Resilient Networked Multiagent Systems: A Distributed Adaptive Control Approach

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DISTRIBUTED SYSTEMS

- Large-scale distributed systems
  - Composed of interconnected heterogeneous subsystems
  - Reliance on local information to achieve global objectives
- Consensus Protocols
  - Basic property
  - Emergent behaviors
- Vulnerable to Failure
  - Increase points of failure
  - Susceptible to attack
  - Catastrophic failure in single subsystem may need to catastrophic system failure
DISTRIBUTED SYSTEMS

- Large-scale distributed systems
- Composed of interconnected heterogeneous submodules
- Reliance on local information to achieve global objectives

Ideal Behavior
- Agent agree on predictable value (centroid)

Catastrophic System failure
**DISTRIBUTED SYSTEMS**

- Large-scale distributed systems
- Failure
  - Relatively small external disturbances

Catastrophic system failure
DISTRIBUTED SYSTEMS

- Large-scale distributed systems
- Composed of interconnected heterogeneous submodules
- Reliance on local information to achieve global objectives
- Consensus Protocols
  - Basic property
  - Emergent behaviors
- Vulnerable to Failure
  - Increase points of failure
  - Susceptible to attack
  - Catastrophic failure in single submodule may need to catastrophic system failure

Adaptation

- Stability in the presence of disturbances
LITERATURE ON DETECTION/ISOLATION

- F. Pasqualetti *et al.* (ACC, 2008 and TAC, 2012)
  - Input observer technique and performance quantified
  - Scalability and computation issues

- S. Sundaram and C. N. Hadjicostis (ACC, 2008)
  - Inspired by parity space methods (fault detection)
  - Restrictive assumption on network connectivity

- M. Franceschelli *et al.* (ACC, 2008)
  - Detection and isolation of misbehaving agents
  - Excitation of system

- I. Shames *et al.* (Automatica, 2011)
  - Input observer technique
  - Detection and isolation of misbehaving agents

- H. J. LaBlanc *et al.* (ACC, 2013)
  - Assumption on the number of misbehaving agents
PROPOSED SOLUTION

- Adaptive
  - Estimation of unknown parameters
  - Control input suppresses uncertainty
- Distributed
  - Same connectivity as consensus needed
  - Limited to only use local information
- State Emulator-Based
  - Local emulator used
  - A “reference model” is not possible
CONTRIBUTION

- Distributed Adaptive Approach to Resilient Networks
  - Computationally simple
  - No restrictive assumptions on
    - Number of misbehaving agents
    - Network connectivity
  - Framework can be extended to general linear systems
**Problem Formulation**

- A networked multiagent system whose agents are under the influence of exogenous disturbances.

\[ \dot{x}_i(t) = u_i(t) + w_i, \quad t \geq 0, \quad x_i(t_0) = x_{i0}, \quad i \in \mathcal{V}(G) \]

- Design an input such that the overall system remains stable and "closely" resembles the undisturbed ideal system.
**PROPOSED SOLUTION**

- A networked multiagent system whose agents are under the influence of exogenous disturbances

\[ \dot{x}_i(t) = u_i(t) + w_i, \ t \geq 0, \ x_i(t_0) = x_{i0}, \ i \in \mathcal{V}(G) \]

- **Augmented consensus protocol**

\[ u_i(t) = - \sum_{i \sim j} (x_i(t) - x_j(t)) - \hat{w}_i, \ i \in \mathcal{V}(G) \]

- **Disturbance estimate update law**

\[ \dot{\hat{w}}_i(t) = \alpha (x_i(t) - \hat{x}_i(t)), \ t \geq 0, \ \hat{w}_i(t_0) = \hat{w}_{i0} \]
**PROPOSED SOLUTION**

- A state emulator is devised in lieu of a reference system

\[
\dot{\hat{x}}_i(t) = -d_i \hat{x}_i(t) + \sum_{i \sim j} x_j(t), \quad t \geq 0, \quad \hat{x}_i(0) = \hat{x}_{i0}
\]

- Undisturbed response captured, indirect disturbances from neighboring agents
- No additional information from neighboring agents needed
**PROPOSED SOLUTION**

- Compact form

\[
\begin{align*}
\dot{x}(t) &= -\mathcal{L}(G)x(t) + A(G)\tilde{x}(t), \ t \geq 0, \ x(t_0) = \hat{x}_0 \\
\dot{\tilde{x}}(t) &= -\Delta(G)\tilde{x}(t) + \tilde{w}(t), \ \tilde{x}(t_0) = \bar{x}_0 \\
\dot{\tilde{w}}(t) &= -\alpha\tilde{x}(t), \ \tilde{w}(t_0) = \tilde{W}_0
\end{align*}
\]

\[\tilde{x}_i(t) \triangleq x_i(t) - \hat{x}_i(t)\]
\[\tilde{w}_i(t) \triangleq w_i - \tilde{w}_i(t)\]

- Stability Requirements

\[
\begin{align*}
\dot{x}(t) &\to \text{span}\{1\} \\
\tilde{x}(t) &\to 0 \\
\tilde{w}(t) &\to 0
\end{align*}
\]
PROPOSED SOLUTION

- Compact form

\[ \dot{y}(t) = -T\mathcal{L}T^{-1}\dot{y}(t) + TA\ddot{x}(t), \quad t \geq 0, \quad \dot{y}(t_0) = \dot{y}_0 \]

- Partitioning

\[ \begin{align*}
\dot{\hat{z}}_1(t) &= A_1\hat{z}_1(t) + A_2\ddot{x}(t), \quad t \geq 0, \quad \hat{z}_1(t_0) = \hat{z}_{10} \\
\dot{\hat{c}}_G(t) &= \sum_{i \in \mathcal{V}(G)} d_i\dot{\ddot{x}}_i(t), \quad \hat{c}_G(t_0) = \hat{c}_G_0
\end{align*} \]
**PROPOSED SOLUTION**

- **Compact form**

- **New Stability Requirements**
  - $\hat{z}(t) \to 0$
  - $\hat{x}(t) \to 0$
  - $\hat{w}(t) \to 0$

- **Compact form**

  $$\dot{\xi}(t) = M\xi(t), \quad t \geq 0, \quad \xi(t_0) = \xi_0$$

  $$M = \begin{bmatrix}
  A_1 & A_2 & 0_{n-1 \times n} \\
  0_{n \times n-1} & -\Delta(G) & I_n \\
  0_{n \times n-1} & -\alpha I_n & 0_{n \times n}
  \end{bmatrix}$$
Proposed Solution

Compact form

\[
\begin{align*}
    \dot{x}(t) &= -\mathcal{L}(\mathcal{G})\hat{x}(t) + \mathcal{A}(\mathcal{G})\tilde{x}(t), \quad t \geq 0, \quad \hat{x}(t_0) = \hat{x}_0, \\
    \dot{\tilde{x}}(t) &= -\Delta(\mathcal{G})\tilde{x}(t) + \tilde{w}(t), \quad \tilde{x}(t_0) = \tilde{x}_0, \\
    \dot{\tilde{w}}(t) &= -\alpha \tilde{x}(t), \quad \tilde{w}(t_0) = \tilde{W}_0,
\end{align*}
\]

Theorem 3.1. Consider the network multiagent system given by (15), (11), and (12). The solution \((\hat{z}_1(t), \hat{x}(t), \tilde{w}(t))\) is exponentially stable for all \((\hat{z}_{10}, \hat{x}_0, \tilde{w}_0) \in \mathbb{R}^{n-1} \times \mathbb{R}^n \times \mathbb{R}^n\).

\[
\begin{align*}
    x(t) &\to \text{Span}\{1\} \\
    \tilde{x}(t) &\to 0 \\
    \tilde{w}(t) &\to 0
\end{align*}
\]
PERFORMANCE OF SOLUTION

- The system should “resemble” the undisturbed system
**PERFORMANCE OF SOLUTION**

- The system should "resemble" the undisturbed system

- The system's non-convergent mode
  
  **Undisturbed system**
  \[
  \dot{c}_G(t) = 0
  \]

  **Disturbed system**
  \[
  \dot{c}_G(t) = \sum_{i \in \mathcal{V}(G)} d_i \dot{x}_i(t), \quad \hat{c}_G(t_0) = \hat{c}_{G0}
  \]

- Emulator error acts like a disturbance
The system should "resemble" the undisturbed system.

**Reduce perturbation**

\[ \| \tilde{x}(t) \|_2 \]

**Energy function**

\[ E(\tilde{x}(t), \tilde{w}(t)) = \frac{1}{2} \tilde{x}^T \tilde{x} + \frac{1}{2\alpha} \tilde{w}^T \tilde{w} \]

**Performance guarantee**

\[ \| \tilde{x} \|_2 \leq \frac{\| \tilde{w}(0) \|_2}{\sqrt{\alpha}} \]
PERFORMANCE OF SOLUTION

- The system should "resemble" the undisturbed system

- **Bounded non-convergent mode**

\[
\hat{c}_G(t) = \int_0^t \sum_{i \in \mathcal{V}(G)} d_i \hat{x}_i(t) \, dt + \hat{c}_0
\]

- **Due to exponential convergence**

\[
\hat{x}(t) \rightarrow 0
\]

- **Rate of convergence computable**
NONLINEAR DISTURBANCES

- **Unknown weights and known basis functions**

\[ \dot{x}_i(t) = u_i(t) - W_i^T \sigma_i(x), \quad t \geq 0, \quad x_i(t_0) = x_{i0} \]

**Assumption 3.1.** The first and second order partial derivatives of \( \sigma_{ij}, \frac{\partial \sigma_{ij}}{\partial x} \) and \( \frac{\partial^2 \sigma_{ij}}{\partial^2 x} \), are bounded \( \forall i \in \mathcal{V}(\mathcal{G}) \) and \( \forall j \in \{1, \ldots, s_i\} \). In addition, \( \sigma_i(x) = \sigma_i(y) \) if \( x_k - c = y_k, \quad \forall k = \{1, \ldots, n\} \) where \( c \in \mathbb{R} \).

\[ u_i(t) = -\sum_{i \sim j} (x_i - x_j) + \hat{W}_i^T(t) \sigma_i(x), \quad i \in \mathcal{V}(\mathcal{G}) \]

- **Adaptive Consensus Protocol**
**NONLINEAR DISTURBANCES**

- **Frequency-limited adaptive framework**

\[
\dot{\hat{x}}(t) = -\Delta(G)\hat{x}(t) + A(G)x(t) + \kappa(\tilde{x}(t) - \tilde{x}_L(t))
\]

- **Modified state emulator**

\[
\dot{\tilde{x}}_L(t) = -\Delta(G)\tilde{x}_L(t) + \eta(\tilde{x}(t) - \tilde{x}_L(t))
\]

- **Exponentially Stable, Bounded** non-convergent mode
**SELF AWARENESS**

- Disconnect from network if disturbance is large
CONCLUDING REMARKS

- Contribution
  - State emulator approach
  - Mitigation of disturbances
  - General distributive adaptive framework

- Ongoing research
  - General linear/nonlinear dynamics
  - Trust parameters
  - Greater class of uncertainties/disturbances
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