Proceedings of the ASME 2015 Dynamic Systems and Control Conference
DSCC 2015
October 28-30, 2015, Columbus, Ohio, USA

DSCC2015-9724

EVENT-TRIGGERED ADAPTIVE CONTROL

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Abstract

In this paper, we present a new adaptive control methodology that allows a desirable command performance while the proposed controller exchanges data with the physical system through a real-time network. Specifically, we utilize tools and methods from event-triggering control theory to schedule data exchange dependent upon errors exceeding user-defined thresholds and show the boundedness of the overall closed-loop system using Lyapunov stability. An illustrative numerical example is provided to demonstrate the efficacy of the proposed adaptive control approach.

1 Introduction

We study the problem of controlling a class of uncertain dynamical systems through a real-time network using adaptive control theory. In the literature, notable contributions related to this problem include [1–3]. Specifically, the authors of [1, 2] develop (neural networks-based) adaptive control approaches to deal with system uncertainties, where their results only consider data transmission from a physical system to the controller but not vice versa. The authors of [3] consider the case where data transmits from a physical system to the controller and from the controller to this physical system (i.e., two-way data exchange is allowed through a real-time network). Even though this approach is promising, their methodology requires the knowledge of a conservative upper bound on the unknown constant gain appearing in their uncertainty parameterization. While this conservative upper bound may be available for some applications, the actual upper bound may change and exceed its conservative estimate, for example, when an aircraft undergoes a sudden change in dynamics as a result of reconfiguration, deployment of a payload, docking, or structural damage [4].

In this paper, we present a new adaptive control methodology that allows a desirable command performance while the proposed controller exchanges data with the physical system through a real-time network. Specifically, we consider a state emulator-based adaptive control framework [5–12] due to the fact that this framework achieves stringent performance specifications without causing high-frequency oscillations in the controller response [11, 12] unlike standard adaptive controllers, utilize tools and methods from event-triggering control theory to schedule data exchange dependent upon errors exceeding user-defined thresholds, and show the boundedness of the overall closed-loop system using Lyapunov stability. An illustrative numerical example is provided to demonstrate the efficacy of the proposed adaptive control approach.

The notation used in this paper is fairly standard. Specifically, $\mathbb{R}$ donetes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices, $\mathbb{R}_+$ denotes the set of positive real numbers, $\mathbb{R}_+^{n\times n}$ denotes the set of $n \times n$ positive-definite real matrices, $\mathbb{S}^{n\times n}$ denotes the set of $n \times n$ symmetric real matrices, $\mathbb{D}^{n\times n}$ denotes the set of $n \times n$ real matrices with diagonal scalar entries, $(\cdot)^{\mathrm{T}}$ denotes transpose, $(\cdot)^{-1}$ denotes inverse, $\mathrm{tr}(\cdot)$ denotes the trace operator, and '$\triangleq$' denotes equality by definition. In addition,we use $\|\cdot\|$ to indicate the Euclidean norm and $\|\cdot\|_{\mathrm{F}}$ for the Frobenius matrix norm. Furthermore, we use $\vee$ to denote the logical operator "or" and $\overline{(\cdot)}$ to denote the logical operator "not". Also, for the signal $x(t) = [x_1(t), x_2(t), ..., x_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ defined for all $t \geq 0$.

2 Mathematical Preliminaries

In this section, we introduce necessary preliminaries to develop the main results of this paper. We begin with the following definition.

Definition 1: Let $\phi : \mathbb{R}^n \to \mathbb{R}$ be a continuously differentiable convex function given by $\phi(\theta) \triangleq \dfrac{(\varepsilon_\theta + 1)\,\theta^T\theta - \theta_{\max}^2}{\varepsilon_\theta\,\theta_{\max}^2}$, where $\theta_{\max} \in \mathbb{R}$ is a projection norm bound imposed on $\theta \in \mathbb{R}^n$ and $\varepsilon > 0$ is a projection tolerance bound. Then, the *projection operator* $\mathrm{Proj} : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ is defined by

$$\mathrm{Proj}(\theta, y) \triangleq \begin{cases} y, & \text{if } \phi(\theta) < 0, \\ y, & \text{if } \phi(\theta) \geq 0 \text{ and } \phi'(\theta) y \leq 0, \\ y - \dfrac{\phi'^{\mathrm{T}}(\theta)\phi'(\theta) y}{\phi'(\theta)\phi'^{\mathrm{T}}(\theta)}\phi(\theta), & \\ & \text{if } \phi(\theta) \geq 0 \text{ and } \phi'(\theta) y > 0, \end{cases} \tag{1}$$

where $y \in \mathbb{R}^n$.

It follows from Definition 1 that $(\theta - \theta^*)^{\mathrm{T}}(\mathrm{Proj}(\theta, y) - y) \leq 0$, $\theta^* \in \mathbb{R}^n$, holds [13]. The definition of the projection operator can be generalized to matrices as $\mathrm{Proj_m}(\Theta, Y) = \big(\mathrm{Proj}(\mathrm{col}_1(\Theta), \mathrm{col}_1(Y)), ..., \mathrm{Proj}(\mathrm{col}_m(\Theta), \mathrm{col}_m(Y))\big)$, where $\Theta \in \mathbb{R}^{n\times m}$, $Y \in \mathbb{R}^{n\times m}$, and $\mathrm{col}_i(\cdot)$ denotes the ith column operator. In this case for a given $\Theta^* \in \mathbb{R}^{n\times m}$, it follows that $\mathrm{tr}\Big[(\Theta - \Theta^*)^{\mathrm{T}}(\mathrm{Proj_m}(\Theta, Y) - Y)\Big] = \sum_{i=1}^m \Big[\mathrm{col}_i(\Theta - \Theta^*)^{\mathrm{T}} \times (\mathrm{Proj}(\mathrm{col}_i(\Theta), \mathrm{col}_i(Y)) - \mathrm{col}_i(Y))\Big] \leq 0$ holds.

Next, we briefly state the standard model reference control problem. For this purpose, consider the uncertain dynamical system given by

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0, \tag{2}$$

where $x(t) \in \mathbb{R}^n$ is the state vector available for feedback, $u(t) \in \mathbb{R}^m$ is the control input, $A \in \mathbb{R}^{n\times n}$ is an unknown system matrix, $B \in \mathbb{R}^{n\times m}$ is an unknown control input matrix, and the pair (A, B) is controllable. We assume that the unknown control input matrix is parameterized as $B = D\Lambda$, where $D \in \mathbb{R}^{n\times m}$ is a known input matrix, and $\Lambda \in \mathbb{R}_+^{m\times m} \cap \mathbb{D}^{m\times m}$ is an unknown control effectiveness matrix.

Now, consider the reference system capturing a desired, ideal closed-loop dynamical system performance given by

$$\dot{x}_{\mathrm{ri}}(t) = A_{\mathrm{r}} x_{\mathrm{ri}}(t) + B_{\mathrm{r}} c(t), \quad x_{\mathrm{ri}}(0) = x_{\mathrm{ri}0}, \tag{3}$$

where $x_{\mathrm{ri}}(t) \in \mathbb{R}^n$ is the ideal reference state vector, $c(t) \in \mathbb{R}^m$ is a given uniformly continuous bounded command, $A_{\mathrm{r}} \in \mathbb{R}^{n\times n}$ is the Hurwitz reference system matrix, and $B_{\mathrm{r}} \in \mathbb{R}^{n\times m}$ is the command input matrix. As it is standard in the model reference adaptive control literature (see, for example, [12]), we assume that there exist gain matrices $K_1 \in \mathbb{R}^{m\times n}$ and $K_2 \in \mathbb{R}^{m\times m}$ such that $A_{\mathrm{r}} = A + DK_1$ and $B_{\mathrm{r}} = DK_2$ hold. Using this assumption, (2) can be rewritten as

$$\dot{x}(t) = A_{\mathrm{r}} x(t) + B_{\mathrm{r}} c(t) + D\Lambda[u(t) + W_1^{\mathrm{T}} x(t) + W_2^{\mathrm{T}} c(t)], \quad x(0) = x_0, \tag{4}$$

where $W_1 \triangleq -K_1^{\mathrm{T}}\Lambda^{-1} \in \mathbb{R}^{n\times m}$ and $W_2 \triangleq -K_2^{\mathrm{T}}\Lambda^{-1} \in \mathbb{R}^{m\times m}$ are unknown matrices. Motivating from the structure of the uncertain terms appearing in (4), let the adaptive feedback control law be given by

$$u(t) = -\hat{W}^{\mathrm{T}}(t)\sigma(x(t), c(t)), \tag{5}$$

where $\sigma(x(t), c(t)) = \left[x^{\mathrm{T}}(t)\ ,\ c^{\mathrm{T}}(t)\right]^{\mathrm{T}} \in \mathbb{R}^{n+m}$ and $\hat{W}(t) \in \mathbb{R}^{(n+m)\times m}$ is the estimate of $W \triangleq \left[W_1^{\mathrm{T}}\ ,\ W_2^{\mathrm{T}}\right]^{\mathrm{T}} \in \mathbb{R}^{(n+m)\times m}$, satisfying the weight update law

$$\dot{\hat{W}}(t) = \gamma\sigma(x(t), c(t))\, e_{\mathrm{o}}^{\mathrm{T}}(t) PD, \quad \hat{W}(0) = \hat{W}_0, \tag{6}$$

where $\gamma \in \mathbb{R}_+$ is the learning rate, $e_o(t) \triangleq x(t) - x_{\mathrm{ri}}(t) \in \mathbb{R}^n$ is the ideal system error, and $P \in \mathbb{R}_+^{n\times n} \cap \mathbb{S}^{n\times n}$ is a solution of the Lyapunov equation

$$0 = A_{\mathrm{r}}^{\mathrm{T}} P + PA_{\mathrm{r}} + R, \tag{7}$$

with $R \in \mathbb{R}_+^{n\times n} \cap \mathbb{S}^{n\times n}$. Note that since A_{r} is Hurwitz, it follows from the converse Lyapunov theory [14] that there exists a unique P satisfying (7) for a given R.

Using (5) in (4) yields

$$\dot{x}(t) = A_{\mathrm{r}} x(t) + B_{\mathrm{r}} c(t) - D\Lambda\tilde{W}^{\mathrm{T}}(t)\sigma(x(t), c(t)), \quad x(0) = x_0, \tag{8}$$

and the ideal system error dynamics is given using (3) and (8) as

$$\dot{e}_{\mathrm{o}}(t) = A_{\mathrm{r}} e_{\mathrm{o}}(t) - D\Lambda \tilde{W}^{\mathrm{T}}(t)\sigma(x(t), c(t)), \quad e_{\mathrm{o}}(0) = e_{\mathrm{o}0}, \quad (9)$$

where $\tilde{W}(t) \triangleq \hat{W}(t) - W \in \mathbb{R}^{(n+m)\times m}$. Note that it can be shown that $e_o(t)$ satisfying (9) asymptotically goes to zero with the standard model reference adaptive controller given by (5), (6), and (7) [12].

Finally, we briefly discuss a state emulator-based adaptive control framework [5–12] that we consider throughout this paper. It is shown in [11, 12] that this framework achieves stringent transient and steady-state system performance specifications without causing high-frequency oscillations in the controller response (unlike standard model reference adaptive controllers). In particular, consider the (modified) reference system so-called the state emulator given by

$$\dot{x}_{\mathrm{r}}(t) = A_{\mathrm{r}} x_{\mathrm{r}}(t) + B_{\mathrm{r}} c(t) + L(x(t) - x_{\mathrm{r}}(t)), \quad x_{\mathrm{r}}(0) = x_{\mathrm{r}0}, \quad (10)$$

where $L \in \mathbb{R}^{n\times n} \cap \mathbb{S}^{n\times n}$ is the state emulator gain. Defining $\tilde{x}(t) \triangleq x_{\mathrm{r}}(t) - x_{\mathrm{ri}}(t) \in \mathbb{R}^n$, the reference system error dynamics capturing the difference between the ideal reference model (3) and the state emulator-based (modified) reference model (10) is given by

$$\dot{\tilde{x}}(t) = A_{\mathrm{r}}\tilde{x}(t) + L(x(t) - x_{\mathrm{r}}(t)). \quad (11)$$

The emulator-based system error dynamics then follows from (8) and (10) as

$$\dot{e}(t) = \tilde{A} e(t) - D\Lambda \tilde{W}^{\mathrm{T}}(t)\sigma(x(t), c(t)), \quad e(0) = e_0, \quad (12)$$

where $e(t) = x(t) - x_{\mathrm{r}}(t) \in \mathbb{R}^n$ is the system state error vector, and $\tilde{A} = A_{\mathrm{r}} - L \in \mathbb{R}^{n\times n}$. Note that it can be shown that $e(t)$ satisfying (12) asymptotically goes to zero with the adaptive controller given by (5), (6), and (7) with $e_o(t)$ replaced with $e(t)$ in (6) and A_{r} replaced with $\tilde{A}$ in (7) [12].

3 Event-Triggered Adaptive Control

This section presents a state emulator-based adaptive control approach using tools and methods from event-triggered control theory. Specifically, when a predefined event occurs, the uncertain dynamical system sends its state signal to the adaptive controller. The kth time instants of the state transmission is represented by the monotonic sequence $\{s_k\}_{k=1}^{\infty}$, where $s_k \in \mathbb{R}_+$. The controller uses this triggered system state signal to compute the control signal using state emulator-based adaptive control architecture. When another predefined event occurs, the updated feedback control input is transmitted to the uncertain dynamical system. The jth time instants of the feedback control transmission is then represented by the monotonic sequence $\{r_j\}_{j=1}^{\infty}$, where $r_j \in \mathbb{R}_+$. Following this discussion, consider the uncertain system given by

$$\dot{x}(t) = Ax(t) + Bu_{\mathrm{s}}(t), \quad x(0) = x_0, \quad (13)$$

where $u_{\mathrm{s}}(t) \in \mathbb{R}^m$ is the sampled control input vector. Now, resorting to the mathematical preliminaries stated in Section 2, (13) can be equivalently written as

$$\dot{x}(t) = A_{\mathrm{r}} x(t) + B_{\mathrm{r}} c(t) + D\Lambda[u_{\mathrm{s}}(t) + W_1^{\mathrm{T}} x(t) + W_2^{\mathrm{T}} c(t)]. \quad (14)$$

Let the adaptive feedback control law be

$$u(t) = -\hat{W}^{\mathrm{T}}(t)\sigma_{\mathrm{s}}(x_{\mathrm{s}}(t), c(t)), \quad (15)$$

where $x_{\mathrm{s}}(t) \in \mathbb{R}^n$ is the sampled state vector, $\sigma_{\mathrm{s}}(x_{\mathrm{s}}(t), c(t)) = \left[x_{\mathrm{s}}^{\mathrm{T}}(t)\ c^{\mathrm{T}}(t)\right]^{\mathrm{T}} \in \mathbb{R}^{n+m}$, and $\hat{W}(t)$ satisfies the weight update law

$$\dot{\hat{W}}(t) = \gamma\,\mathrm{Proj_m}[\hat{W}^{\mathrm{T}}(t), \sigma_{\mathrm{s}}(x_{\mathrm{s}}(t), c(t)) e_{\mathrm{s}}^{\mathrm{T}}(t) P D], \quad \hat{W}(0) = \hat{W}_0, \quad (16)$$

with $e_{\mathrm{s}}(t) = x_{\mathrm{s}}(t) - x_{\mathrm{r}}(t) \in \mathbb{R}^n$ being the error of the triggered system state vector, and $P \in \mathbb{R}_+^{n\times n} \cap \mathbb{S}^{n\times n}$ being the solution of the Lyapunov equation

$$0 = \tilde{A}^{\mathrm{T}} P + P\tilde{A} + R. \quad (17)$$

Note that using (15) in (14) yields

$$\begin{aligned}\dot{x}(t) = {} & A_{\mathrm{r}} x(t) + B_{\mathrm{r}} c(t) + D\Lambda(u_{\mathrm{s}}(t) - u(t)) - D\Lambda\tilde{W}^{\mathrm{T}}(t) \\ & \cdot\sigma(x(t), c(t)) - D\Lambda\hat{W}^{\mathrm{T}}(t)(\sigma_{\mathrm{s}}(x_{\mathrm{s}}(t), c(t)) \\ & -\sigma(x(t), c(t))), \quad (18)\end{aligned}$$

Next, consider the ideal reference model in (3) and the state emulator-based reference model with the event-triggered system state given by

$$\dot{x}_{\mathrm{r}}(t) = A_{\mathrm{r}} x_{\mathrm{r}}(t) + B_{\mathrm{r}} c(t) + L e_{\mathrm{s}}(t), \quad x_{\mathrm{r}}(0) = x_{\mathrm{r}0}. \quad (19)$$

Now, the reference system error dynamics and the system error dynamics are respectively given by

$$\dot{\tilde{x}}(t) = A_{\mathrm{r}}\tilde{x}(t) + L e_{\mathrm{s}}(t), \quad (20)$$

$$\begin{aligned}\dot{e}(t) = {} & \tilde{A} e(t) - D\Lambda\hat{W}^{\mathrm{T}}(t)(\sigma_{\mathrm{s}}(x_{\mathrm{s}}(t), c(t)) - \sigma(x(t), c(t))) \\ & -D\Lambda\tilde{W}^{\mathrm{T}}(t)\sigma(x(t), c(t))) + D\Lambda(u_{\mathrm{s}}(t) - u(t)) \\ & -L(x_{\mathrm{s}}(t) - x(t)), \quad e(0) = e_0. \quad (21)\end{aligned}$$

In the reminder of this section, we first state user-defined event-triggering rules and discuss "no-Zeno behavior" (Section 3.1) and then show the boundedness of the overall closed-loop system using Lyapunov stability (Section 3.2).

3.1 User-Defined Event-Triggering Rules

For a given sensing threshold $\varepsilon_x \in \mathbb{R}_+$ (i.e., the threshold to allow data transmission from the uncertain dynamical system to the controller) and actuation threshold $\varepsilon_u \in \mathbb{R}_+$ (i.e., the threshold to allow data transmission from the controller to the uncertain dynamical system), we define three logic rules in a similar fashion to [3] as

$$E_1 : \|x_s(t) - x(t)\| \leq \varepsilon_x, \tag{22}$$

$$E_2 : \|u_s(t) - u(t)\| \leq \varepsilon_u, \tag{23}$$

$$E_3 : \text{The controller receives } x_s. \tag{24}$$

When the inequality (22) is violated at the s_k moment of the kth time instant, the uncertain dynamical system triggers the system state signal information such that $x_s(t)$ is sent to the controller. Likewise, when (23) is violated or the controller receives a new transmitted system state from the uncertain dynamical system (i.e., when $\bar{E}_2 \vee E_3$ is true), then the adaptive controller sends a new control input $u_s(t)$ to the uncertain dynamical system at the r_j moment of the jth time instant. For the purpose of the following lemma, we first consider $r_i^k \in (s_k, s_{k+1})$ to be the ith time instant when E_2 is violated over (s_k, s_{k+1}), and since $\{s_k\}_{k=1}^{\infty}$ is a subsequence of $\{r_j\}_{j=1}^{\infty}$, it follows that $\{r_j\}_{j=1}^{\infty} = \{s_k\}_{k=1}^{\infty} \bigcup \{r_i^k\}_{k=1,i=1}^{\infty,m_k}$, where $m_k \in \mathbb{N}$ is the number of violation times of E_2 over (s_k, s_{k+1}).

Now, consider the uncertain dynamical given by (13), the state emulator-based reference system given by (19), and the adaptive feedback control law given by (15) with the weight update law given by (16). Letting the transmission of the system state to the controller occur when $\bar{E}_1$ is true and letting the transmission of the control signal to the uncertain dynamical system occur when $\bar{E}_2 \vee E_3$ is true, it can be shown that

$$s_{k+1} - s_k > 0, \quad \forall k \in \mathbb{N}, \tag{25}$$

$$r_{i+1}^k - r_i^k > 0, \quad \forall i \in \{0, ..., m_k\}, \forall k \in \mathbb{N}. \tag{26}$$

Note that for this result, the boundedness of the closed-loop dynamical system is necessary and it is shown in the next subsection. In summary, this result show that the intersample times for the system state vector and feedback control vector are bounded away from zero, and hence, the proposed event-triggered adaptive control approach does not yield to a Zeno behavior.

3.2 Boundedness of the Overall Closed-Loop Dynamical System

In this section, we show the uniform ultimate boundedness of the closed-loop dynamical system subject to the proposed event-triggered adaptive control architecture. For this purpose, consider the uncertain dynamical system given by (13), the ideal reference system given by (3), the state emulator-based reference system given by (19), and the adaptive feedback control law given by (15) with the weight update law given by (16). Then, using the Lyapunov function candidate given by

$$\mathcal{V}(e, \tilde{W}, \tilde{x}) = e^{\mathrm{T}} P e + \gamma^{-1} \mathrm{tr} \tilde{W}^{\mathrm{T}} \tilde{W} \Lambda + \beta \tilde{x}^{\mathrm{T}} \tilde{P} \tilde{x}, \tag{27}$$

where $\beta \in \mathbb{R}_+$ is positive constant and $\tilde{P} \in \mathbb{R}_+^{n\times n} \cap \mathbb{S}^{n\times n}$ is the solution to the Lyapunov equation

$$0 = A_{\mathrm{r}}^{\mathrm{T}} \tilde{P} + \tilde{P} A_{\mathrm{r}} + \tilde{R}, \tag{28}$$

with $\tilde{R} \in \mathbb{R}_+^{n\times n} \cap \mathbb{S}^{n\times n}$, it can be shown that $\dot{\mathcal{V}}(e(t), \tilde{W}(t), \tilde{x}(t)) \leq 0$ when $\|e(t)\| \geq \psi_1$ and $\|\tilde{x}(t)\| \geq \psi_2$, where $\psi_1 \triangleq \frac{\frac{d_3}{2\sqrt{d_1}} + \sqrt{d_5 + \frac{d_3^2}{4d_1} + \frac{d_4^2}{4d_2}}}{\sqrt{d_1}}$ and $\psi_2 \triangleq \frac{\frac{d_4}{2\sqrt{d_2}} + \sqrt{d_5 + \frac{d_3^2}{4d_1} + \frac{d_4^2}{4d_2}}}{\sqrt{d_2}}$ with $d_1 \triangleq \lambda_{\min}(R) - \alpha \|\tilde{P}\|_{\mathrm{F}}^2 \|L\|_{\mathrm{F}}^2$, $d_2 \triangleq \beta \lambda_{\min}(\tilde{R}) - \frac{\beta^2}{\alpha}$, $d_3 \triangleq 2\|PD\|_{\mathrm{F}} \|\Lambda\|_{\mathrm{F}} \tilde{w}^* \varepsilon_x - 2\|PD\|_{\mathrm{F}} \|\Lambda\|_{\mathrm{F}} \|W\|_{\mathrm{F}} \varepsilon_x + 2\|PD\|_{\mathrm{F}} \|\Lambda\|_{\mathrm{F}} \varepsilon_u - 2\|P\|_{\mathrm{F}} \|L\|_{\mathrm{F}} \varepsilon_x$, $d_4 \triangleq 2\|PD\|_{\mathrm{F}} \|\Lambda\|_{\mathrm{F}} \tilde{w}^* \varepsilon_x + 2\beta \|\tilde{P}\|_{\mathrm{F}} \|L\|_{\mathrm{F}} \varepsilon_x$, and $d_5 \triangleq 2\|PD\|_{\mathrm{F}} \|\Lambda\|_{\mathrm{F}} \tilde{w}^* \tilde{\varepsilon}_x \varepsilon_x$. Note that $\|\tilde{W}(t)\|_{\mathrm{F}} \leq \tilde{w}^*$ due to utilizing the projection operator in the weight update law given by (16). Hence, the closed-loop solution $(e(t), \tilde{W}(t), \tilde{x}(t))$ is uniformly ultimately bounded.

4 Illustrative Numerical Example

In order to illustrate the proposed event-triggered adaptive control approach, consider a second-order system given by

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0.2 & -0.2 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_s(t) \tag{29}$$

with zero initial conditions. For this example, let $x_1(t)$ represent an angle in radians and $x_2(t)$ represent an angular rate in radians per second. We choose a second-order ideal reference system that has a natural frequency of 0.40 rad/s and a damping ratio of 0.707. Furthermore, we set $R = I_2$ and $\tilde{R} = I_2$. Figure 1 shows the results of the proposed state emulator-based adaptive controller architecture when $\gamma = 40$ and $L = 5I$, where a desirable command tracking performance is achieved with a reasonable control signal. Note that the results in Figure 1 can be easily compared with a standard model reference adaptive controller by setting L to zero. To see this, we set $\gamma = 2.5$ and $L = 0$ in Figure 2, where a desirable command tracking performance is achieved with the expense of a highly oscillative control signal. This shows the importance of employing a state emulator-based framework due to its capability of achieving stringent performance specifications without causing high-frequency oscillations in the controller response unlike standard adaptive controllers, as previously noted. Finally, the state and control event triggers for the cases in Figures 1 and 2 are given in Figure 3, where we can observe that the state emulator approach has a recognizable effect in reducing state and event triggers.

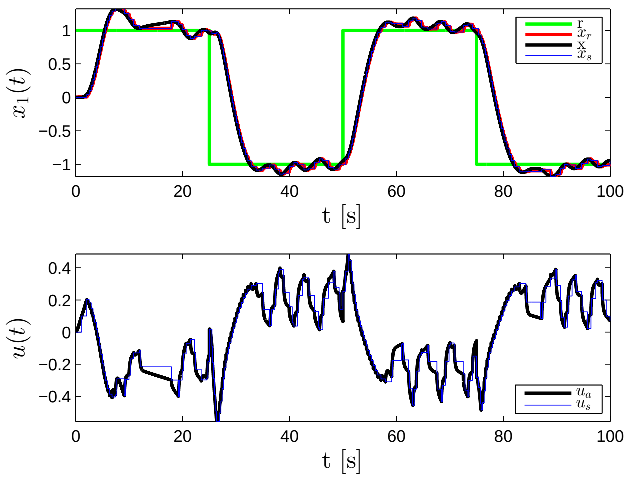

Figure 1. Command following performance using event-triggered adaptive control approach with $\varepsilon_x = 0.1$, $\varepsilon_u = 0.1$, $\gamma = 40$, and $L = 5I$.

5 Conclusion

To contribute to the previous studies in adaptive control of uncertain dynamical systems, we considered a scenario in which the dynamical system and its controller communicates through a real-time network. Utilizing results from event-triggering control theory to schedule data exchange dependent upon errors exceeding user-defined thresholds, we analyzed the boundedness of the overall closed-loop system using tools and methods from Lyapunov stability. Finally, we illustrated the efficacy of the proposed event-triggered adaptive control approach in a numerical example.

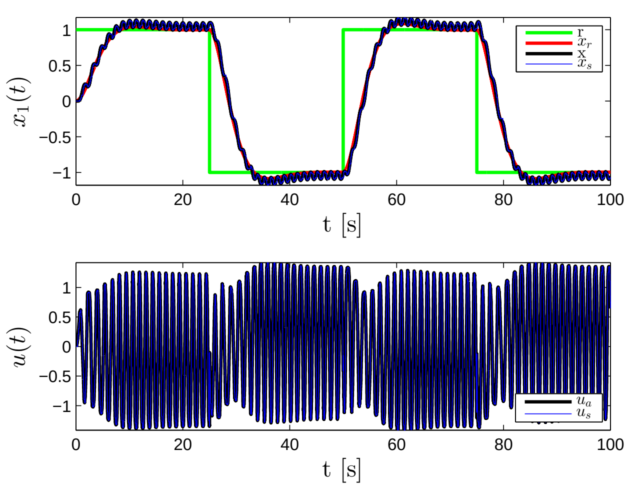

Figure 2. Command following performance using event-triggered adaptive control approach with $\varepsilon_x = 0.1$, $\varepsilon_u = 0.1$, $\gamma = 2.5$, and $L = 0$.

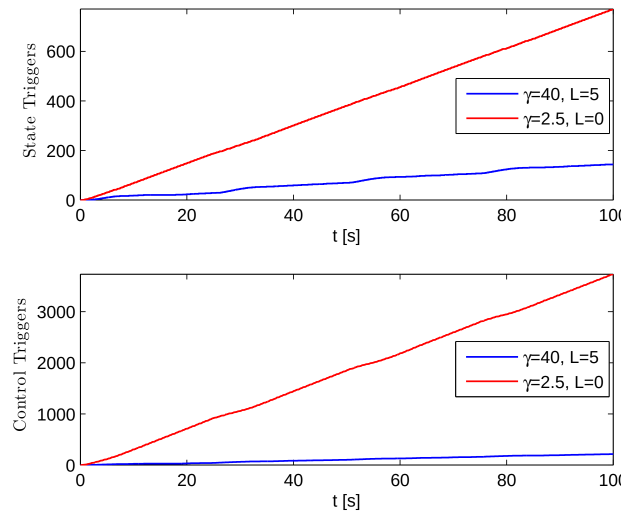

Figure 3. State and control event triggers for the cases in Figures 1 and 2.

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