexpected system changes lead to modeling inaccu-

racies. If not mitigated, the uncertainties present in the system model can result in poor performance or system instability. Therefore, it is of great importance in control design to achieve a desired performance when dealing with dynamical systems subject to system uncertainties.

The model reference adaptive control (MRAC) framework was first introduced by Whitaker et al. in order to improve system performance in the presence of system uncertainties11,15. In this framework, the controller gains are updated based on the difference (error) between a reference model capturing the desired closed-loop dynamical system behavior and the uncertain system. Therefore, the control gains are changed in order for the uncertain system to track the reference model. This tracking error can be reduced by increasing the adaptive learning rate of the adaptive controller and, therefore, for sufficiently high learning rates large uncertainties can be mitigated. This is especially important in high performance aircraft systems

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since system faults, structural damage, or sudden changes in control surfaces effectiveness can result in large changes in system parameters. However, high learning rates can result in adaptive control signals with high-frequency content and, for practical applications, can result in violation of actuator amplitude and rate saturation constraints,\textsuperscript{8} excite unmodeled dynamics\textsuperscript{4,12}, and lead to system instability. With the exception of a few MRAC frameworks\textsuperscript{5,6}, this design trade-off must be considered when designing an adaptive controller.

In Ref. 6, the authors use a low-pass filter to suppress the high-frequency oscillations caused by fast adaptation. The authors in Ref. 19 present an adaptive control approach that preserves system robustness while utilizing high learning rates. However, both approaches require knowledge of a conservative upper bound on the unknown constant gain in their uncertainty parameterization. This conservative bound may not be available or can change if the system undergoes an unexpected change.

The authors in Refs. 9, 13 present a modification to the reference system in order to address poor transient performance. Specifically, the modification architecture involves the system error between the uncertain dynamical system and the modified reference system scaled by a modification gain. In the limiting case as the modification gain goes to infinity, it is shown in Ref. 9 that the uncertain system converges to the reference system. However, as shown in Refs. 17, 18, even though this modification can effectively suppress uncertainties, in the presence of exogenous low-frequency persistent disturbances the tracking performance is poor.

In this paper, we apply two newly proposed frequency limiting adaptive control frameworks\textsuperscript{17,18,20} to the NASA generic transport model (GTM)\textsuperscript{1}. The framework presented in Refs. 17, 18 limits the high frequency content of the adaptive control signal through a modification of the reference system. Specifically, this modification causes the high-frequency content of the system error to rapidly converge to zero. The adaptive framework presented in Ref. 20 also limits high frequency content. However, it is through a modification of the adaptive weight update law that this is achieved. The numerical study presented in the paper shows that the frequency limiting adaptive control frameworks result in an increase in system performance. Furthermore, the frequency limiting modifications are shown to allow for high adaptive learning rates without exhibiting high-frequency oscillations in the control signal or the system states.

\section{II. Frequency Limiting Adaptive Control Frameworks}

We begin by presenting a brief review of the model reference adaptive control problem. Consider the nonlinear uncertain dynamical system given by

\begin{align}
\dot{x}_p(t) &= A_p x_p(t) + B_p u(t) + B_p \delta_p(x_p(t)), \quad x_p(0) = x_{p0}, \quad t \geq 0, \quad (1)
\end{align}

where, for $t \geq 0$, $x_p(t) \in \mathbb{R}^{n_p}$ is the accessible state vector, $u(t) \in \mathbb{R}^m$ is the control input, and $\delta_p : \mathbb{R}^{n_p} \to \mathbb{R}^m$ represents system uncertainty, $A_p \in \mathbb{R}^{n_p \times n_p}$ is a known system matrix, $B_p \in \mathbb{R}^{n_p \times m}$ is a known control input matrix, and $\Lambda \in \mathbb{R}^{m \times m}$ is an unknown positive definite and diagonal control effectiveness matrix.

Furthermore, we assume that the pair $(A_p, B_p)$ is controllable and the uncertainty is parameterized as

\begin{align}
\delta_p(x_p) &= W_p^T \sigma_p(x_p), \quad x_p \in \mathbb{R}^{n_p}, \quad (2)
\end{align}

where $W_p \in \mathbb{R}^{s \times m}$ is an unknown weighting matrix and $\sigma_p : \mathbb{R}^{n_p} \to \mathbb{R}^s$ is a known basis function of the form $\sigma_p(x_p) = [\sigma_{p1}(x_p), \sigma_{p2}(x_p), \ldots, \sigma_{ps}(x_p)]^T$.

To address command following, let $c(t) \in \mathbb{R}^{n_c}$, $t \geq 0$, be a given bounded piecewise continuous command and $x_c(t) \in \mathbb{R}^{n_c}$, $t \geq 0$, be the integrator state satisfying

\begin{align}
\dot{x}_c(t) &= E_p x_p(t) - c(t), \quad x_c(0) = x_{c0}, \quad t \geq 0, \quad (3)
\end{align}

where $E_p \in \mathbb{R}^{n_c \times n_p}$ is a weighting matrix that allows us to choose a linear combination of $x_p(t)$, $t \geq 0$, to be tracked by $c(t)$, $t \geq 0$. Now, augmenting (1) with (3) yields

\begin{align}
\dot{x}(t) &= Ax(t) + B Au(t) + BW_p^T \sigma_p(x_p(t)) + B c(t), \quad x(0) = x_0, \quad t \geq 0, \quad (4)
\end{align}
where \( x(t) \triangleq [x_p^T(t), x^T_c(t)]^T \in \mathbb{R}^n \), \( n = n_p + n_c \), is the (augmented) state vector, \( x_0 \triangleq [x_{p0}^T, x_{c0}^T]^T \in \mathbb{R}^n \),

\[
A \triangleq \begin{bmatrix} A_p & 0_{n_p \times n_c} \\ E_p & 0_{n_c \times n_c} \end{bmatrix} \in \mathbb{R}^{n \times n},
\]

(5)

\[
B \triangleq \begin{bmatrix} B_p^T, 0_{n_c \times m} \end{bmatrix}^T \in \mathbb{R}^{n \times m},
\]

(6)

\[
B_r \triangleq \begin{bmatrix} 0_{n_p \times n_c}, -I_{n_c \times n_c} \end{bmatrix}^T \in \mathbb{R}^{n \times n_c}.
\]

(7)

Next, consider the feedback control law given by

\[
u(t) = u_n(t) + u_a(t),
\]

(8)

where \( u_n(t) \in \mathbb{R}^m \) and \( u_a(t) \in \mathbb{R}^m \) are the nominal and adaptive control laws, respectively. Furthermore, let the nominal control law be given by

\[
u_n(t) = -Kx(t),
\]

(9)

where \( K \in \mathbb{R}^{m \times n} \) is such that \( A_r := A - BK \) is Hurwitz. Using (8), (9), and (4) yields

\[
\dot{x}(t) = A_r x(t) + B_r c(t) + BA [u_n(t) + W^T \sigma(x(t))], \quad x(0) = x_0, \quad t \geq 0,
\]

(10)

where \( W^T \triangleq [\Lambda^{-1}W_p^T, (\Lambda^{-1} - I_{m \times m})K_r] \in \mathbb{R}^{(s+n) \times m} \) is an unknown (aggregate) weight matrix and \( \sigma^T(x(t)) \triangleq [\sigma_p^T(x_p(t)), x^T_c(t)] \in \mathbb{R}^{s+n} \) is a known (aggregate) basis function. Now, using (10), let the adaptive control law be given by

\[
u_a(t) = -\hat{W}^T(t) \sigma(x(t)),
\]

(11)

where \( \hat{W}(t) \in \mathbb{R}^{(s+n) \times m} \) is the estimate of \( W \) satisfying the update law

\[
\dot{\hat{W}}(t) = \gamma \sigma(x(t)) \hat{e}^T(t) PB, \quad \hat{W}(0) = \hat{W}_0, \quad t \geq 0,
\]

(12)

where \( \gamma > 0 \) is the learning rate, \( e(t) \triangleq x(t) - x_r(t) \) is the system error with \( x_r(t) \in \mathbb{R}^n \) representing the reference state vector satisfying the reference system

\[
\dot{x}_r(t) = A_r x_r(t) + B_r c(t), \quad x_r(0) = x_{r0}, \quad t \geq 0,
\]

(13)

and the \( n \times n \) matrix \( P = P^T > 0 \) is a solution of the Lyapunov equation

\[
0 = A_r^T P + PA_r + R,
\]

(14)

with \( R = R^T \geq 0 \).

Now, using (10), (11), and (13) the system error dynamics are given by

\[
\dot{e}(t) = A_r e(t) - B A \hat{W}^T(t) \sigma(x(t)), \quad e(0) = e_0, \quad t \geq 0,
\]

(15)

where \( \hat{W}(t) \triangleq \hat{W}(t) - W \in \mathbb{R}^{(s+n) \times m} \) is the weight error and \( e_0 \triangleq x_0 - x_{r0} \). It can be shown though Lyapunov stability analysis that the system error \( e(t) \) and the weight error \( \hat{W}(t) \) are Lyapunov stable, and hence, are bounded for all \( t \geq 0 \) and \( \lim_{t \to \infty} e(t) = 0 \).

However, as discussed in the Introduction, high learning rates can lead to unrealistic control signals with high-frequency oscillations that can violate actuator amplitude and rate saturation constraints. Furthermore, unmodeled system dynamics can be excited with high learning rates and control signals containing high-frequency content. In the following sections two modifications to the standard model reference adaptive control architecture are presented. In particular, these modifications aim to reduce the high-frequency content exhibited in adaptive control signals when high learning rates are used. Therefore, high learning rates can be used for fast adaption without the adverse consequences of high-frequency oscillations.

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A. Frequency Limitation through Reference Model Modification

In this section, the adaptive control architecture proposed in Refs. 17,18 is summarized. This architecture aims to reduce the high-frequency content of the adaptive control signal by limiting the frequency-content of the system error dynamics (15) during transient-time (i.e., the learning phase). Since the update law (12) is driven by the system error \( e(t) \), reduction in the frequency-content of the error signal will reduce the high-frequency oscillations that may be present in the adaptive control signal.

To begin, let \( e_L(t) \in \mathbb{R}^n \) be a low-pass filtered system error of \( e(t) \) given by

\[
\dot{e}_L(t) = A_r e_L(t) + \eta (e(t) - e_L(t)), \quad e_L(0) = 0, \quad t \geq 0,
\]

where \( \eta \in \mathbb{R}_+ \) is a filter gain. Note that since \( e_L(t) \) is a low-pass filtered system error of \( e(t) \), the filter gain \( \eta \) is chosen such that \( \eta \leq \eta^* \), where \( \eta^* \in \mathbb{R}_+ \) is a design parameter.

Next, we add a mismatch term to the system error dynamics (15) in order to enforce a distance condition between the trajectories of the system error \( e(t) \) and the trajectories of its low-pass filtered version \( e_L(t) \). This leads to a minimization problem involving an error criterion capturing the Euclidean distance between \( e(t) \) and \( e_L(t) \). In particular, consider the cost function given by

\[
J(e, e_L) = \frac{1}{2} \| e - e_L \|^2
\]

and note that the negative gradient of (17) with respect to \( e \) is given by

\[
\frac{\partial[-J(e(t), e_L(t))]}{\partial e(t)} = -(e(t) - e_L(t)), \quad t \geq 0,
\]

which gives the structure of the proposed mismatch term.

Using the idea presented in Refs. [2,11,14,20,21], we now add (18) to the system error dynamics given by (15). For this purpose, we modify the reference system (13) as

\[
\dot{x}_r(t) = A_r x_r(t) + B_r c_t + \kappa (e(t) - e_L(t)), \quad x_r(0) = x_{r0}, \quad t \geq 0,
\]

where \( \kappa > 0 \), and hence, using (10), (11), and (19) the system error dynamics is given by

\[
\dot{e}(t) = A_r e(t) - B \Lambda \hat{W}(t) \sigma(x(t)) \kappa (e(t) - e_L(t)), \quad e(0) = e_0, \quad t \geq 0.
\]

Note that the update law (12) is driven by the system error \( e(t) = x(t) - x_r(t) \), where \( x_r(t) \) is obtained from (19) (not (13)). To prevent any confusion in notation, we redefine the ideal (unmodified) reference system given by (13) as

\[
\dot{x}_r(t) = A_r x_r(t) + B_r c(t), \quad x_r(0) = x_{r0}, \quad t \geq 0
\]

where \( x_r(t) \in \mathbb{R}^n \) represents the ideal reference state vector.

Finally, let \( \hat{x}(t) \triangleq x(t) - x_r(t) \) be the deviation error from the ideal reference trajectory \( x_r(t) \) with \( x_r(t) \), once again, satisfying (19). Then, the system error, weight update error, low-pass filtered system error, and the deviation error dynamics are, respectively, given by (20), (16), and

\[
\dot{\hat{W}}(t) = \gamma \sigma(x(t)) e^T(t) P B, \quad \hat{W}(0) = \hat{W}_0, \quad t \geq 0
\]

\[
\dot{\hat{x}}(t) = A_r \hat{x}(t) + \kappa (e(t) - e_L(t)), \quad \hat{x}(0) = 0
\]

where \( \hat{W}(t) \triangleq \hat{W}(t) - W \) and \( \hat{W}_0 \triangleq \hat{W}_0 - W \). The next theorem presents the first consequence of the presented modification. For the statement of this theorem \( \| x \|_{\mathcal{L}_\infty} \triangleq \sup_{t \in [0, \infty)} \| x(t) \|_2 \) and \( \| \cdot \|_F \) denotes the Frobenius matrix norm.

**Theorem II.1 (17,18)** Consider the nonlinear uncertain dynamical system given by (1) with uncertainty given by (2), the (modified) reference system given by (19), and the feedback control law given by (8) with (9), (11), and (12). Then, the solution \( (e(t), \hat{W}(t), e_L(t), \hat{x}(t)) \) of the closed-loop system given by (20), (16), (22), and (23) is Lyapunov stable for all \((e_0, \hat{W}_0, 0, 0) \in \mathbb{R}^n \times \mathbb{R}^{(n+n)\times m} \times \mathbb{R}^n \times \mathbb{R}^n \) and \( t \in \mathbb{R}_+ \), and \( \lim_{t \to \infty} (x(t) - x_r(t)) = 0 \). In addition,
\[ \|x(\cdot) - x_r(\cdot)\|_{\infty} \leq \frac{\varepsilon_Y}{\lambda_{\min}(P)} \left( 1 + \sqrt{\frac{\kappa \lambda_{\max}(P)}{2\xi \lambda_{\min}(R)}} \right), \]  

where \( \xi \in (0, 1) \) and \( \varepsilon_Y \triangleq \gamma^{-1} ||\hat{W}_0 \Lambda^{1/2}||_F^2 + \lambda_{\max}(P)||e_0||^2 \).

Note that even though the proposed weight update law utilizes the error between the modified reference system and the uncertain system, Theorem II.1 guarantees that \( \lim_{t \to \infty} (x(t) - x_r(t)) = 0 \). Thus, the uncertain system asymptotically converges to the ideal reference system.

The following result shows that the high-frequency content of the system error is effectively suppressed as the design parameter \( \kappa \) of the modified reference system (19) increases.

**Theorem II.2 (17,18)** Consider the system error dynamics given by (20) and the low-pass filtered system error dynamics given by (16). Then, for sufficiently large \( \kappa \) and \( \gamma \),

\[ e_H(t, \kappa) = e^{-\kappa t}e_{i_0} + O(\kappa^{-1}), \]

where \( e_{i_0} \triangleq e(0) - e_L(0) = e_0 \).

Theorem II.2 shows how the proposed architecture limits the high-frequency content of the system error. Specifically, the solution \( e_H(t) \) is globally exponentially stable for a sufficiently high \( \kappa \), and hence, vanishes exponentially.

### B. Frequency Limitation through Weight Update Law Modification

In this section, the adaptive control architecture proposed in Ref. 20 in summarized. As the framework presented above, this architecture aims to reduce the high frequency content of the adaptive control signal. However, instead of achieving this goal through a modification of the error dynamics and reference system, this framework modifies the adaptive update law. Specifically, it introduces a \( \sigma \)-modification-like term to reduce the high frequency-content of the weight estimate and, therefore, the adaptive control signal.

Let \( \hat{W}_t(t) \in \mathbb{R}^{s \times m}, t \geq 0 \), be a low-pass filter weight estimate of \( \hat{W}(t), t \geq 0 \), given by

\[ \dot{\hat{W}}_t(t) = \Gamma_t[\hat{W}(t) - \hat{W}_t(t)], \quad \hat{W}_t(0) = \hat{W}_0, \quad t \geq 0, \]  

where \( \Gamma_t \in \mathbb{R}^{s \times s} \) is a positive-definite filter gain matrix. Note that since \( \hat{W}_t(t), t \geq 0 \), is a low-pass filter weight estimate of \( \hat{W}(t), t \geq 0 \), the filter gain matrix \( \Gamma_t \) is chosen such that \( \lambda_{\max}(\Gamma_t) \leq \gamma_{t,\max}, \) where \( \gamma_{t,\max} > 0 \) is a design parameter. We add a modification term to the standard update law given by (12) in order to enforce a distance condition between the trajectories of the weight estimate \( \hat{W}(t), t \geq 0 \), and the trajectories of its low-pass filtered version \( \hat{W}_t(t), t \geq 0 \). This leads to a minimization problem involving an error criterion capturing the Frobenius distance between \( \hat{W}(t), t \geq 0 \), and \( \hat{W}_t(t), t \geq 0 \).

Specifically, consider the cost function given by

\[ \mathcal{J}(\hat{W}, \hat{W}_t) = \frac{1}{2} ||\hat{W} - \hat{W}_t||^2_F \]  

and note that the negative gradient of (27) with respect to \( \hat{W} \) is given by

\[ \frac{\partial [-\mathcal{J}(\hat{W}(t), \hat{W}_t(t))]}{\partial \hat{W}(t)} = -(\hat{W}(t) - \hat{W}_t(t)), \quad t \geq 0, \]

which gives the structure of the proposed modification term. As before, we now construct the proposed update law by adding (28) to (12) to obtain the modified update law

\[ \dot{\hat{W}}(t) = \Gamma[\beta(x(t))e^T(t)PB - \sigma(\hat{W}(t) - \hat{W}_t(t))], \quad \hat{W}(0) = \hat{W}_0, \quad t \geq 0, \]  

where \( \Gamma \in \mathbb{R}^{s \times s} \) is a positive-definite learning rate matrix and \( \sigma > 0 \) is a modification gain.

The adaptive control literature contains many examples of modification terms that are introduced to the standard update law given by (12) in order to achieve a specific desired behavior\(^2,3,7,10,14\). For example,
the $\sigma$-modification term takes the form $-\sigma(\hat{W}(t) - W^*)$, where $\sigma > 0$ and $W^*$ is an approximation of the ideal weight and was introduced in order to increase the system’s robustness to external disturbances and unmodeled dynamics. This modification term effectively results in a gradient descent of the weight estimate $\hat{W}$ to $W^*$. The proposed modification terms given by (28) is of the same structure as the $\sigma$-modification term with $W^*$ replaced by $\hat{W}_t(t)$, $t \geq 0$. Note that this new modification architecture allows the update law to learn using its low-frequency content, since the weight estimate is driven to its filtered state. Therefore, the modification term suppresses the high-frequency content that may be present in the adaptive control signal. Furthermore, as shown in Ref. 20, the proposed update law does not affect the asymptotic stability of the system error dynamics.

Define $e(t) \triangleq x(t) - x_m(t)$, $t \geq 0$, $\hat{W}(t) \triangleq \hat{W}(t) - W$, $t \geq 0$, and $\hat{W}_t(t) \triangleq \hat{W}_t(t) - W$, $t \geq 0$. Then, the system error, weight update error, and filtered weight update error dynamics are, respectively, given by

$$
\dot{e}(t) = A_ce(t) - B\hat{W}^T(t)\sigma(x(t)), \quad e(0) = e_0, \quad t \geq 0,
$$

$$
\dot{\hat{W}}(t) = \Gamma_0 [\sigma(x(t))e^T(t)PB - \sigma(\hat{W}(t) - \hat{W}_t(t))], \quad \hat{W}(0) = \hat{W}_0,
$$

$$
\dot{\hat{W}}_t(t) = \Gamma_1 [\hat{W}(t) - \hat{W}_t(t)], \quad \hat{W}_t(0) = \hat{W}_{t,0}.
$$

**Theorem II.3 (20)** Consider the nonlinear uncertain dynamical system given by (10), the reference system given by (13), and the feedback control law given by (8), (9), and (11). Furthermore, let the update law be given by (29). Then, the solution $(e(t), \hat{W}(t), \hat{W}_t(t))$ of the closed-loop system given by (30), (31), and (32) is Lyapunov stable for all $(e_0, \hat{W}_0, \hat{W}_{t,0}) \in \mathbb{R}^n \times \mathbb{R}^{s \times m} \times \mathbb{R}^{s \times m}$ and $t \geq 0$, and $x(t) \rightarrow x_r(t)$ as $t \rightarrow \infty$. In addition, for all $t \geq 0$, the system error, weight update error, and filtered weight update error satisfy the transient performance bounds given by

$$
\|e(t)\|_2 \leq \left[ 1 \over \lambda_{\min}(P) \right] \left( \lambda_{\max}(P) \|e_0\|_2^2 + \|\Gamma_0^{-1}\|_F \|\hat{W}_0\|_F^2 + \sigma \|\Gamma_1^{-1}\|_F \|\hat{W}_{t,0}\|_F^2 \right)^{\frac{1}{2}},
$$

$$
\|\hat{W}(t)\|_F \leq \left[ 1 \over \lambda_{\min}(\Gamma_0) \right] \left( \lambda_{\max}(P) \|e_0\|_2^2 + \|\Gamma_0^{-1}\|_F \|\hat{W}_0\|_F^2 + \sigma \|\Gamma_1^{-1}\|_F \|\hat{W}_{t,0}\|_F^2 \right)^{\frac{1}{2}},
$$

$$
\|\hat{W}_t(t)\|_F \leq \left[ 1 \over \lambda_{\min}(\Gamma_1) \right] \left( \lambda_{\max}(P) \|e_0\|_2^2 + \|\Gamma_0^{-1}\|_F \|\hat{W}_0\|_F^2 + \sigma \|\Gamma_1^{-1}\|_F \|\hat{W}_{t,0}\|_F^2 \right)^{\frac{1}{2}}.
$$

### III. Application to the NASA Generic Transport Model

In this section, we apply the proposed frequency limiting adaptive control architectures to the NASA Generic Transport Model (GTM). The GTM is a high-fidelity scaled transport aircraft model developed by NASA Langley Research Center. The simulation model includes actuator dynamics, including rate and amplitude saturation. In addition, the model can be used to simulate structural damage to the aircraft and changes in control effectiveness. It is shown that the proposed frequency limiting architecture can be implemented to increase tracking performance of the system in pitch and roll rate commands. In particular, it is shown that the adaptive controllers are able to improve tracking performance in the event of unexpected structural damage or change in control effectiveness. In addition, the frequency limiting modifications are shown to allow for higher adaptive learning rates than the standard model reference adaptive control architecture.

A pitch rate and roll rate command tracking nominal controller is first designed for a linear model using a robust servomechanism LQR approach that incorporates integral control. The linear model was obtained by linearizing the system about the trimmed operating point given by an angle of attack of 5 degrees and an altitude of 1000 feet. The linearized GTM model is 7th-ordered and its state vector is given by $x(t) = [p_i(t), q_i(t), v(t), w(t), p(t), q(t), r(t)]$, where $p_i(t), q_i(t)$ are integrator states, $v(t), w(t)$ are velocity components of the vehicle about the $y$ and $z$ body axes, and $p(t), q(t)$, and $r(t)$ are body angular rate about the pitch, roll, and yaw body axes, respectively. In addition, the feedback input to the system is given as $u(t) = [u_1(t), u_2(t), u_3(t), u_4(t), u_5(t)]$, where $u_1(t)$ is the input to the elevator channel, $u_2(t)$ is the input to the aileron channel, $u_3(t)$ is the input to the rudder channel, $u_4(t)$ is the input to the left spoiler channel, and $u_5(t)$ is the input to the right spoiler channel.
The uncertain model given by (10) is modified slightly as

\[ \dot{x}(t) = Ax(t) + BA(GKx(t) + u_a(t) + W^T\sigma(x(t))) + Bc(t), \quad x(0) = x_0, \quad t \geq 0, \tag{36} \]

where

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & -1 & 0 \\
0 & 0 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & -0.5206 & 0 & 11.1178 & 0 & -124.4157 \\
0 & 0 & -0.0342 & -2.6469 & 0 & 119.5622 & 0 \\
0 & 0 & -0.7489 & -0.0003 & -6.2366 & -0.0073 & 2.2293 \\
0 & 0 & -0.0133 & -0.2477 & 0.0012 & -3.5807 & 0.0350 \\
0 & 0 & 0.2039 & 0 & -0.3503 & -0.0288 & -1.4579
\end{bmatrix},
\tag{37}
\]

\[
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & -0.0236 & 0.4139 & 0.0152 & -0.0152 \\
-0.5996 & 0.1860 & -0.0900 & 0.2521 & 0.2521 \\
0 & -0.8702 & 0.2666 & -0.4945 & 0.4945 \\
-0.7147 & 0.0814 & -0.0071 & 0.0140 & 0.0140 \\
0 & -0.0290 & -0.4122 & -0.0466 & 0.0466
\end{bmatrix},
\tag{38}
\]

\[
K = \begin{bmatrix}
33.0851 & 1.6509 & 0.0283 & 0.1705 & -0.2278 & -7.6022 & -0.4511 \\
-1.5924 & 45.6361 & 0.4432 & -0.0030 & -6.5512 & 0.5294 & -7.5856 \\
1.1923 & 7.5703 & 0.1539 & 0.0102 & -0.6602 & -0.1312 & -7.5138
\end{bmatrix},
\tag{39}
\]

and \(G \in \mathbb{R}^{M \times 3}\) is a control allocation matrix in which \(M > 3\) denotes the number of independent control channels and is given by

\[
G = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}.
\tag{40}
\]

The command input is given as \(c(t) = [c_1(t), c_2(t)], \quad t \geq 0,\) where \(c_1(t)\) is the pitch rate command given by

\[
c_1(t) = \begin{cases} 
8 \text{ [deg/sec]}, & 2.5 \leq t \leq 5 \text{ or } 12.5 \leq t < 15, \\
-8 \text{ [deg/sec]}, & 5 \leq t \leq 7.5 \text{ or } 15 \leq t < 17.5, \\
0 \text{ [deg/sec]}, & \text{otherwise},
\end{cases}
\tag{41}
\]

and \(c_2(t)\) is the roll rate command given by

\[
c_2(t) = \begin{cases} 
8 \text{ [deg/sec]}, & 5 \leq t \leq 7.5 \text{ or } 15 \leq t < 17.5, \\
-8 \text{ [deg/sec]}, & 7.5 \leq t \leq 10 \text{ or } 17.5 \leq t < 20, \\
0 \text{ [deg/sec]}, & \text{otherwise}.
\end{cases}
\tag{42}
\]
noted, \( \Lambda = I_5 \). Finally, all of the control surfaces of the aircraft have an imposed rate limit of 50 degrees per second.

Three adaptive controllers are designed for this study. The first controller, adaptive controller A, was designed using the architecture summarized in Section II.A. Adaptive controller B was designed using the architecture summarized in Section II.B. The last controller, adaptive controller C, was designed by combining both of the presented architectures. Table 1 shows the design parameters selected for each adaptive controller.

<table>
<thead>
<tr>
<th>Adaptive Controller</th>
<th>( \eta )</th>
<th>( \kappa )</th>
<th>( \sigma )</th>
<th>( \Gamma_f )</th>
<th>( \Gamma )</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>25</td>
<td>10</td>
<td>0</td>
<td>0</td>
<td>diag[0.1, 0.1, 0.01, 0.01, 5, 5, 1]</td>
</tr>
<tr>
<td>B</td>
<td>0</td>
<td>0</td>
<td>0.5</td>
<td>0.1</td>
<td>diag[5, 5, 0.01, 0.01, 100, 100, 100]</td>
</tr>
<tr>
<td>C</td>
<td>25</td>
<td>10</td>
<td>0.5</td>
<td>0.1</td>
<td>diag[5, 5, 0.01, 0.01, 100, 100, 100]</td>
</tr>
</tbody>
</table>

In order to compare the performance of each of the controllers objectively, the following performance measures were calculated for each design:

\[
\int_0^\infty |e_p(t)|dt, \quad \int_0^\infty |e_q(t)|dt, \quad (43)
\]

where \( e_p(t) \triangleq p_r(t) - p(t) \), \( e_q(t) \triangleq q_r(t) - q(t) \), and \( p_r(t) \) and \( q_r(t) \) are the reference pitch and roll rate, respectively. The results of all of the designs are shown in Table 2.

| Case 1 | \( \int_0^\infty |e_p(t)|dt \) | \( \int_0^\infty |e_q(t)|dt \) |
|--------|----------------|----------------|
| Nominal Controller | 11.10 | 10.10 |
| Adaptive Controller A | 11.08 | 10.10 |
| Adaptive Controller B | 11.02 | 10.11 |
| Adaptive Controller C | 11.06 | 10.10 |

| Case 2 | \( \int_0^\infty |e_p(t)|dt \) | \( \int_0^\infty |e_q(t)|dt \) |
|--------|----------------|----------------|
| Nominal Controller | 31.91 | 10.35 |
| Adaptive Controller A | 29.31 | 10.50 |
| Adaptive Controller B | 29.86 | 10.61 |
| Adaptive Controller C | 28.61 | 10.46 |

| Case 3 | \( \int_0^\infty |e_p(t)|dt \) | \( \int_0^\infty |e_q(t)|dt \) |
|--------|----------------|----------------|
| Nominal Controller | 14.02 | 16.87 |
| Adaptive Controller A | 14.07 | 16.52 |
| Adaptive Controller B | 14.12 | 15.90 |
| Adaptive Controller C | 14.03 | 16.50 |

| Case 4 | \( \int_0^\infty |e_p(t)|dt \) | \( \int_0^\infty |e_q(t)|dt \) |
|--------|----------------|----------------|
| Nominal Controller | 8.94 | 14.6 |
| Adaptive Controller A | 8.85 | 13.98 |
| Adaptive Controller B | 8.82 | 13.37 |
| Adaptive Controller C | 8.87 | 14.10 |
A. Case 1

Figures 1-4 show the response of each system when no damage is present, which corresponds to a baseline response. Note that even though no uncertainty caused by structural damage or changes in control effectiveness is present, all adaptive controllers where able to slightly improve the system tracking performance. Figure 5 shows the response of the system when the standard model reference adaptive control architecture is used with the same learning rates as adaptive controllers B and C. Note the poor tracking results, particularly in the pitch rate, due to oscillations in the system states. A similar trend is seen if the learning rates from adaptive controller A are used. Therefore, it can be concluded that the presented modifications do indeed allow for high learning rates while suppressing the high-frequency content of the adaptive control signal.

B. Case 2

Figures 6-9 show the response of each system if the left outboard trailing edge flap is removed at $t = 10$. The primary effects of this structural change are a rolling moment bias that increases with the angle of attack and the available roll control authority is reduced\(^1\). Note that all adaptive controllers were able to significantly reduce the tracking error in pitch rate while only marginally increasing the tracking error in roll rate.

C. Case 3

Figures 10-13 show the response of each system if the entire elevator from the left stabilizer is removed at $t = 10$. The primary effects of this structural change are a rolling moment bias that increases with the angle of attack, the available pitch and roll control authorities are reduced and longitudinal stability is reduced\(^1\). Note that adaptive controller B was able to significantly reduce the tracking error in roll rate while only marginally increasing the tracking error in pitch rate.

D. Case 4

Figures 10-13 show the response of each system when $\Lambda = 0.75I_5$. This case simulates a reduction of control effectiveness over all control surfaces. Note that adaptive controller B was able to reduce the tracking error in pitch rate and roll rate.

IV. Conclusion

In this paper, we implemented two newly proposed frequency limiting adaptive control frameworks to the NASA GTM model. Specifically, improved system performance through high adaptive rates in the presence of unexpected structural damage and uncertainty in control effectiveness is shown. It is also shown that the modifications present in the frameworks successfully suppress the high-frequency content often associated with high learning rates in adaptive controllers which are necessary in order to rapidly reduce and maintain system trajectory tracking errors for safety-critical systems.

V. Acknowledgment

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References

\(^1\)GM documentation for release v0903. NASA Langley Research Center, 2009.

Figure 1. Case 1: Angular rates, aerodynamic angles, and control surface positions as functions of time when the nominal controller is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system with the nominal controller, and the red line corresponds to the system's angle of attack).
Figure 2. Case 1: Angular rates, aerodynamic angles, and control surface positions as functions of time when adaptive controller A is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, and the red line corresponds to the system’s angle of attack).

Figure 3. Case 1: Angular rates, aerodynamic angles, and control surface positions as functions of time when adaptive controller B is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, and the red line corresponds to the system’s angle of attack).
Figure 4. Case 1: Angular rates, aerodynamic angles, and control surface positions as functions of time when adaptive controller C is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, and the red line corresponds to the system’s angle of attack).

Figure 5. Case 1: Angular rates, aerodynamic angles, and control surface positions as functions of time when the standard model reference adaptive control architecture is used with the same learning rates of adaptive controllers B and C (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the plant and the red line corresponds to the system’s angle of attack).
Figure 6. Case 2: Angular rates, aerodynamic angles, and control surface positions as functions of time when the nominal controller is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, with the nominal controller, and the red line corresponds to the system’s angle of attack).

Figure 7. Case 2: Angular rates, aerodynamic angles, and control surface positions as functions of time when adaptive controller A is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system,, and the red line corresponds to the system’s angle of attack).
Figure 8. Case 2: Angular rates, aerodynamic angles, and control surface positions as functions of time when adaptive controller B is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, and the red line corresponds to the system’s angle of attack).

Figure 9. Case 2: Angular rates, aerodynamic angles, and control surface positions as functions of time when adaptive controller C is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, and the red line corresponds to the system’s angle of attack).
Figure 10. Case 3: Angular rates, aerodynamic angles, and control surface positions as functions of time when the nominal controller is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, with the nominal controller, and the red line corresponds to the system’s angle of attack).

Figure 11. Case 3: Angular rates, aerodynamic angles, and control surface positions as functions of time when adaptive controller A is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, and the red line corresponds to the system’s angle of attack).
Figure 12. Case 3: Angular rates, aerodynamic angles, and control surface positions as functions of time when adaptive controller B is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, and the red line corresponds to the system's angle of attack).

Figure 13. Case 3: Angular rates, aerodynamic angles, and control surface positions as functions of time when adaptive controller C is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, and the red line corresponds to the system's angle of attack).
Figure 14. Case 4: Angular rates, aerodynamic angles, and control surface positions as functions of time when the nominal controller is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, with the nominal controller, and the red line corresponds to the system’s angle of attack).

Figure 15. Case 4: Angular rates, aerodynamic angles, and control surface positions as functions of time when adaptive controller A is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, and the red line corresponds to the system’s angle of attack).
Figure 16. Case 4: Angular rates, aerodynamic angles, and control surface positions as functions of time when adaptive controller B is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, and the red line corresponds to the system's angle of attack).

Figure 17. Case 4: Angular rates, aerodynamic angles, and control surface positions as functions of time when adaptive controller C is used (dotted lines correspond to the reference trajectory/right-sided surfaces, solid lines correspond to the uncertain system, and the red line corresponds to the system's angle of attack).