Mitigating the Effects of Sensor Uncertainties in Networked Multiagent Systems

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Abstract—Networked multiagent systems consist of interacting agents that locally exchange information, energy, or matter. Since they do not in general have a centralized entity to monitor the activity of each agent, resilient distributed control system design for networked multiagent systems is essential in providing high system performance, reliability, and operation in the presence of system uncertainties. An important class of such system uncertainties that can significantly deteriorate the achievable closed-loop system performance is sensor uncertainties, which can arise due to low sensor quality, sensor failure, sensor bias, or detrimental environmental conditions. This paper presents a novel distributed adaptive control architecture for networked multiagent systems to mitigate the effect of sensor uncertainties. Specifically, we consider agents having high-order, linear dynamics with sensor interactions corrupted by unknown exogenous disturbances. We show that the proposed adaptive control architecture guarantees asymptotic stability of the closed-loop dynamical system when the exogenous disturbances are time-invariant and uniform ultimate boundedness when the exogenous disturbances are time-varying. A numerical example is provided to illustrate the efficacy of the proposed distributed adaptive control architecture.

I. INTRODUCTION

Networked multiagent systems (e.g., communication networks, power systems, and process control systems) consist of interacting agents that locally exchange information, energy, or matter [1]–[4]. Since these systems do not in general have a centralized architecture that monitors the activity of each agent, resilient distributed control system design for networked multiagent systems is essential in providing high system performance, reliability, and operation in the presence of system uncertainties [5]–[9]. An important class of such system uncertainties that can significantly deteriorate achievable closed-loop dynamical system performance is sensor uncertainties. In particular, sensor uncertainties can arise due to low sensor quality, sensor failure, sensor bias, or detrimental environmental conditions [10]–[13]. If relatively cheap sensor suites are used for low-cost, small-scale unmanned vehicle applications, then this can result in inaccurate sensor measurements. Alternatively, sensor measurements can be corrupted by malicious attacks if these dynamical systems are controlled through large-scale, multilayered communication networks as in the case of cyber-physical systems.

Early approaches that deal with sensor uncertainties focus on classical fault detection, isolation, and recovery schemes (see, for example, [14], [15]). In these approaches sensor measurements are compared with an analytical model of the dynamical system by forming a residual signal and analyzing this signal to determine if a fault has occurred. However, in practice it is difficult to identify a single residual signal per failure mode, and as the number of failure modes increase this becomes prohibitive. In addition, a common underlying assumption of the classical fault detection, isolation, and recovery schemes is that all dynamical system signals remain bounded during the fault detection process, which may not always be a valid assumption. More recently, the authors of [16]–[18] consider the fundamental limitations of attack detection and identification methods for linear systems. However, their approach is not only computationally expensive but also it is not linked to the controller design. In [19], adversarial attacks on actuator and sensors are modeled as exogenous disturbances. However, the presented control methodology cannot address situations where more than half of the sensors are compromised and the set of attacked nodes change over time. Finally, the authors in [20] analyze a case where the interaction between networked agents are corrupted by exogenous disturbances. However, their approach is limited to agents having scalar dynamics and is not linked to the controller design in order to mitigate the effect of such sensor uncertainties.

In this paper, we present a novel distributed adaptive control architecture for networked multiagent systems to mitigate the effect of sensor uncertainties. Specifically, we consider multiagent systems having high-order, linear dynamics with agent interactions corrupted by unknown exogenous disturbances. We show that the proposed adaptive control architecture guarantees asymptotic stability of the closed-loop dynamical system when the exogenous disturbances are time-invariant and uniform ultimate boundedness when the exogenous disturbances are time-varying. A numerical example is provided to illustrate the efficacy of the proposed distributed adaptive control architecture.

II. MATHEMATICAL PRELIMINARIES AND PROBLEM FORMULATION

A. Mathematical Preliminaries

The notation used in this paper is fairly standard. Specifically, \( \mathbb{R} \) denotes the set of real numbers, \( \mathbb{R}^n \) denotes the set of \( n \times 1 \) real column vectors, \( \mathbb{R}^{n \times m} \) denotes the set of \( n \times m \) real matrices, \( \mathbb{R}_+ \) denotes the set of positive real numbers, \( \mathbb{R}_+^n \) (resp., \( \mathbb{R}_+^{n 	imes m} \)) denotes the set of \( n \times n \) positive definite (resp., nonnegative-definite) real matrices, \( \mathcal{O}_n \) denotes the \( n \times 1 \) zero vector, \( \mathbf{1}_n \) denotes the \( n \times 1 \) ones vector, \( \mathbf{0}_{n \times n} \) denotes the \( n \times n \) zero matrix, and \( \oplus \) denotes equality by definition. In addition, we write \( (\cdot)^T \) for the transpose operator, \( (\cdot)^{-1} \) for the inverse operator, \( \text{det}(\cdot) \) for the determinant operator, \( \| \cdot \|_2 \) for the Euclidean norm, and \( \otimes \) for the Kronecker product. Furthermore, we write \( \lambda_{\text{min}}(A) \) (resp., \( \lambda_{\text{max}}(A) \)) for the minimum (resp., maximum) eigenvalue of the square matrix.
$A, \lambda_i(A)$ for the $ith$ eigenvalue of the square matrix $A$ (with eigenvalues ordered from minimum to maximum value), $\text{spec}(A)$ for the spectrum of the square matrix $A$ including multiplicity, $[A]_{ij}$ for the $(i,j)$th entry of the matrix $A$, and $x$ (resp., $\bar{x}$) for the lower bound (resp., upper bound) of a bounded signal $x(t) \in \mathbb{R}^n$, $t \geq 0$, that is, $\bar{x} \leq \|x(t)\|_2$, $t \geq 0$ (resp., $\|x(t)\|_2 \leq \bar{x}$, $t \geq 0$).

Next, we recall some basic notions from graph theory, where we refer the reader to [3], [21] for further details. Specifically, graphs are broadly adopted in the multiagent systems literature to encode interactions between networked systems. An undirected graph $G$ is defined by a set $V_G = \{1, \ldots, N\}$ of nodes and a set $E_G \subset V_G \times V_G$ of edges. If $(i, j) \in E_G$, then the nodes $i$ and $j$ are neighbors and the neighboring relation is indicated by $i \sim j$. The degree of a node is given by the number of its neighbors. In particular, letting $d_i$ denote the degree of node $i$, then the degree matrix of a graph $G$, denoted by $\mathcal{D}(G) \in \mathbb{R}^{N \times N}$, is given by

$$\mathcal{D}(G) \triangleq \text{diag}[d], \quad d = [d_1, \ldots, d_N]^T.$$  \hspace{1cm} (1)

A path $i_0i_1 \cdots i_L$ is a finite sequence of nodes such that $i_{k-1} \sim i_k$, $k = 1, \ldots, L$, and if every pair of distinct nodes has a path, then a graph $G$ is connected. We write $A(G) \in \mathbb{R}^{N \times N}$ for the adjacency matrix of a graph $G$ defined by

$$[A(G)]_{ij} \triangleq \begin{cases} 1, & \text{if} \ (i, j) \in E_G, \\ 0, & \text{otherwise}, \end{cases}$$  \hspace{1cm} (2)

and $B(G) \in \mathbb{R}^{N \times M}$ for the (node-edge) incidence matrix of a graph $G$ defined by

$$[B(G)]_{ij} \triangleq \begin{cases} 1, & \text{if node} \ i \ \text{is the head of edge} \ j, \\ -1, & \text{if node} \ i \ \text{is the tail of edge} \ j, \\ 0, & \text{otherwise}, \end{cases}$$  \hspace{1cm} (3)

where $M$ is the number of edges, $i$ is an index for the node set, and $j$ is an index for the edge set.

The graph Laplacian matrix, denoted by $\mathcal{L}(G) \in \mathbb{R}^{N}$, is defined by $\mathcal{L}(G) \triangleq \mathcal{D}(G) - A(G)$ or, equivalently, $\mathcal{L}(G) = B(G)B(G)^T$, where the spectrum of the Laplacian of a connected, undirected graph $G$ can be ordered as

$$0 = \lambda_1(\mathcal{L}(G)) < \lambda_2(\mathcal{L}(G)) \leq \cdots \leq \lambda_N(\mathcal{L}(G)),$$  \hspace{1cm} (4)

with $1_N$ as the eigenvector corresponding to the zero eigenvalue $\lambda_1(\mathcal{L}(G))$, and $\mathcal{L}(G)1_N = 0_N$ and $e\mathcal{L}(G)1_N = 1_N$. Finally, we partition the incidence matrix as $B(G) = [B_L(G)^T, B_F(G)^T]^T$, where $B_L(G) \in \mathbb{R}^{N_L \times M}$, $B_F(G) \in \mathbb{R}^{N_F \times M}$. $N = N_L + N_F$, and $N_L$ and $N_F$, respectively denote the cardinalities of the leader and follower groups [3]. Furthermore, without loss of generality, we assume that the leader agents are indexed first and the follower agents are indexed last in the graph $G$ so that $\mathcal{L}(G) = B(G)B(G)^T$ is given by

$$\mathcal{L}(G) = \begin{bmatrix} L(G) & G(G)^T \\ G(G) & F(G) \end{bmatrix},$$  \hspace{1cm} (5)

where $L(G) \triangleq B_L(G)B_L(G)^T$, $G(G) \triangleq B_F(G)B_F(G)^T$, and $F(G) \triangleq B_F(G)B_F(G)^T$. Note that $F(G) \in \mathbb{R}^{N_F \times N_F}$ for a connected, undirected graph $G$ and satisfies $F(G)1_{N_F} = -G(G)1_{N_L}$. This implies that each row sum of $-F(G)^{-1}G(G)$ is equal to 1.

Fig. 1. A networked multiagent system with agents lying on an agent layer and their local controllers lying on a control layer.

**B. Problem Formulation**

Consider a networked multiagent system consisting of $N$ agents with the dynamics of agent $i, i = 1, \ldots, N$, given by

$$\dot{x}_i(t) = Ax_i(t) + Bu_i(t), \quad x_i(0) = x_{i0}, \quad t \geq 0,$$  \hspace{1cm} (6)

where $x_i(t) \in \mathbb{R}^n, t \geq 0$, is the state vector of agent $i$, $u_i(t) \in \mathbb{R}^m, t \geq 0$, is the control input of agent $i$, and $A \in \mathbb{R}^{n \times n}$ and $B \in \mathbb{R}^{n \times m}$ are system matrices. We assume that the pair $(A, B)$ is controllable and the control input $u_i(\cdot)$, $i = 1, \ldots, N$, is restricted to the class of admissible controls consisting of measurable functions such that $u_i(t) \in \mathbb{R}^m, t \geq 0$. In addition, we assume that the agents can locally exchange information according to a connected, undirected graph $G$ with nodes and edges representing agents and interagent information exchange links, respectively, resulting in a static network topology; that is, the time evolution of the agent nodes does not result in edges appearing or disappearing in the network.

Here, we consider a networked multiagent system, where the agents lie on an agent layer and their local controllers lie on a control layer as depicted in Figure 1. Specifically, agent $i, i = 1, \ldots, N$, sends its state measurement to its local controller at a given control layer and this controller sends its control input to agent $i$ lying on the agent layer. In addition, we assume that the compromised state

$$\tilde{x}_i(t) = x_i(t) + \delta_i(t), \quad i = 1, \ldots, N,$$  \hspace{1cm} (7)

is available to the local controller of agent $i$, $i = 1, \ldots, N$, where $\tilde{x}_i(t) \in \mathbb{R}^n, t \geq 0$, and $\delta_i(t) \in \mathbb{R}^n, t \geq 0$, captures sensor uncertainties. In particular, if $\delta_i(\cdot)$ is nonzero, then the state vector $x_i(t)$, $t \geq 0$, of agent $i, i = 1, \ldots, N$, is corrupted with a faulty or malicious signal $\delta_i(\cdot)$. Alternatively, if $\delta_i(\cdot)$ is zero, then $\tilde{x}_i(t) = x_i(t)$, $t \geq 0$, and the compromised state vector is available to the local controller of agent $i$, $i = 1, \ldots, N$.

Given the two-layer networked multiagent system hierarchy, we are interested in the problem of asymptotically (or approximately) driving the state vector of each agent $x_i(t), i = 1, \ldots, N$, $t \geq 0$, to the state vector of a (virtual) leader $x_0(t) \in \mathbb{R}^n, t \geq 0$, that lies on the control layer subject to the dynamics

$$\dot{x}_0(t) = Ax_0(t), \quad x_0(0) = x_{00}, \quad t \geq 0,$$  \hspace{1cm} (8)

where $A_0 \in \mathbb{R}^{n \times n}$ is Lyapunov stable and is given by $A_0 \triangleq A - BK_0$ with $K_0 \in \mathbb{R}^{n \times n}$.

For the case where the uncompromised state vector is available to the local controller of agent $i$, $i = 1, \ldots, N$, that is, $\delta_i(\cdot) \equiv 0$, then the controller

$$u_i(t) = -K_0x_i(t) - cK\sum_{i \sim j}(x_i(t) - x_j(t))$$
+l_i(x_i(t) - x_0(t)) \right], \quad (9)

guarantees that \( \lim_{t \to \infty} x_i(t) = x_0(t) \) for all \( i = 1, \ldots, N \), where \( l_i = 1 \) for a set of \( N_L \) agents that have access to the state of the leader \( x_0(t) \), \( t \geq 0 \), and \( l_i = 0 \) for the remaining \( N_F \) agents with \( N = N_L + N_F \), and where \( K \in \mathbb{R}^{m \times n} \) and \( c \in \mathbb{R}_+ \) denote an appropriate feedback gain matrix and coupling strength, respectively, such that

\[
A_{\xi} \triangleq A_0 - \eta cBK,
\]

is Hurwitz for all \( \eta \in \text{spec}(F(G)) \). To see this, let \( \xi_i(t) \triangleq x_i(t) - x_0(t) \) and note that using (6), (8), and (9),

\[
\dot{\xi}_i(t) = A_0\xi_i(t) - cBK\left[ \sum_{i \neq j} (\xi_i(t) - \xi_j(t)) + l_i\xi_i(t) \right],
\]

\[
\xi_i(0) = \xi_{i0}, \quad t \geq 0,
\]

with \( \xi_{i0} \triangleq x_{i0} - x_0 \). In addition, defining \( \xi(t) \triangleq [\xi_1^T(t), \ldots, \xi_N^T(t)]^T \), (11) can be written in compact form as

\[
\dot{\xi}(t) = [I_N \otimes A_0 - cF(G) \otimes BK]\xi(t), \quad \xi(0) = \xi_0, \quad t \geq 0.
\]

Now, using the results in [9], [22], [23], it can be shown that \( I_N \otimes A_0 - cF(G) \otimes BK \) in (12) is Hurwitz when (10) is Hurwitz for all \( \eta \in \text{spec}(F(G)) \). Hence, \( \lim_{t \to \infty} \xi_i(t) = 0 \), that is, \( \lim_{t \to \infty} x_i(t) = x_0(t) \) for all \( i = 1, \ldots, N \).

For \( \delta(t) \neq 0 \), our objective is to design a local controller for each agent \( i = 1, \ldots, N \), of the form

\[
u_i(t) = -K_0\hat{x}_i(t) - cK\left[ \sum_{i \neq j} (\hat{x}_i(t) - \hat{x}_j(t)) \right] + l_i(\hat{x}_i(t) - x_0(t)) + v_i(t), \quad (13)
\]

where \( v_i(t) \in \mathbb{R}^m, \quad t \geq 0 \), is a local corrective signal that suppresses or counteracts the effect of \( \delta_i(t), \quad t \geq 0 \), to asymptotically (or approximately) recover the ideal system performance (i.e., \( \lim_{t \to \infty} x_i(t) = x_0(t) \) for all \( i = 1, \ldots, N \)) achieved when the state vector is available for feedback. Thus, assuming that (10) is Hurwitz for all \( \eta \in \text{spec}(F(G)) \) implies that there exists an ideal system performance that can be recovered by designing the local corrective signals \( v_i(t) \in \mathbb{R}^m, \quad t \geq 0 \), for each agent \( i = 1, \ldots, N \). Although we consider this specific problem in this paper, the proposed approach dealing with sensor uncertainties can be used in many other problems that exist in the networked multiagent systems literature [3], [4].

### III. Adaptive Leader Following with Time-Invariant Sensor Uncertainties

In this section, we design the local corrective signal \( v_i(t), \quad i = 1, \ldots, N, \quad t \geq 0 \), in (13) to achieve asymptotic adaptive leader following in the presence of time-invariant sensor uncertainties, that is, \( \delta_i(t) \triangleq \delta_i, \quad i = 1, \ldots, N, \quad t \geq 0 \). For this problem, we propose the corrective signal

\[
v_i(t) = K_0\hat{\delta}_i(t) + cK\left[ \sum_{i \neq j} (\hat{\delta}_i(t) - \hat{\delta}_j(t)) + l_i\hat{\delta}_i(t) \right], \quad (14)
\]

where

\[
\dot{\hat{\delta}}_i(t) = -\gamma A^T P \hat{x}_i(t) - \dot{x}_i(t) - \hat{\delta}_i(t),
\]

\[
\hat{\delta}_i(0) = \hat{\delta}_{i0}, \quad t \geq 0,
\]

\[
\hat{x}_i(t) = A_0\hat{x}_i(t) - cBK\left[ \sum_{i \neq j} (\hat{x}_i(t) - \hat{x}_j(t)) \right] + l_i(\hat{x}_i(t) - x_0(t)) + (\gamma A^T P + \mu I_n)(\hat{x}_i(t) - \hat{x}_i(t) - \hat{\delta}_i(t)),
\]

\[
\hat{x}_i(0) = \hat{x}_{i0}, \quad t \geq 0,
\]

\[
\hat{\delta}_i(t) \in \mathbb{R}^n, \quad t \geq 0, \quad \text{is the estimate of the sensor uncertainty} \quad \delta_i(t), \quad t \geq 0, \quad \hat{x}_i(t) \in \mathbb{R}^n, \quad t \geq 0, \quad \text{is the state estimate of the compromised state vector} \quad x_i(t), \quad t \geq 0, \quad \gamma \in \mathbb{R}_+ \quad \text{and} \quad \mu \in \mathbb{R}_+ \text{ are design gains, and } \quad P \in \mathbb{R}^n \text{ is a solution to the linear matrix inequality (LMI) given by}
\]

\[
I_N \otimes (A_0^T P + PA_0 - 2\mu P) - cF(G) \otimes (K^T B^T P + PBK) < 0.
\]

For the next result, we note from (6) and (13) that

\[
x_i(t) = Ax_i(t) - BK_0x_0(t) - cBK\left[ \sum_{i \neq j} (x_i(t) - x_j(t)) \right] + l_i(x_i(t) - x_0(t)) - B(cI_K + K_0)\delta_i + Bv_i(t),
\]

\[
x_i(0) = x_{i0}, \quad t \geq 0.
\]

Now, let \( x(t) \triangleq [x_1^T(t), \ldots, x_N^T(t)]^T, \quad \delta \triangleq [\delta_1^T, \ldots, \delta_N^T]^T, \quad \text{and} \quad v(t) \triangleq [v_1(t), \ldots, v_N(t)]^T, \) and note that (18) can be written in a compact form as

\[
\dot{x}(t) = [I_N \otimes A_0 - cF(G) \otimes BK]x(t) + (cG(G) \otimes BK)x_0(t) - (cF(G) \otimes BK)\dot{x}(t) + I_N \otimes BK_0\delta + (I_N \otimes B)v(t),
\]

\[
x(0) = x_0, \quad t \geq 0.
\]

Using (7) and (19), the dynamics for \( \hat{x}(t), \quad t \geq 0, \) with \( \hat{x}(t) \triangleq [\hat{x}_1^T(t), \ldots, \hat{x}_N^T(t)]^T \), can also be written in compact form as

\[
\dot{\hat{x}}(t) = [I_N \otimes A_0 - cF(G) \otimes BK]\hat{x}(t) + (cG(G) \otimes BK)x_0(t) - (cF(G) \otimes BK)\dot{\hat{x}}(t) + I_N \otimes BK_0\delta(t) + (I_N \otimes B)v(t), \quad \hat{x}(0) = \hat{x}_0, \quad t \geq 0.
\]

Next, letting \( \hat{\phi}(t) \triangleq [\phi_1^T(t), \ldots, \phi_N^T(t)]^T \), a compact form for the dynamics of \( \hat{x}(t), \quad t \geq 0, \) is given by

\[
\dot{\hat{\phi}}(t) = [I_N \otimes A_0 - cF(G) \otimes BK]\hat{\phi}(t) + (cG(G) \otimes BK)x_0(t) - (cF(G) \otimes BK)\hat{\phi}(t) + I_N \otimes BK_0\delta(t) + (I_N \otimes B)v(t) + \phi(t), \quad \hat{x}(0) = \hat{x}_0, \quad t \geq 0,
\]

where \( \phi(t) \triangleq [\phi_1^T(t), \ldots, \phi_N^T(t)]^T \) with \( \phi_i(t) \triangleq -\hat{\delta}_i(t) + u_i(c_i(t)). \) Finally, define

\[
e_{i}(t) \triangleq \hat{x}_i(t) - \hat{x}_i(t) - \delta_i(t),
\]

\[
\hat{\delta}_i(t) \triangleq \delta_i - \delta_i(t).
\]
and note that it follows from (22) and (23) that
\[
\dot{\delta}(t) = (I_N \otimes A_0)\dot{\delta}(t), \quad \delta(0) = \delta_0, \quad t \geq 0,
\]
where \( A_0 = I_N \otimes A_0 - cF(G) \otimes BK \).

The next theorem presents the main result of this paper.

\textbf{Theorem 1.} Consider the networked multiagent system consisting of \( N \) agents on a connected, undirected graph \( G \), where the dynamics of agent \( i \), \( i = 1, \ldots, N \), is given by (6). In addition, assume that the local controller \( u_i(t) \), \( i = 1, \ldots, N \), \( t \geq 0 \), for each agent is given by (13) with the corrective signal \( v_i(t) \), \( i = 1, \ldots, N \), \( t \geq 0 \), given by (14). Moreover, assume that the sensor uncertainties are time-invariant, that is, \( \delta_i(t) = \delta_i, t \geq 0 \), and \( \det(A) \neq 0 \). Then, the zero solution \( (\delta(t), \dot{\delta}(t)) = (0, 0) \) of the closed-loop system given by (24) and (25) is Lyapunov stable for all \( (\delta_0, \dot{\delta}_0) \in \mathbb{R}^{nN} \times \mathbb{R}^{nN} \) and \( \lim_{t \to \infty} \delta(t) = 0 \) if \( \lim_{t \to \infty} \dot{\delta}(t) = 0 \).

\textbf{Remark 1.} It follows from (6), (13), and (14) that
\[
\dot{x}_i(t) = A_0 x_i(t) - cBK \left[ \sum_{j \neq i} (x_i(t) - x_j(t)) + l_i(x_i(t) - x_0(t)) \right] - BK \delta_i(t)
\]
and, which, using the boundedness of \( \delta_i(t), i = 1, \ldots, N, t \geq 0 \), and the assumption that (10) is Hurwitz for all \( \eta_i \in \spec(F(G)) \), implies that \( x_i(t) \) is bounded for all \( t \geq 0 \) and \( i = 1, \ldots, N \). Hence, using (7), \( \dot{x}_i(t) \) is bounded for all \( t \geq 0 \) and \( i = 1, \ldots, N \). Furthermore, since \( e_i(t), t \geq 0 \), \( \dot{x}_i(t), t \geq 0 \), and \( \delta_i(t), t \geq 0 \), are bounded for all \( i = 1, \ldots, N \), it follows that \( \dot{x}_i(t) \) is bounded for all \( t \geq 0 \) and \( i = 1, \ldots, N \).

\textbf{Remark 2.} Since, by Theorem 1, \( \lim_{t \to \infty} \dot{\delta}(t) = 0 \), \( i = 1, \ldots, N, t \geq 0 \), it follows from (26) that each agent subject to the dynamics given by (6) asymptotically recovers the ideal system performance (i.e., \( \lim_{t \to \infty} x_i(t) = x_0(t) \) for all \( i = 1, \ldots, N \)), which is a direct consequence of the discussion given in Section 2.2 and the assumption that (10) is Hurwitz for all \( \eta_i \in \spec(F(G)) \). Adding (27) and (28) yields
\[
I_N \otimes (A_0^T P + PA_0 - 2\mu P)
\]
which can be equivalently written as
\[
I_N \otimes (A_0^T P + PA_0 - 2\mu P) - 2cF(G) \otimes (PBQ^{-1} B^T P)(T^{-1} \otimes I_N) = 0.
\]

IV. ADAPTIVE LEADER FOLLOWING WITH TIME-VARYING SENSOR UNCERTAINTIES

In this section, we generalize the results of the previous section by designing the local corrective signal \( v_i(t), i = 1, \ldots, N, t \geq 0 \), in (13) to achieve approximate adaptive leader following in the presence of time-varying sensor uncertainties \( \delta_i(t), i = 1, \ldots, N, t \geq 0 \). We assume that the time-varying sensor uncertainties are bounded and have bounded rates of change; that is, \( \|\delta_i(t)\|_2 \leq \bar{\delta}, t \geq 0 \), and \( \|\dot{\delta}_i(t)\|_2 \leq \bar{\delta}_r, t \geq 0 \), for all \( i = 1, \ldots, N \), with unknown \( \bar{\delta} \) and \( \bar{\delta}_r \).

For the statement of our next result, it is necessary to introduce the projection operator \([26]\). Specifically, let \( \phi : \mathbb{R}^n \to \mathbb{R}^n \) be a continuously differentiable convex function given by
\[
\phi(\theta) \triangleq \frac{(\varepsilon_\theta + 1)\theta^T \theta - \theta^2_{\text{max}}}{\varepsilon_\theta \theta_{\text{max}}},
\]
where \( \theta_{\text{max}} \in \mathbb{R} \) is a projection norm bound imposed on \( \theta \in \mathbb{R}^n \) and \( \varepsilon_\theta > 0 \) is a projection tolerance bound. Then, the projection operator \( \text{Proj} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n \) is defined by
\[
\text{Proj}(\theta, y) \triangleq \begin{cases} 
  y, & \text{if } \phi(\theta) < 0, \\
  y - \phi(\theta) \phi'(\theta) y, & \text{if } \phi(\theta) \geq 0 \text{ and } \phi'(\theta) y \leq 0, \\
  y - \frac{\phi''(\theta)}{\phi'(\theta)^2} \phi'(\theta) y, & \text{if } \phi(\theta) \geq 0 \text{ and } \phi'(\theta) y > 0,
\end{cases}
\]
where \( y \in \mathbb{R}^n \). Note that it follows from the definition of the projection operator that

\[
(\theta^* - \theta)^T \text{Proj}(\theta, y) - y \geq 0, \quad \theta^* \in \mathbb{R}^n. \tag{36}
\]

Next, for the controller given by (13), we use the local corrective signal

\[
v_i(t) = K_0 \delta_i(t) + K \left( \sum_{i \neq j} (\delta_i(t) - \delta_j(t)) + I_i \delta_i(t) \right), \tag{37}
\]

where

\[
\dot{\delta}_i(t) = \gamma \text{Proj}(t, -A^T P(x_i - \hat{x}_i - \dot{\delta}_i(t))),
\]

\[
\hat{x}_i(t) = A_0 \hat{x}_i(t) - cB K \left( \sum_{i \neq j} (\hat{x}_i - \hat{x}_j) \right) + I_i (\hat{x}_i(t) - x_0(t)) + (\gamma A^T P + \mu I_n),
\]

\[
\hat{x}_i(t) = \hat{x}_i(t) - \hat{x}_i(t) - \delta(t), t \geq 0, \] for each agent given by (37), (38), (39) are bounded.

Next theorem presents the second main result of this paper.

**Theorem 2.** Consider the networked multiagent system consisting of \( N \) agents on a connected, undirected graph \( G \), where the dynamics of agent \( i, i = 1, \ldots, N \), is given by (6). In addition, assume that the local controller \( u_i(t), i = 1, \ldots, N, t \geq 0 \), for each agent is given by (13) with the corrective signal \( v_i(t), i = 1, \ldots, N, t \geq 0 \), given by (37). Moreover, assume that the sensor uncertainties are time-varying and \( \text{det}(A) \neq 0 \). Then, the closed-loop system dynamics given by (42) and (43) are uniformly bounded for all \( (e_0, \lambda_i) \) with the ultimate bounds

\[
\|e_i(t)\|_2 \leq \left[ \frac{\lambda_{\max}(P)}{\lambda_{\min}(P)} \eta_i^2 + \frac{1}{\gamma \lambda_{\min}(P)} \eta_i^2 \right]^{\frac{1}{2}}, \quad t \geq T, \tag{44}
\]

\[
\|\delta_i(t)\|_2 \leq \gamma \lambda_{\max}(P) \eta_i^2 + \delta_i^2, \quad t \geq T, \tag{45}
\]

where \( \eta_i = \frac{d_i}{\sqrt{\lambda_{\min}(G)}} \bigg( \frac{d_i}{\sqrt{\lambda_{\min}(G)}} \bigg) \), \( \eta_i^2 = \lambda_{\max}(R) \), \( \eta_i \geq \delta_i \), and \( \delta_i \geq 2N \gamma^{-1} \delta_i \).

**Remark 4.** A similar remark to Remark 1 holds for Theorem 2. Namely, all signals used to construct the local controller \( u_i(t), i = 1, \ldots, N, t \geq 0 \), for each agent given by (13) with the local corrective signal \( v_i(t), i = 1, \ldots, N, t \geq 0 \), given by (37), (38), (39) are bounded.

V. ILLUSTRATIVE NUMERICAL EXAMPLE

In this section, we present a numerical example to demonstrate the utility and efficacy of the proposed distributed control architecture for networked multiagent systems to mitigate the effect of sensor uncertainties. Specifically, consider a group of \( N = 4 \) agents subject to a connected, undirected graph \( G \) given in Figure 1, where the dynamics of agent \( i \) satisfy

\[
\begin{bmatrix}
\dot{x}_i^1(t) \\
\dot{x}_i^2(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 0.3 \\
-2 & 0
\end{bmatrix}
\begin{bmatrix}
x_i^1(t) \\
x_i^2(t)
\end{bmatrix} +
\begin{bmatrix}
0 \\
1
\end{bmatrix} u(t), \quad i = 1, \ldots, 4,
\]

where \( x_i^1(0) = [0, 0]^T, x_i^2(0) = [1, 2]^T, x_i^3(0) = [3, 1]^T, \) and \( x_i^4(0) = [2, 2]^T \). Note that \( \text{det}(A) \neq 0 \), where \( A \) is the system matrix of (46). For this example, we are interested in the problem of asymptotically driving the state vector of each agent \( x_i(t), i = 1, \ldots, 4, t \geq 0 \), to the state vector of a leader \( x_0(t), t \geq 0 \), subject to the dynamics given by

\[
\begin{bmatrix}
\dot{y}_i^1(t) \\
\dot{y}_i^2(t)
\end{bmatrix} =
\begin{bmatrix}
0 & 0.3 \\
-2 & 0
\end{bmatrix}
\begin{bmatrix}
y_i^1(t) \\
y_i^2(t)
\end{bmatrix} +
\begin{bmatrix}
1 \\
1
\end{bmatrix},
\]

for this problem, \( A_0 = A - BK_0 \) holds with \( K_0 = [0, 0] \) as a direct consequence of (46) and (47).

To design the proposed local controllers, let \( K = Q^{-1} B^T P \) and set \( Q = 0.1 I_2, \mu = 3.2, \) and \( c = 6 \geq \min(h) \), so that the LMI given by (33) for \( P = S^{-1} \) (see Remark 3) is satisfied with

\[
P =
\begin{bmatrix}
0.0539 & 0.0105 \\
0.0105 & 0.0520
\end{bmatrix}, \tag{48}
\]

and hence, \( K = [0.1048, 0.5199] \). Note with this selection of \( K, (10) \) is Hurwitz for all \( \eta_i \in \text{spec}(F(G)) \). Next, consider a time-invariant sensor uncertainty given by (7) with \( \delta_i = [10, 7]^T, \delta_i = [8, 5]^T, \delta_i = [6, 3]^T, \) and \( \delta_i = [4, 1]^T \). The system performance for the case when the compromised state vector is available to the local controller of agent \( i, i = 1, \ldots, 4 \), (i.e., \( \delta_i(t) \neq 0 \)), is shown in Figure 2 using (9) (i.e., \( v_i(t) \equiv 0, i = 1, \ldots, 4 \)). Now, to illustrate the results of Theorem 1, we use the proposed local controller \( u_i(t), i = 1, \ldots, 4, t \geq 0 \), for each agent given by (13) and the local corrective signal \( v_i(t), i = 1, \ldots, 4, t \geq 0 \), given by
system when the exogenous disturbances are time-invariant guarantee asymptotic stability of the closed-loop dynamical and showed that the proposed adaptive control architectures invariant and time-varying sensor uncertainties in networked performance. In this paper, we presented two distributed state vector of the leader agent in Figure 3. As expected, the proposed distributed control (14) with controller given by (13) and the local corrective signal given by (14) when Fig. 3. System performance for a group of agents with the proposed local controller given by (9) (i.e., $v_i(t) \equiv 0$, $i = 1, \ldots, 4$) when the compensated state vector is available for feedback.

Fig. 2. System performance for a group of agents with the local controller given by (13) and the local corrective signal given by (14) when the compensated state vector is available for feedback.

(14) with $\gamma = 620$. For this case, the system performance in the presence of time-invariant sensor uncertainties is shown in Figure 3. As expected, the proposed distributed control architecture of Theorem 1 allows the state vector of each agent $x_i(t)$, $i = 1, \ldots, 4$, $t \geq 0$, to asymptotically track the state vector of the leader $x_0(t)$, $t \geq 0$.

VI. CONCLUSION

Sensor uncertainties in networked multiagent systems, which may result from low sensor quality, sensor failure, sensor bias, or detrimental environmental conditions, can significantly deteriorate achievable closed-loop system performance. In this paper, we presented two distributed adaptive control architectures to mitigate the effect of time-invariant and time-varying sensor uncertainties in networked multiagent systems with agents having high-order, linear dynamics. Specifically, we modeled agent uncertainty as networked agents using unknown exogenous disturbances and showed that the proposed adaptive control architectures guarantee asymptotic stability of the closed-loop dynamical system when the exogenous disturbances are time-invariant and uniform ultimate boundedness when the exogenous disturbances are time-varying.

REFERENCES