ADAPTIVE ARCHITECTURES FOR DISTRIBUTED CONTROL OF MODULAR SYSTEMS

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MODULAR SYSTEMS

- Large-scale modular systems
  - Composed of interconnected heterogeneous submodules
  - Reliance on local information to achieve global objectives

- Modeling challenge
  - First principles of physics
  - Idealized assumptions
  - Model simplifications

⇒ Models that do not adequately capture the physical subsystems

- Distributed control objective
  - Achieve & maintain performance goals under uncertainties
**WHY ADAPTIVE CONTROL**

- **Fixed-gain controllers**
  - Tradeoff performance vs uncertainty
  - Cannot improve themselves
  - Tuned to worst-case
  - Not to a physical system

- Assumes future will be like present
  - Does not take into account
    - *Environmental changes*
    - *Change in dynamics*
    - *Structural damage*

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**Graph**

- Performance Error vs Modeling Accuracy
- Point 1: 
  - $k$ kg
  - $1$ m
- Point 2:
  - $2$ kg

**Question**

What if $m = 0.8$ kg?
Why Adaptive Control

- Adaptive controllers
  - Achieve a given performance
  - Do not excessively rely on models
  - Improve themselves under adverse conditions

Fixed-gain controllers require greater modeling accuracy.
Adaptive controller tunes itself to the physical system.
**WHY ADAPTIVE CONTROL**

- **Design challenge**
  - Worst-case performance bounds depend on system uncertainty
  - Control engineers cannot assign a priori bounds during a design
  - This issue can lead to a modular system failure
LITERATURE ON DISTRIBUTED ADAPTATION

a) Traditional approaches
- No communication is allowed
- No asymptotic stability result

b) Approaches with ctrl exchange
- Every local controller accesses to desired trajectories of all others
- Cost of communication

c) Approaches with state exchange
- Every local state needs to be shared with all other subsystems
- Cost of communication
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**CONTRIBUTION OF THE RESEARCH**

- Distributed control of modular systems
- Graph-theoretic approach to model interactions
- Submodules communicate with each other, locally
- Adaptive control with performance guarantees, a priori
**PROBLEM FORMULATION**

- A modular system consisting of **N uncertain heterogeneous submodules** defined on a graph such that **the physical interactions between submodules are unknown**

\[
S_i: \quad \dot{x}_i(t) = A_i x_i(t) + B_i u_i(t) + D_i \left[ \sum_{i \sim j} \Delta_{ij}(x_j(t)) + \delta_i(x_i(t)) \right], \quad x_i(0) = x_{i0}.
\]

- **Assumptions** on the system uncertainties

\[
\Delta_{ij}(x_j) = F_{ij}^T \alpha_{ij}(x_j)
\]
\[
\delta_i(x_i) = G_i^T \beta_i(x_i)
\]
\[
B_i = D_i \Lambda_i
\]
PROBLEM FORMULATION

- A modular system consisting of $N$ uncertain heterogeneous submodules defined on a graph such that the physical interactions between submodules are unknown.

- Allowing local communication through graph each local submodule has to follow desired trajectories

$$S_{ri} : \quad \dot{x}_{ri}(t) = A_{ri}x_{ri}(t) + B_{ri}c_{i}(t), \quad x_{ri}(0) = x_{ri0}$$

with a priori given performance bounds

- $\Delta_{ij}(x_j) = F_{ij} \alpha_{ij}(x_j)$
- $\delta_{i}(x_{i}) = G_{i}^{T} \beta_{i}(x_{i})$
- $B_{i} = D_{i} \Lambda_{i}$
PROBLEM FORMULATION

A modular system consisting of N uncertain heterogeneous submodules defined on a graph such that the physical interactions between submodules are unknown.

Allowing local communication through graph each local submodule has to follow desired trajectories with a priori given performance bounds.

\[ S_{ri} : \dot{x}_{ri}(t) = A_{ri}x_{ri}(t) + B_{ri}c_i(t), \quad x_{ri}(0) = x_{ri0} \]

\[ \Delta_{ij}(x_j) = r_{ij} \alpha_{ij}(x_j) \]

\[ \delta_i(x_i) = G_i^T \beta_i(x_i) \]

\[ A_{ri} = A_i + B_iK_{1i} \]

\[ B_{ri} = B_iK_{2i} \]
DISTRIBUTED ADAPTIVE CONTROL

- Grouping uncertain terms with the local controller

\[ \dot{x}_i(t) = A_{ri}x_i(t) + B_{ri}c_i(t) + D_i\Lambda_i\tilde{u}_i(t) \]

\[ \tilde{u}_i(t) \triangleq u_i(t) + \Lambda_i^{-1}\sum_{i \sim j} F_{ij}^T \alpha_{ij}(x_j(t)) + \Lambda_i^{-1}G_i^T \beta_i(x_i(t)) \]

\[ -K_{1i}x_i(t) - K_{2i}c_i(t) \]

\[ W_i^T \sigma_i(\cdot) \triangleq \Lambda_i^{-1}\sum_{i \sim j} F_{ij}^T \alpha_{ij}(x_j(t)) + \Lambda_i^{-1}G_i^T \beta_i(x_i(t)) \]

\[ -K_{1i}x_i(t) - K_{2i}c_i(t) \]

- Distributed adaptive controller

\[ C_i : u_i(t) \triangleq -\hat{W}_i^T(t)\sigma_i(x_i(t), x_j(t), c_i(t)) \]

Received information from local neighbors
DISTRIBUTED ADAPTIVE CONTROL

- Traditional adaptive control, revisited

\[
\dot{e}_i(t) \triangleq A_{ri}e_i(t) - D_i\Lambda_i\tilde{W}_i^T(t)\sigma_i(x_i(t), x_j(t), c_i(t))
\]

\[
\mathcal{V}(\cdot) \triangleq \sum_{i=1}^{N} \mathcal{V}_i(e_i, \tilde{W}_i)
\]

\[
\mathcal{V}_i(e_i, \tilde{W}_i) = e_i^T P_i e_i + \gamma_i^{-1} \text{tr}(\tilde{W}_i\Lambda_i^{1/2})^T(\tilde{W}_i\Lambda_i^{1/2})
\]

\[
\mathcal{V}(\cdot) = -\sum_{i=1}^{N} e_i^T(t)R_i e_i(t)
\]

- Hence, \( \lim_{t \to \infty} e_i(t) = 0 \), \( e_i(t) \triangleq x_i(t) - x_{ri}(t) \), if we let

\[
\dot{\tilde{W}}_i(t) \triangleq \gamma_i\sigma_i(x_i(t), x_j(t), c_i(t))(x_i(t) - x_{ri}(t))^T P_i D_i
\]

\[
0 = A_{ri}^T P_i + P_i A_{ri} + R_i
\]

Received information from local neighbors
DISTRIBUTED ADAPTIVE CONTROL

Traditional setup enforces to use high gain all time to achieve a given performance – not practical to use available system bandwidth all time

\[ \hat{e}_i(t) \triangleq A_{ri}e_i(t) - D_i \Lambda_i \hat{W}_i(t) \sigma_i(x_i(t), x_j(t), c_i(t)) \]

\[ \nu(\cdot) \triangleq \sum_{i=1}^{N} \nu_i(e_i, \hat{W}_i) \]

\[ \nu_i(e_i, \hat{W}_i) = e_i^T P_i e_i + \gamma_i^{-1} \text{tr}(\hat{W}_i \Lambda_i^2)^T (\hat{W}_i \Lambda_i^2) \]

\[ \dot{\hat{W}}_i(t) \triangleq \sigma_i \mu_i \]

\[ \|e_i(t)\|_{L_\infty} \leq \left[ \frac{\lambda_{\max}(P_i) \|e_{i0}\|_2^2 + \gamma_i^{-1} \|\hat{W}_{i0} \Lambda_i^{\frac{1}{2}}\|_F^2}{\lambda_{\min}(P_i)} \right]^{\frac{1}{2}} \]

\[ \|e(t)\|_{L_\infty} \leq N \max_{i,i \in \mathcal{V}_G} \|e_i(t)\|_{L_\infty} \]

\[ e(t) \triangleq [e_1^T(t), \ldots, e_N^T(t)]^T \]
**SET-THEORETIC ADAPTIVE CONTROL**

- Introduce a state-dependent rate adjustment approach for distributed adaptation to achieve a priori known performance.
- Restricted potential functions \( \phi(\|z\|_H) \), \( \|z\|_H \triangleq \sqrt{z^T H z} \), defined on \( \mathcal{D}_\epsilon \triangleq \{\|z\|_H : \|z\|_H \in [0, \epsilon)\} \) with respect to:

\[
\begin{align*}
  \text{i)} & \quad \text{If } \|z\|_H = 0, \text{ then } \phi(\|z\|_H) = 0 \\
  \text{ii)} & \quad \text{If } \|z\|_H \in \mathcal{D}_\epsilon \text{ and } \|z\|_H \neq 0, \text{ then } \phi(\|z\|_H) > 0 \\
  \text{iii)} & \quad \text{If } \|z\|_H \to \epsilon, \text{ then } \phi(\|z\|_H) \to \infty \\
  \text{iv)} & \quad \phi(\|z\|_H) \text{ is continuously differentiable on } \mathcal{D}_\epsilon \\
  \text{v)} & \quad \text{If } \|z\|_H \in \mathcal{D}_\epsilon, \text{ then } \phi_d(\|z\|_H) > 0, \text{ where } \phi_d(\|z\|_H) \triangleq \frac{d\phi(\|z\|_H)}{d\|z\|^2_H}
\end{align*}
\]
A candidate restricted potential function satisfying all conditions: 

\[ \phi(\|z\|_H) = \frac{\|z\|_H^2}{\epsilon - \|z\|_H}, \quad \|z\|_H \in D_\epsilon \]

\[ \phi(\|z\|_H), \quad \phi_d(\|z\|_H) \]

\[ \|z\|_H \]

\[ \phi \]

\[ \phi_d \]
**STABILITY WITH PERFORMANCE GUARANTEES**

- Consider the restricted Lyapunov function

\[
\mathcal{V}_i(e_i, \tilde{W}_i) = \phi(\|e_i\|_{P_i}) + \gamma_i^{-1} \text{tr} \left( \tilde{W}_i \Lambda_i^{1/2} \right)^T \left( \tilde{W}_i \Lambda_i^{1/2} \right),
\]

\[
\mathcal{V}_i : \mathcal{D}_{e_i} \times \mathbb{R}^{\dim(\tilde{W}_i)} \to \mathbb{R}_+ , \quad \mathcal{D}_{e_i} \triangleq \{ e_i(t) : \|e_i(t)\|_{P_i} < \epsilon \}
\]

- Using

\[
\frac{d\phi(\|e_i(t)\|_{P_i})}{dt} = 2\phi_d(\|e_i(t)\|_{P_i}) e_i^T(t) P_i \dot{e}_i(t),
\]

we obtain

\[
\dot{\mathcal{V}}_i(e_i(t), \tilde{W}_i(t)) = -\phi_d(\|e_i(t)\|_{P_i}) e_i^T(t) R_i e_i(t) \leq 0
\]

if we let

\[
\dot{\tilde{W}}_i(t) \triangleq \gamma_i \phi_d(\|e_i\|_{P_i}) \sigma_i(x_i(t), x_j(t), c_i(t)) e_i^T(t) P_i D_i
\]

- The level sets

\[
\Omega_i \triangleq \{ (e_i(t), \tilde{W}_i(t)) \in \mathcal{D}_{e_i} \times \mathbb{R}^{\dim(\tilde{W}_i)} : \mathcal{V}_i(e_i, \tilde{W}_i) \leq \xi_i \}
\]

are compact and invariant for any \( \xi_i > 0 \)

- Follows from LaSalle-Yoshizawa theorem that

\[
\lim_{t \to \infty} e_i(t) = 0
\]
Consider the restricted Lyapunov function
\[ \mathcal{V}_i(e_i, \tilde{W}_i) = \phi(\|e_i\|_{P_i}) + \gamma_i^{-1} \text{tr} \left( \tilde{W}_i \Lambda_i^2 \right) \left( \tilde{W}_i \Lambda_i^2 \right)^T, \]
where \( \mathcal{V}_i : D_{e_i} \times \mathbb{R}^{\text{dim}(\tilde{W}_i)} \to \mathbb{R}_+ \) and \( D_{e_i} \triangleq \{ e_i(t) : \|e_i(t)\|_{P_i} < \epsilon \} \).

Using
\[ \frac{d\phi(\|e_i(t)\|_{P_i})}{dt} = 2\phi_d(\|e_i(t)\|_{P_i}) e_i^T(t) P_i \dot{e}_i(t), \]
we obtain
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The level sets
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are compact and invariant for any \( \xi_i > 0 \).

Follows from LaSalle-Yoshizawa theorem that
\[ \lim_{t \to \infty} e_i(t) = 0 \]
Consider the restricted Lyapunov function

\[ \mathcal{V}_i(e_i, \tilde{W}_i) = \phi(\|e_i\|_{P_i}) + \gamma_i^{-1} \text{tr} \left( \tilde{W}_i \Lambda_i^{\frac{1}{2}} \right)^T \left( \tilde{W}_i \Lambda_i^{\frac{1}{2}} \right), \]

\[ \gamma_i : \mathcal{D}_{e_i} \times \mathbb{R}^{\text{dim}(\tilde{W}_i)} \to \mathbb{R}_+, \quad \mathcal{D}_{e_i} \triangleq \{ e_i(t) : \|e_i(t)\|_{P_i} < \epsilon \} \]

Using

\[ \frac{d}{dt} \phi(\|e_i(t)\|_{P_i}) = 2\phi_d(\|e_i(t)\|_{P_i}) e_i^T(t) P_i \dot{e}_i(t), \]

we obtain

\[ \dot{\mathcal{V}}_i(e_i(t), \tilde{W}_i(t)) = -\phi_d(\|e_i(t)\|_{P_i}) e_i^T(t) R_i e_i(t) \leq 0 \]

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The level sets

\[ \Omega_i \triangleq \{ (e_i(t), \tilde{W}_i(t)) \in \mathcal{D}_{e_i} \times \mathbb{R}^{\text{dim}(\tilde{W}_i)} : \mathcal{V}_i(e_i, \tilde{W}_i) \leq \xi_i \} \]

are compact and invariant for any \( \xi_i > 0 \)

Proposed setup enforces to use high gain only when it is absolutely necessary, unlike traditional setup

Error trajectory always stays inside a priori known user defined bounds.

\[ \lim_{t \to \infty} e_i(t) = 0 \]
ILLUSTRATIVE EXAMPLE

- 8 coupled inverted pendulums considered
- Dampers and springs connecting inverted pendulums are nonidentical and unknown
- Torque generated by each pendulum motor is subject to uncertain and heterogeneous control effectiveness
ILLUSTRATIVE EXAMPLE

Illustrative Example

- Coupled inverted pendulums considered
- Dampers and springs connecting inverted pendulums are nonidentical and unknown
- Torque generated by each pendulum motor is subject to uncertain and heterogeneous control effectiveness

Graphs showing the response of the system variables over time.

\[ \theta_i(t), \quad \theta_r(t) \]

\[ \theta_i(t) \]

\[ u_i(t) \]

\[ t, \text{ sec} \]
ILLUSTRATIVE EXAMPLE

- Coupled inverted pendulums
- Dampers and springs nonidentical and unknown
- Torque generated by each pendulum motor subject to uncertain and heterogeneous control effectiveness

Graphs showing:
- $\phi_d(t)$ and $\|e_i(t)\|_{P_i}$
- $\epsilon_i$ and $\|e_i(t)\|_{P_i}$
CONCLUDING REMARKS

- Contribution to modular systems
  - Achieve asymptotic stability via local communications only that are graph-theoretically modeled
- Contribution to adaptive control
  - Uncertainty-independent performance bounds determined by control engineers, a priori
- Ongoing research
  - If bounds on the user defined functions are tight and there exists significant amount of measurement noise in states, then adaptation gains can be very large - Limit frequency content to achieve robustness in this case
  - Incorporating state constraints with control constraints
  - Consideration of communication dropouts and time-delays in communication between submodules
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