Scalability of Closed-Loop System Responses in Adaptive Control Schemes

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***Abstract*—Although adaptive control offers a unique way to control dynamical systems without excessive reliance on their mathematical models, it is well known that nonidentical command profiles result in nonidentical closed-loop responses with such controllers, and therefore, it is of practical interest to study theoretical ways to construct predictable closed-loop responses when adaptive controllers are in the loop. To that end, we introduce a new concept called *scalability* to adaptive control in this paper. In particular, we analyze how to scale learning rates of adaptive weight update laws of various adaptive control schemes with respect to given command profiles to achieve a predictable closed-loop response. Illustrative numerical examples are provided to demonstrate the proposed concept and a possible utilization for the evaluation of adaptive controllers.**

I. INTRODUCTION

The model reference adaptive control concept, which was originally proposed by Whitaker *et al.* [1], [2], is based on the exigence to deal with uncertainties, which are inevitable when capturing a physical system with a mathematical model. For this purpose, various direct and indirect model reference adaptive control frameworks emerged in the past decades [3]–[6]. This paper considers a well-known class of direct model reference adaptive controllers employing in general three major components, namely, a reference system (model), an update law, and a controller. The reference system captures a desired closed-loop dynamical system behavior for which its output (respectively, state) is compared with the output (respectively, state) of the uncertain dynamical system. This comparison results in an error signal used to drive the update law online. The controller then adapts feedback gains to minimize this error signal using the information received from the update law.

In the adaptive control community it is well known that high learning rates in the adaptive weight update laws may lead to unacceptable control input signals due to high-frequency content [7]. Hence, modifications of adaptive controllers such as pseudo control hedging [8], frequency-limited adaptive control [9], and $\mathcal{L}_1$ adaptive control [10] have been introduced in order to employ high adaptive gains. Additionally, it is known that high excitation of the regressor vectors in the adaptive weight update law have a similar effect as high learning rates. Thus, tuning the adaptive weights over the whole envelope of permissible system states is a difficult task.

In this paper, a concept called scalability is introduced in order to analyze the closed-loop responses of model reference adaptive controllers in case of nonidentical, scalable command profiles. The analysis focuses on the influence of the learning rates of the adaptive weight update laws on the system's responses, establishing a correlation of the scaling coefficient of the command profile with the magnitude of the learning rates. Using this relation, predictable responses can be achieved for scalable command profiles. This paper follows previous simulation results [11] and provides mathematical background on how to scale the learning rates to achieve predictable performance for various well-known adaptive control methodologies.

In the literature, there have been approaches that consider tuning of the learning rates as a method to manipulate learning phase and robustness of an adaptive controller. For example, in [12] the recursive least-squares (RLS) method is used by an indirect adaptive update law in order to tune the nonlinear dynamic inversion controller of a hybrid adaptive control architecture. In a similar way, [13] employs covariances in order to update the modification gain of an adaptive controller. A method to update the adaptive learning rate based on notions of persistency of excitation is introduced in [3]. Additionally, in [14] the learning rates are adjusted based on a function of the tracking error in order to tune the adaptive weights to respond to degraded system performance. Note that all these methods are predominantly designed in order to improve the performance of adaptive controllers, especially in the transient phase, but do not address predictability of the system response over the envelope of permissible commands. Architectures such as $\mathcal{L}_1$ adaptive control [10] or the command governor framework [15] achieve approximate scalability for different references and hence, are possible candidates for analysis employing the concept introduced in this paper.

Furthermore, in [16] a guideline for tuning the adaptive learning rate comprising worst-case values of the ideal weights, the smallest reference model time constant, the input matrix, the solution of the Lyapunov equation, and the square of the maximum norm of the reference. Although the square of the maximum reference is related to the square of the scaling coefficient of the reference as utilized for the concept proposed here, [16] uses the maximum reference in order to optimize the free parameters of a MRAC architecture for one specific case. In contrast, in the paper at hand a scalability notion is introduced for a class of uncertainties with linear regressor vectors in order to show that scalability can be achieved for nonidentical, scalable command profiles, and hence, predictability in adaptive control is shown.

A. Nomenclature

The notation used in this paper is fairly standard. Specifically, $\Re$ denotes the set of real numbers, $\Re^n$ denotes the set of $n \times 1$ real column vectors, $\Re^{n\times m}$ denotes the set of $n \times m$ real matrices, $(\cdot)^{\mathrm{T}}$ denotes transpose, $(\cdot)^{-1}$ denotes inverse, and "$\triangleq$" denotes equality by definition. In addition, we write $\lambda_{\min}(A)$ (resp., $\lambda_{\max}(A)$) for the minimum (resp., maximum) eigenvalue of the Hermitian matrix A, $\det(A)$ for the determinant of the Hermitian matrix A, $\Re_+^{n\times n}$ for a positive definite matrix, $I_n \in \Re_+^{n\times n}$ for the unity matrix, and $\|\cdot\|_2$ for the Euclidian norm.

B. Organization

The paper is arranged as follows. In Section II, the model reference adaptive control (MRAC) problem formulation is given and the scaling approach for this case is introduced. Then, an example of the longitudinal motion of an aircraft is given to illustrate the result. In Section III, the scaling approach is extended to other well-known adaptive control schemes including σ-modification and e-modification adaptive control architectures [17], [18], adaptive control architectures with optimal control modifications [19], frequency-limited adaptive controllers [9], adaptive control architectures employing closed-loop reference models [20], [21], and command governor-based adaptive controllers [15]. Finally, conclusions and an outlook are given in Section IV.

II. SCALABLE PERFORMANCE IN MODEL REFERENCE ADAPTIVE CONTROL

In this section, scalability is shown in the standard model reference adaptive control (MRAC) architecture.

A. MRAC Problem Formulation

Consider the uncertain dynamical system given by

$$\dot{x}(t) = Ax(t) + B\Lambda u(t) + B\Delta(x(t)), \quad x(0) = x_0, \tag{1}$$

where $x(t) \in \Re^n$ is the accessible state vector, $u(t) \in \Re^m$ is the control input vector, $\Delta(x(t)) : \Re^n \to \Re^m$ is an *uncertainty*, $A \in \Re^{n\times n}$ is a known system matrix, $\Lambda \in \Re_+^{m\times m}$ is an *unknown* control effectiveness matrix, and $B \in \Re^{n\times m}$ is a known control input matrix. We assume that the pair (A, B) is controllable. Additionally, we assume

$$\Delta(x(t)) = \Lambda \begin{bmatrix} W_{\mathrm{x}}^{\mathrm{T}} & W_{\mathrm{c}}^{\mathrm{T}} & w_\kappa \end{bmatrix} \omega(t), \tag{2}$$

where $W_{\mathrm{x}} \in \Re^{n\times m}$ represents an uncertainty in the system matrix, $W_{\mathrm{c}} \in \Re^{l\times m}$ represents an uncertainty in the command input matrix, $\omega = \begin{pmatrix} x(t)^{\mathrm{T}} & c(t)^{\mathrm{T}} & \kappa \end{pmatrix} \in \Re^{n+l+1}$ is a known regressor vector, $c(t) \in \Re^l$ is the uniformly continuous bounded command, κ is a constant, and $w_\kappa \in \Re^m$ represents a constant disturbance. The reference system is given by

$$\dot{x}_r(t) = A_r x_r(t) + B_r c(t), \quad x_r(0) = x_{r0}, \tag{3}$$

where $x_r(t) \in \Re^n$ is the reference model state vector, $A_r \in \Re^{n\times n}$ is the desired Hurwitz system matrix, and $B_r \in \Re^{n\times l}$ is the command input matrix. The control signal $u(t)$ is given as

$$u(t) = u_{\mathrm{nom}}(t) - u_{\mathrm{ad}}(t), \tag{4}$$

where $u_{\mathrm{ad}}(t) \in \Re^m$ is the adaptive control input and $u_{\mathrm{nom}}(t) \in \Re^m$ denotes the nominal control input given by

$$u_{\mathrm{nom}}(t) = -K_x x(t) + K_c c(t), \tag{5}$$

where $K_x \in \Re^{m\times n}$ is the nominal feedback matrix and $K_c \in \Re^{m\times l}$ is the nominal feedforward matrix chosen such that

$$A - BK_x = A_r, \tag{6}$$
$$BK_c = B_r. \tag{7}$$

Using (2), (4), and (5) in (1), yields

$$\dot{x}(t) = A_r x(t) + B_r c(t) + B\Lambda W^{\mathrm{T}} \omega(t) - B\Lambda u_{\mathrm{ad}}(t), \tag{8}$$

where

$$W \triangleq \begin{bmatrix} W_{\mathrm{x}}^{\mathrm{T}} - \Lambda^* K_x & W_{\mathrm{c}}^{\mathrm{T}} + \Lambda^* K_c & w_\kappa \end{bmatrix}^{\mathrm{T}}, \tag{9}$$
$$\Lambda^* \triangleq \begin{bmatrix} I_m - \Lambda^{-1} \end{bmatrix}. \tag{10}$$

Furthermore, we define the adaptive control law

$$u_{\mathrm{ad}}(t) = \hat{W}^{\mathrm{T}}(t)\omega(t), \tag{11}$$

where $\hat{W}(t) \in \Re^{(n+l+1)\times m}$ is the adaptive weight matrix satisfying the adaptive weight update law

$$\dot{\hat{W}}(t) = \Gamma\omega(t)e^{\mathrm{T}}(t)PB, \quad \hat{W}(0) = \hat{W}_0, \tag{12}$$
$$e(t) \triangleq x(t) - x_r(t) \tag{13}$$

is the tracking error, and $P \in \Re^{n\times n}$ is the positive definite solution of the Lyapunov equation

$$Q + A_r^{\mathrm{T}} P + PA_r = 0, \tag{14}$$

where $Q \in \Re^{n\times n}$ is a positive definite design matrix. Finally, the uncertain dynamical system (1) can now be given as

$$\dot{x}(t) = A_r x(t) + B_r c(t) - B\Lambda\tilde{W}^{\mathrm{T}}(t)\omega(t),\ x(0) = x_0, \tag{15}$$

where $\tilde{W}(t) \triangleq \hat{W}(t) - W \in \Re^{(n+l+1)\times m}$ is the adaptive weight estimation error.

Remark 1: Note that the formulation of the uncertainty in (2) captures all kinds of linear uncertainties including uncertainties in the system matrix, the command input matrix, and constant disturbances and biases. Although the uncertain dynamical system (15) consequently is an uncertain linear system, the overall system including the adaptive control scheme is inherently nonlinear.

B. Scalability

Now, following a similar approach like in the motivational scalar example, we assume that the control engineer has found an appropriate adaptive control performance for a certain command history $c_0(t)$ and a specified learning rate Γ_0, resulting in the adaptive weight update law

$$\dot{\hat{W}}(t) = \Gamma_0 \omega(t) e^{\mathrm{T}}(t) P B, \quad \hat{W}(0) = \hat{W}_0, \tag{16}$$

For any scaled command profiles $c(t) = \alpha c_0(t)$ with scalar scaling command coefficients $\alpha \neq 0$ given a Lyapunov design matrix Q it is possible to achieve scaled system responses by choosing $\Gamma = \Gamma_0/\alpha^2$. To show this, we define $z(t) \triangleq x(t)/\alpha$, $z_0 \triangleq x_0/\alpha$, $z_{\mathrm{r}}(t) \triangleq x_{\mathrm{r}}(t)/\alpha$, $z_{\mathrm{r}0} \triangleq x_{\mathrm{r}0}/\alpha$, $e_{\mathrm{z}}(t) \triangleq z(t) - z_{\mathrm{r}}(t) = e(t)/\alpha$, $\kappa = \alpha$ and

$$\omega_z(t) \triangleq \alpha \omega(t) = \alpha \begin{pmatrix} z(t) & c_0(t) & 1 \end{pmatrix}^{\mathrm{T}}. \tag{17}$$

By applying this transformation to the uncertain dynamical system (15) and the weight update law (12), we have

$$\dot{z}(t) = A_r z(t) + B_r c_0(t) - B\Lambda \tilde{W}^{\mathrm{T}} \omega_z(t), \; z(0) = z_0, \tag{18}$$

$$\dot{z}_r(t) = A_r z(t) + B_r c_0(t), \quad z_r(0) = z_{r0}, \tag{19}$$

$$\dot{\hat{W}}(t) = \Gamma_0 \omega_z(t) e_z^{\mathrm{T}}(t) P B, \quad \hat{W}(0) = \hat{W}_0. \tag{20}$$

Note that the equations (3), (15), and (16) hold for any $\alpha \neq 0$. Further, note that the uncertain system (15,18) and the reference system (3,19) are scalable in the sense that state histories can be given by a nominal system response scaled by α. Stability of both original system (18), (19), (20) and scaled system (18), (19), (20) can be shown using Lyapunov stability theory as further detailed in [3]–[6].

Remark 2: Note that the adaptive weight response is not scaled and is identical given any α. Therefore, the adaptive weight update law is invariant to the scaling of the system's command. This departs from a traditional adaptive framework since large regressor vectors $\omega(t)$ have the same negative effects, namely undesirable oscillations [7], on most adaptive control architectures as excessively large learning rates Γ, which is obvious from (12). The results detailed above demonstrate that by introducing a scaling factor the adaptive learning rate can be adjusted accordingly such that a large command $c(t)$ does not lead to high excitation of the adaptive weights. A certain level of predictability is now obtained for the overall system behavior, resulting in an important step towards validation and verification of adaptive controllers.

C. MRAC Example

A short period model of the F16 aircraft is used to demonstrate the results derived above. The system matrices used are given by

$$A = \begin{bmatrix} -1.84 & 0.90 \\ -30.78 & -3.75 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ -12.75 \end{bmatrix}, \tag{21}$$

$$W = \begin{bmatrix} -3.07 \\ -0.37 \end{bmatrix}, \tag{22}$$

where $x_1(t)$ represents the angle of attack in rad and $x_2(t)$ represents the pitch rate in rad/s. Furthermore,

$$w(t) = [x_1(t), x_2(t)]^{\mathrm{T}}, \quad K_x = \begin{bmatrix} 1.49 & -0.04 \end{bmatrix}^{\mathrm{T}}, \tag{23}$$

$K_r = -1.51$, $\Lambda = 1$, and the learning rate is chosen as $\Gamma_0 = 100$. For the figures, the command profile is given by $c(t) = \alpha c_\alpha(t)$, where

$$c_\alpha(t) = 2^\circ, \; 0s \le t < 5s, \;\; 10s \le t < 15s, \tag{24}$$

$$c_\alpha(t) = -2^\circ, \; 5s \le t < 10s, \; 15s \le t < 20s. \tag{25}$$

Figures 1 and 2 show the response of the system with $\alpha = 1$ and $\alpha = 2$, respectively. The system responses with different

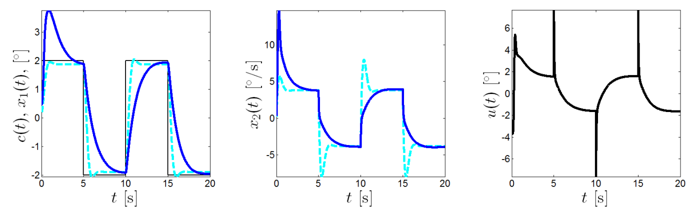

Fig. 1. Response of reference system and the uncertain system when $\alpha = 1$ (where the dotted lines indicate the reference system response).

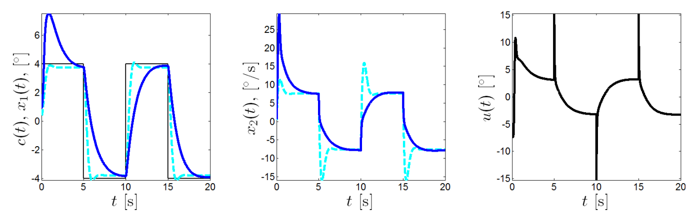

Fig. 2. Response of reference system and the uncertain system when $\alpha = 2$ (where the dotted lines indicate the reference system response).

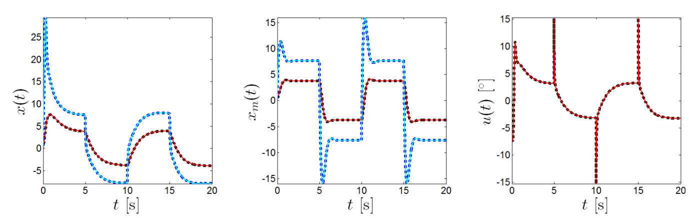

Fig. 3. Scaled comparison of the system response with $\alpha = 2$ (solid line) and $\alpha = 1$ (dotted line) when the scaling factor is used.

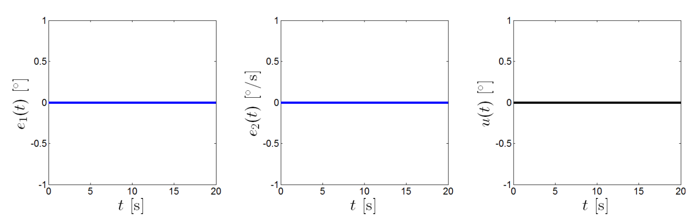

Fig. 4. Scaled difference in system responses when the scaling factor is used.

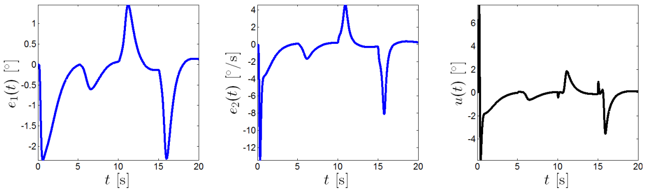

Fig. 5. Scaled difference in system responses when the scaling factor is not used.

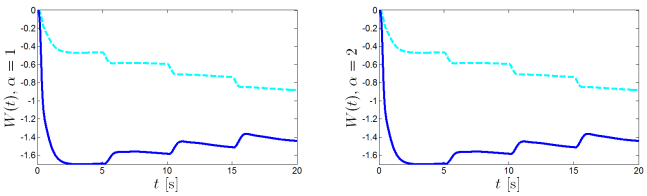

Fig. 6. Adaptive weights as a function of time when $\alpha = 1$ (left) and $\alpha = 2$ (right).

scaling command coefficients can be seen to be qualitatively identical. This is illustrated further in Figures 3 and 4, which shows that the error between the scaled systems is zero. However, if the standard MRAC architecture is used without the scaling factor, the responses using the same learning rate and different command do not display a scaled response, as expected. This is illustrated by Figure 5 with large scaled errors in both the system states and input. Finally, Figure 6 shows that the adaptive weight response is invariant to the scaling factor as discussed in Remark 2.

This numerical example highlights the key advantage of introducing the proposed scaling factor. By scaling the learning rates predictable performance with respect to a well tuned nominal system response is achieved. However, if not used the system response is not predictable for nonidentical commands.

III. EXTENSIONS TO OTHER MRAC SCHEMES

The scalability notion is applicable to all MRAC based schemes under the assumption that the states are applicable. In particular, in this section it is shown that the σ-modification and e-modification adaptive control architectures [17], [18], adaptive control architectures with optimal control modifications [19], frequency-limited adaptive controllers [9], adaptive control architectures employing closed-loop reference models [20], [21], and command governor-based adaptive controllers [15] can all be modified in order to achieve predictable performances as shown previously.

A. σ, e, and optimal control modification architectures

These robustness modifications have been introduced in order to avoid the phenomena of parameter drift and increase the robustness with respect to unmodeled dynamics. All three architectures modify the adaptive weight update law by augmenting it with a “damping-like” term.

In [17] the standard MRAC adaptive weight update law was modified as

$$\dot{\hat{W}}(t) = \Gamma\omega(t)e^{\mathrm{T}}(t)PB - \sigma\hat{W}(t), \quad \hat{W}(0) = \hat{W}_0, \qquad (26)$$

where $\sigma > 0$ is a damping coefficient used to ”pull” the estimated adaptive weights towards the origin. It was claimed that this σ-modification prevented the estimated adaptive weight from becoming unbounded. By introducing a scaling factor, as done in Section II-B, the σ-modifed adaptive weight update law is given as

$$\dot{\hat{W}}(t) = \Gamma_0\omega_z(t)e_z^{\mathrm{T}}(t)PB - \sigma\hat{W}(t), \quad \hat{W}(0) = \hat{W}_0. \qquad (27)$$

where $\Gamma = \Gamma_0/\alpha^2$, $\omega_z(t)$ and $e_z(t)$ are defined in the previous section. It can be readily seen that the adaptive weight response, as before, is invariant to the scaling factor α. Furthermore, scalability of the system states and inputs is also evident since the reference system (3) and the uncertain system (15) are not modified.

In [18] the standard MRAC adaptive law was further modified by replacing σ in (26) with a time-varying damping coefficient given by $\sigma_e \left\|e(t)\right\|_2$. Therefore, the effect of the modification was determined by the norm of the system's tracking error. The so-called e-modification adaptive weight update law is given as

$$\dot{\hat{W}}(t) = \Gamma\omega(t)e^{\mathrm{T}}(t)PB - \sigma_e\left\|e(t)\right\|_2\hat{W}(t), \qquad (28)$$

$\hat{W}(0) = \hat{W}_0$, where $\sigma_e > 0$. Similar to (26), by introducing a scaling factor (28) can be rewritten as

$$\dot{\hat{W}}(t) = \Gamma_0\omega_z(t)e_z^{\mathrm{T}}(t)PB - \sigma_0\left\|e_z(t)\right\|_2\hat{W}(t), \qquad (29)$$

$\hat{W}(0) = \hat{W}_0$, where $\sigma_e = \sigma_0/\alpha$ and, as before, $\Gamma = \Gamma_0/\alpha^2$. Note that, as seen for the σ-modification case, (28) is invariant to α and, therefore, scalability results.

Finally, consider the optimal control modification introduced in [19], which was developed in the same spirit as the modifications summarized above. The modification added to the adaptive law in this case is based on a ”minimization of the $\mathcal{L}_2$ norm of the tracking error bounded away from some lower bound formulated as an optimal control problem” [19] and is given by

$$\dot{\hat{W}}(t) = \Gamma\omega(t)[e^{\mathrm{T}}(t)P - \sigma\omega^{\mathrm{T}}(t)\hat{W}(t)B^{\mathrm{T}}PA_r^{-1}]B, \qquad (30)$$

$\hat{W}(0) = \hat{W}_0$. As before, by introducing the scaling factor α (30) can now be given as

$$\dot{\hat{W}}(t) = \Gamma_0\omega_z(t)[e_z^{\mathrm{T}}(t)P - \sigma\omega_z^{\mathrm{T}}(t)\hat{W}(t)B^{\mathrm{T}}PA_r^{-1}]B, \qquad (31)$$

$\hat{W}(0) = \hat{W}_0$. As anticipated, (31) is invariant to α and scalability, as introduced in this paper, results.

It should be noted that all the adaptive control architectures considered in this section are obtained with simple augmentations of the standard MRAC adaptive weight update law. In general, if the augmentation is invariant to the scaling factor α then the modified adaptive control framework will be scalable in the sense introduced in this paper.

B. Frequency-Limited Adaptive Control

The frequency limited adaptive control architecture introduced in [9] employs a gradient based modification term and a low pass filter. It is claimed that the modification term filters high-frequency content out of the adaptive weight update law, allowing for the controller to be tuned with high learning rates in order to enable robust and fast adaptation. The adaptive weight update law is given by

$$\dot{\hat{W}}(t) = \Gamma\omega(t)e^{\mathrm{T}}(t)PB - \sigma[\hat{W}(t) - \hat{W}_f(t)], \quad (32)$$

$\hat{W}(0) = \hat{W}_0$, where $\sigma > 0$ is a modification gain and $\hat{W}_f(t) \in \Re^{(n+l+1)\times m}$ is the low-pass filtered weight estimate of $\hat{W}(t)$, satisfying

$$\dot{\hat{W}}_f(t) = \Gamma_f[\hat{W}(t) - \hat{W}_f(t)],\ \hat{W}_f(0) = \hat{W}_0, \quad (33)$$

where $\Gamma_f \in \Re^{(n+l+1)\times(n+l+1)}$ is a positive definite filter gain matrix such that $\lambda_{\max}(\Gamma_f) \le \gamma_{\mathrm{f,max}}$ and $\gamma_{\mathrm{f,max}} > 0$ is a design parameter.

The adaptive weight update law (33) can incorporate the scaling factor α as

$$\dot{\hat{W}}(t) = \Gamma_0\omega_z(t)e_z^{\mathrm{T}}(t)PB - \sigma[\hat{W}(t) - \hat{W}_f(t)], \quad (34)$$

$\hat{W}(0) = \hat{W}_0$, where $\Gamma = \Gamma_0/\alpha^2$, $e_z(t) = e(t)/\alpha$, and $\omega_z(t) = \omega(t)/\alpha$. Note that once again the adaptive weight update law is invariant with respect to the scaling factor α. Therefore, as discussed in the previous section, it can be concluded that a system employing this adaptive control framework will have predictably scalable responses.

C. Reference Model Modification

In [20], [21] the reference model was modified by feeding back the tracking error in order to improve the transient performance of MRAC controllers. Therefore, the uncertain dynamical system (15) and the adaptive weight update law (12) are not changed and can be scaled as shown in Section II-B. However, the reference model is given by

$$\dot{x}_r(t) = A_r x_r(t) + B_r c(t) + Le(t), \quad x_r(0) = x_{r0}, \quad (35)$$

where $L \in \Re^{n\times n}$ is a positive definite matrix. The scaling factor can then be introduced to the modified reference model by employing, as before, the relations $z_r(t) = x_r(t)/\alpha$, $z_{r0} = x_{r0}/\alpha$, $e_z(t) = e(t)/\alpha$, and $c(t) = \alpha c_0(t)$, resulting in

$$\dot{z}_r(t) = A_r z_r(t) + B_r c_0(t) + Le_z(t), \quad z_r(0) = z_{r0}. \quad (36)$$

Hence, scalability for adaptive control architectures with modified reference models is obtained.

D. Command Governor Adaptive Control

Here, the scalability notion is applied to the command governor framework for adaptive control [15]. The command governor framework is displayed in Figure 7. Basically, an additional command is used in order to cancel uncertainties in the transient phase.

Hence, the overall command is given by

$$c(t) \triangleq c_D(t) + c_g(t), \quad (37)$$

where $c_D(t) \in \Re^m$ is the bounded, desired tracking command (the original $c(t)$ from the sections above). The additional command $c_g(t) \triangleq K_c^{-1}\left[B^{\mathrm{T}}B\right]^{-1}B^{\mathrm{T}}g(t) \in \Re^m$, $\det(K_c) \ne 0$ is based on a linear system, which is defined as

$$\begin{aligned}\dot{\xi}(t) &= -\lambda\xi(t) + \lambda e(t), \quad \xi(0) = 0, \quad (38)\\ g(t) &= \lambda\xi(t) + [A_r - \lambda I_n]\,e(t), \quad (39)\end{aligned}$$

where $\xi(t) \in \Re^n$ denotes the command governor states, $g(t) \in \Re^n$ is the command governor output, and $\lambda > 0$ is the command governor gain. Since the additional command is applied on both reference model and nominal controller, the error dynamics of the system do not change and therefore, we have

$$\dot{e}(t) = A_r e(t) - B\Lambda\tilde{W}^{\mathrm{T}}(t)\omega(t), \quad e(0) = x_0 - x_{r0}, \quad (40)$$

which can be written as

$$\Lambda\tilde{W}^{\mathrm{T}}(t)\omega(t) = [B^{\mathrm{T}}B]^{-1}B^{\mathrm{T}}\{A_r e(t) - \dot{e}(t)\}. \quad (41)$$

Applying (37), (38), (39), and (41) onto the uncertain system dynamics (15) using $G = B\left[B^{\mathrm{T}}B\right]^{-1}B^{\mathrm{T}}$, we have

$$\dot{x}(t) = A_r x(t) + B_r c_D(t) + G\{\lambda\xi(t) - \lambda e(t) - \dot{e}(t)\}, \quad (42)$$

$x(0) = x_0$. In [15] it is shown that $\lambda\xi(t) - \lambda e(t) - \dot{e}(t) = 0$ for $\lambda \to \infty$ and that the overall system is stable.

Remark 3: Although the reference model is modified, the closed loop uncertain system still tracks the desired reference model given by

$$\dot{x}_{r,D}(t) = A_r x_{r,D}(t) + B_r c_D(t), \quad x_{r,D}(0) = x_{r0} \quad (43)$$

as the last term of (42) is appoximately zero for large λ.

Remark 4: The command governor gain λ can be used to determine a trade off between command governor and adaptive control. Furthermore, note that no adaptive control would be necessary for $\lambda \to \infty$, which is of no practical relevance. For more information about the command governor refer to [15].

Now, considering scalability, assume there was a $c_0(t)$ with a certain reference performance and a learning rate Γ_0. Then, applying a command profile $c_D(t) = \alpha c_0(t)$ and a scaled adaptive gain $\Gamma = \Gamma_0/\alpha^2$, scalability can be achieved. Using $\xi_z(t) = \xi(t)/\alpha$, $g_z(t) = g(t)/\alpha$, and $e_z(t) = e(t)/\alpha$, we have

$$\begin{aligned}\dot{\xi}_z(t) &= -\lambda\xi_z(t) + \lambda e_z(t), \quad \xi_z(0) = 0 \quad (44)\\ g_z(t) &= \lambda\xi_z(t) + [A_r - \lambda I_n]\,e_z(t). \quad (45)\end{aligned}$$

Hence, $c_{g,z}(t) = \alpha c_g(t)$ and $c(t) = \alpha c_0 + \alpha c_g$ holds, which implies that the reference model is also scalable as shown in Section II-B. The transformed uncertain system dynamics are

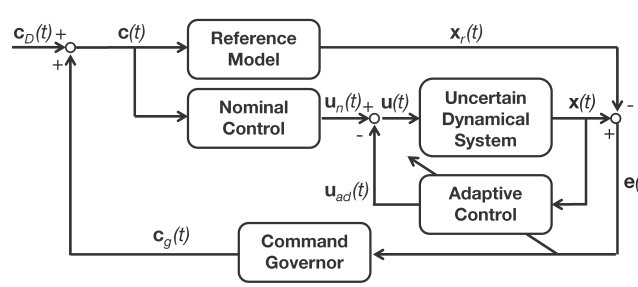

Fig. 7. Command Governor Framework

given by (using $f_z(t) \triangleq \lambda\xi_z(t) - \lambda e_z(t) - \dot{e}_z(t)$)

$$\dot{z}(t) = A_r z(t) + B_r c_0(t) + G f_z(t),\ z(0) = z_0, \qquad (46)$$

which shows scalability of the uncertain system's dynamics. Additionally, the invariance of the adaptive weight update law (20) to the scaling factor stays untouched. Consequently, the scalability approach introduced in this paper also holds for the command governor framework.

IV. CONCLUSIONS AND FUTURE WORKS

The contribution of this paper is to introduce the scalability notion to adaptive control theory as a measure of obtaining predictable system performance for given nonidentical but scalable command profiles. For several MRAC schemes, the use of the scalability approach for obtaining predictable system responses for systems with linear uncertainties is illustrated. By scaling the learning rates Γ, predictable scaled responses can be obtained and hence, the performance of one well-tuned case can be extended to different command profiles. Thus, the system's responses to nonidentical commands may be predicted and used in order to verify the adaptive controller's performance with respect to design objectives.

In order to focus further on the validation and verification, it is of grand interest to achieve a certain level of predictability for systems incorporating nonlinear regressor vector components or radial basis functions. Therefore, future work will be directed towards deriving reasonable conditions for obtaining approximate scalability of adaptive control architectures, which may be employed for validation and verification purposes. Additionally the approach will be further investigated in order to evaluate whether predictability can be achieved for changes in the command signal other than pure scaling.

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