Resilient Networked Multiagent Systems: A Distributed Adaptive Control Approach[†]

Gerardo De La Torre, Tansel Yucelen, and John Daniel Peterson

***Abstract*— Control algorithms of networked multiagent systems are generally computed distributively without having a centralized entity monitoring the activity of agents; and therefore, adverse events such as attacks to the communication network and/or failure of agent-wise components can easily result in system instability and prohibit the accomplishment of system-level objectives. This paper studies resilient coordination of networked multiagent systems in the presence of misbehaving agents, i.e., agents that are subject to such adverse events. In particular, we consider a class of adverse conditions consisting of exogenous disturbances and interagent uncertainties, and present a distributed adaptive control architecture to retrieve the nominal networked multiagent system behavior. Departing from the existing relevant literature that make specific assumptions on the graph topology and/or the fraction of misbehaving agents, we show that the considered class of adverse conditions can be mitigated through a new adaptive control methodology that utilizes a local state emulator – even if all agents are misbehaving. A illustrative numerical example is provided to demonstrate the theoretical findings.**

I. Introduction

Multiagent systems consist of groups of agents that sense their environment and locally communicate with each other through a network to implement distributed control strategies and accomplish system level-objectives, which are not easy and efficient enough for a single agent to accomplish [1], [2]. Although these systems are envisioned to autonomously function in place of humans for repetitive, demanding, and safety-critical missions the current level of controls technology is incapable of providing the needed usability and resiliency of multiagent systems. Much prior work in distributed control of multiagent systems has utilized fixed-gain control strategies that are not capable of recovering the desired system performance in the presence of unforeseen adversaries [3], [4]. Furthermore, since control algorithms of multiagent systems are computed distributively without having a centralized entity monitoring the activity of agents, adverse events such as attacks to the communication network and/or failure of agent-wise components can easily result in system instability and prohibit the accomplishment of system-level objectives [4], [5].

The fragile nature of multiagent systems has triggered the development of detection and isolation algorithms during the last few years, including the notable results in [6]–[12]. For achieving resilient multiagent system behavior, [6] and [7] make a specific assumption on the network connectivity (other than the standard assumption on the connectedness of networked agents) that takes inspiration from parity space methods for fault detection [13]. The proposed framework of [8] requires that at most a fraction of any normal agent's neighbors to be adversaries, or misbehaving agents, in order to achieve resilient multiagent system behavior. The authors of [9] excites the multiagent system in order to detect and isolate the misbehaving agents from the network topology. Like [6] and [7], a computationally expensive and not scalable algorithm is proposed in [10] and [11] based on input observers technique [14], where the effect of misbehaving agents on the overall multiagent system performance is also quantified, and an extension of this work is given in [12] focuses on the detection and isolation of misbehaving agents.

In this paper, we develop a new distributed adaptive control architecture to ensure resilient coordination of networked multiagent systems in the presence of misbehaving agents. Specifically, we consider a class of adverse conditions affecting agents that consist of exogenous disturbances and interagent uncertainties. We show that such adverse conditions can be mitigated through a new adaptive control methodology that utilizes a local state emulator. Departing from the existing relevant literature [6]–[12] that make specific assumptions on the graph topology and/or the fraction of misbehaving agents, the proposed framework can achieve performance recovery on an arbitrary but connected communication topology and even if all agents are misbehaving.

A. Organization

The organization of the paper is as follows. Section II presents mathematical preliminaries. Section III introduces the proposed distributed adaptive control approach to enable resilient coordination of networked multiagent systems. Section IV presents a numerical example and, conclusions are summarized in Section V.

II. Mathematical Preliminaries

We first introduce several results that are necessary for the development of the main results of this paper.

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Theorem 2.1 [15]. Consider the singularly perturbed system given by

$$\dot{x} = f(t, x, z, \epsilon), \quad (1)$$

$$\epsilon \dot{z} = g(t, x, z, \epsilon). \quad (2)$$

Assume that the following assumptions are satisfied for all $(t, x, \epsilon) \in [0, \infty) \times B_r \times [0, \epsilon_0]$.

i) $f(t, 0, 0, \epsilon) = 0$ and $g(t, 0, 0, \epsilon) = 0$.

ii) The equation $0 = g(t, x, z, 0)$ has an isolated root $z = h(t, x)$ such that $h(t, 0) = 0$.

iii) The functions f, g, h and their partial derivatives up to the second order are bounded for $z - h(t, x) \in B_0$

iv) The origin of the reduced system $\dot{x} = f(t, x, h(t, x), 0)$ is exponentially stable.

v) The origin of the boundary-layer system $\frac{dy}{\tau} = g(t, x, y + h(t, x), 0)$ is exponentially stable, uniformly in (t, x).

Then, there exists $\epsilon^\star > 0$ such that for all $\epsilon < \epsilon^\star$, the origin of (1)-(2) is exponentially stable.

Next, we recall some of the basic notions from graph theory and networked multiagent systems [16,17]. Specifically, graphs are broadly used in networked multiagent systems to encode interactions between a group of agents. An *undirected* graph $\mathcal{G}$ is defined by a set $\mathcal{V}_\mathcal{G} = \{1, \ldots, n\}$ of *nodes* and a set $\mathcal{E}_\mathcal{G} \subset \mathcal{V}_\mathcal{G} \times \mathcal{V}_\mathcal{G}$ of *edges*. If the *unordered* pair $(i, j) \in \mathcal{E}_\mathcal{G}$, then nodes i are j are *neighbors* and the neighboring relation is indicated with $i \sim j$. The set of neighbors of node i is denoted by $\mathcal{N}_\mathcal{G}(i) = \{j|(i \sim j) \in \mathcal{E}(\mathcal{G})\}$. The *degree* of a node is given by the number of its neighbors. Letting d_i be the degree of node i, then the *degree* matrix of a graph $\mathcal{G}$, $\Delta(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by $\Delta(\mathcal{G}) \triangleq \mathrm{diag}(d), d = [d_1, \ldots, d_n]^\mathrm{T}$. The *adjacency* matrix of a graph $\mathcal{G}$, $\mathcal{A}(\mathcal{G}) \in \mathbb{R}^{n\times n}$, is given by

$$[A(\mathcal{G})]_{ij} \triangleq \begin{cases} 1, & \text{if } (i, j) \in \mathcal{E}_\mathcal{G}, \\ 0, & \text{otherwise.} \end{cases} \quad (3)$$

The *Laplacian* matrix of a graph, $\mathcal{L}(\mathcal{G}) \in \bar{\mathbb{S}}_+^{n\times n}$, is given by $\mathcal{L}(\mathcal{G}) \triangleq \Delta(\mathcal{G}) - \mathcal{A}(\mathcal{G})$, where the spectrum of the Laplacian for a connected, undirected graph can be ordered as $0 = \lambda_1(\mathcal{L}(\mathcal{G})) < \lambda_2(\mathcal{L}(\mathcal{G})) \leq \cdots \leq \lambda_n(\mathcal{L}(\mathcal{G}))$, with $\mathbf{1}_n$ as the eigenvector corresponding to the zero eigenvalue $\lambda_1(\mathcal{L}(\mathcal{G}))$ and $\mathcal{L}(\mathcal{G})\mathbf{1}_n = \mathbf{0}_n$ holds.

Networked multiagent systems can be modeled by a graph $\mathcal{G}$, where nodes and edges, represent agents and interagent information exchange links, respectively. In particular, let $x_i(t) \in \mathbb{R}^N$ denote the state of agent i at time $t \geq 0$, whose dynamics are described by

$$\dot{x}_i(t) = u_i(t), \quad t \geq 0, \quad x_i(t_0) = x_{i0}, \quad i \in \mathcal{V}(\mathcal{G}), \quad (4)$$

with $u_i(t) \in \mathbb{R}^N$ being the control input of agent i. In this paper, we consider agents having dynamics of the form given by (4) to focus the main results of this paper[1]. In addition, we focus on the consensus problem without loss of much generality when presenting the main contributions of this paper. In particular, if agent i is allowed to access the relative state information with respect to its neighbors, a solution to the standard consensus problem can be given by

$$u_i(t) = -\sum_{i\sim j}(x_i(t) - x_j(t)), \quad i \in \mathcal{V}(\mathcal{G}), \quad (5)$$

for a connected and undirected graph[2]. The networked multiagent system given by (4) and (5) can now be described in the form given by $\dot{x}(t) = -(\mathcal{L}(\mathcal{G})\otimes \mathrm{I}_N)x(t),\ t \geq 0,\ x(t_0) = x_0$, where $x(t) = [x_1^\mathrm{T}(t), \cdots, x_n^\mathrm{T}(t)]^\mathrm{T}$ denotes the aggregated state vector. For ease of exposition, we consider the case of $N = 1$. However, all results presented in this paper can be trivially extended to the general case. Finally, we note that $x(t) \to [(\mathbf{1}_n\mathbf{1}_n^\mathrm{T}/n) \otimes \mathrm{I}_N]x_0$ as $t \to \infty$, since the graph $\mathcal{G}$ is assumed to be connected and undirected. That is, the networked multiagent system is said to reach a consensus since $x_1 = x_2 = \cdots = x_n$.

III. Resilient Coordination of Networked Multiagent Systems

This section introduces the proposed adaptive control approach to enable resilient coordination of networked multiagent systems in the presence of misbehaving agents.

A. Resiliency Against Exogenous Disturbances

We first consider a networked multiagent system whose agents are under the influence of exogenous disturbances. The agent dynamics given by (4) are augmented to incorporate the effect of these disturbances as

$$\dot{x}_i(t) = u_i(t) + w_i,\ t \geq 0,\ x_i(t_0) = x_{i0},\ i \in \mathcal{V}(\mathcal{G}), \quad (6)$$

where $w_i \in \mathbb{R}$ is an unknown disturbance applied to agent i. We say that an agent is misbehaving if there exists a time such that $w_i \neq 0$, $i \in \mathcal{V}(\mathcal{G})$. In order to mitigate the effect of these exogenous disturbances, the nominal consensus protocol given by (5) is modified as

$$u_i(t) = -\sum_{i\sim j}(x_i(t) - x_j(t)) - \hat{w}_i, \quad i \in \mathcal{V}(\mathcal{G}), \quad (7)$$

where $\hat{w}_i$ is an estimate of the disturbance of agent i, and is updated as

$$\dot{\hat{w}}_i(t) = \alpha(x_i(t) - \hat{x}_i(t)), \quad t \geq 0, \quad \hat{w}_i(t_0) = \hat{w}_{i0}, \quad (8)$$

where $\alpha > 0$, $\hat{w}_{i0} = 0$, and $\hat{x}_i(t)$ is a state emulator [19,20] given by

$$\dot{\hat{x}}_i(t) = -\mathrm{d}_i\hat{x}_i(t) + \sum_{i\sim j} x_j(t), \quad t \geq 0, \quad \hat{x}_i(0) = \hat{x}_{i0}. \quad (9)$$

The undisturbed response of the system is captured by $\hat{x}_i(t)$. However, notice that indirect disturbances from neighboring agents still affect the emulator system. Using (6) and (9), the dynamics of the emulator state error are given by $\dot{\tilde{x}}_i(t) = -d_i\tilde{x}_i(t) + \tilde{w}_i(t),\ t \geq 0,\ \tilde{x}_i(t_0) = \tilde{x}_{i0}$, where

[1]For agents having complex dynamics, one can design low-level feedback controllers (or assume their existence) for suppressing existing dynamics and enforcing (4) (see, for example, Example 6.3 of [18]).

[2]Throughout this paper, we assume that the graph $\mathcal{G}$ is connected and undirected.

$\tilde{x}_i(t) \triangleq x_i(t) - \hat{x}_i(t)$ and $\tilde{w}_i(t) \triangleq w_i - \hat{w}_i(t)$. In addition, the exogenous disturbance estimate error dynamics are given by $\dot{\tilde{w}}_i(t) = -\alpha \tilde{x}_i(t),\ t \geq 0,\ \tilde{w}_i(t_0) = \tilde{w}_{i0}$. Now, the networked multiagent system can be described in a compact form as

$$\hat{z}(t) \ \rightarrow \ 0 \quad (10)$$

$$\tilde{x}(t) \ \rightarrow \ 0 \quad (11)$$

$$\tilde{w}(t) \ \rightarrow \ 0 \quad (12)$$

where $\hat{x}(t) = [\hat{x}_1(t), \ldots, \hat{x}_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$, $\tilde{x}(t) = [\tilde{x}_1(t), \ldots, \tilde{x}_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$, $\tilde{w}(t) = [\tilde{w}_1(t), \ldots, \tilde{w}_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$, denote the aggregated emulator state, emulator error, and disturbance estimate error, respectively. Next, we consider the state transformation given by

$$\hat{y}(t) = T\hat{x}(t) = [\hat{z}_{12}(t), \hat{z}_{13}(t), \ldots, \hat{c}_{\mathcal{G}}(t)]^{\mathrm{T}} \quad (13)$$

where $\hat{z}_{1i}(t) = \hat{x}_1(t) - \hat{x}_i(t)$ and $\hat{c}_{\mathcal{G}}(t) = \sum_{i \in \mathcal{V}(\mathcal{G})} \hat{x}_i(t)$. Under this state transformation and using (**??**), it follows that

$$\dot{\hat{y}}(t) = -T\mathcal{L}T^{-1}\hat{y}(t) + T\mathcal{A}\tilde{x}(t),\ t \geq 0,\ \hat{y}(t_0) = \hat{y}_0. \quad (14)$$

Furthermore, since the graph is undirected and connected, (14) can be partitioned as

$$\dot{\hat{z}}_1(t) = A_1\hat{z}_1(t) + A_2\tilde{x}(t),\ t \geq 0,\ \hat{z}_1(t_0) = \hat{z}_{10}, \quad (15)$$

$$\dot{\hat{c}}_{\mathcal{G}}(t) = \sum_{i \in \mathcal{V}(\mathcal{G})} \mathrm{d}_i\tilde{x}_i(t),\ \hat{c}_{\mathcal{G}}(t_0) = \hat{c}_{\mathcal{G}0}, \quad (16)$$

$$\dot{\hat{c}}_{\mathcal{G}}(t) = 0 \quad (17)$$

where $z_1 = [z_{12}, z_{13}, \ldots, z_{1n}]^{\mathrm{T}}$, $A_1 \in \mathbb{R}^{n-1 \times n-1}$ and $A_2 \in \mathbb{R}^{n-1 \times n-1}$ are the matrices obtained by removing the n^{th} row and column from $T\mathcal{L}(\mathcal{G})T^{-1}$ and the matrix obtain by removing the n^{th} row from $T\mathcal{A}(\mathcal{G})$, respectively.

Remark 3.1. The state transformation given by (13) effectively decouples the system's convergent and non-convergent modes. While, $\hat{z}_1(t)$, the distance between agents, will be shown to converge to 0 the system's centroid, $\hat{c}_{\mathcal{G}}(t)$, will (in general) not converge to zero. Though this property results from the Laplacian matrix's structure it must still be ensure that the centroid of the system remains bounded.

Theorem 3.1. Consider the network multiagent system given by (15), (**??**), and (**??**). The solution $(\hat{z}_1(t), \tilde{x}(t), \tilde{w}(t))$ is exponentially stable for all $(\hat{z}_{10}, \tilde{x}_0, \tilde{w}_0) \in \mathbb{R}^{n-1} \times \mathbb{R}^n \times \mathbb{R}^n$.

Proof. Note that the system is equivalently described by $\dot{\xi}(t) = M\xi(t),\ t \geq 0,\ \xi(t_0) = \xi_0$, where

$$M = \left[\begin{array}{c|cc} A_1 & A_2 & 0_{n-1\times n} \\ \hline 0_{n\times n-1} & -\Delta(\mathcal{G}) & \mathrm{I}_n \\ 0_{n\times n-1} & -\alpha\mathrm{I}_n & 0_{n\times n} \end{array}\right], \quad (18)$$

and $\xi(t) = [\hat{z}_1(t)^{\mathrm{T}}, \tilde{x}(t)^{\mathrm{T}}, \tilde{w}(t)^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^{3n-1}$. The spectrum of M is described as $\mathrm{spec}(M) = \mathrm{spec}(A_1) \cup \mathrm{spec}(\begin{bmatrix} -\Delta(\mathcal{G}) & \mathrm{I}_n \\ -\alpha\mathrm{I}_n & 0 \end{bmatrix})$. Since the graph is assumed to be connected $\mathrm{spec}(A_1) < 0$. Furthermore, note that the characteristic polynomial of $\begin{bmatrix} -\Delta(\mathcal{G}) & \mathrm{I}_n \\ -\alpha\mathrm{I}_n & 0 \end{bmatrix}$ is given as $Z(\lambda) = \prod_{i=1}^{n}(\lambda^2 + \lambda d_i + \alpha)$. By using the Routh–Hurwitz stability criterion we can conclude that the zeros of $Z(\lambda)$ are in the left half of the complex plane since $d_i > 0$ and $\alpha > 0$ [21]. Therefore, the solution $(\hat{z}_1(t), \tilde{x}(t), \tilde{w}(t))$ is exponentially stable for all $(\hat{z}_{10}, \tilde{x}_0, \tilde{w}_0) \in \mathbb{R}^{n-1} \times \mathbb{R}^n \times \mathbb{R}^n$. ■

Remark 3.2. Theorem 3.1 shows that the modified consensus protocol given by (7) results in consensus despite the presence of disturbances. Furthermore, the estimates of the disturbances converge to the actual exogenous disturbances.

Notice from (**??**) and (16) that $\tilde{x}(t)$ acts as a vanishing perturbation to an ideal consensus equation. Furthermore, if $\|\tilde{x}(t)\|_2$ is sufficiently small, then agents not only achieve consensus but the final agreement point stays close to the original centroid of the system – the point that would have been reached in the undisturbed case. That is, the effect of the disturbances on the overall system performance can be related to $\|\tilde{x}(t)\|_2$. In addition, α has a direct affect on the bound of $\|\tilde{x}(t)\|_2$. To see this, consider the energy function $E(\tilde{x}(t), \tilde{w}(t)) = \frac{1}{2}\tilde{x}^{\mathrm{T}}\tilde{x} + \frac{1}{2\alpha}\tilde{w}^{\mathrm{T}}\tilde{w}$. Taking the time derivative yields $\dot{E}(\tilde{x}(t), \tilde{w}(t)) = -\tilde{x}^{\mathrm{T}}\Delta(\mathcal{G})\tilde{x} \leq 0$. Therefore, $E(\tilde{x}(t), \tilde{w}(t)) \leq E(\tilde{x}(0), \tilde{w}(0)) = \frac{\|\tilde{w}\|_2^2}{2\alpha}$ and $\|\tilde{x}\|_2 \leq \frac{\|\tilde{w}(0)\|_2}{\sqrt{\alpha}}$. As α is increased, the magnitude of the vanishing perturbation term $\|\tilde{x}\|_2$ becomes smaller. Meaning, during transient-time the state emulator system (**??**) and the emulator centroid, $\hat{c}(t)$, are affected less by disturbances. Finally, note that $\hat{c}_{\mathcal{G}}(t) = \int_0^t \sum_{i \in \mathcal{V}(\mathcal{G})} \mathrm{d}_i\tilde{x}_i(t)\ \mathrm{d}t + \hat{c}_0$ remains bounded since $\tilde{x}(t)$ exponentially converges to 0.

B. Resiliency Against Interagent Disturbances

We now consider nonlinear interagent disturbances (uncertainties). It is first shown in Theorem 3.2 that a standard model reference adaptive control architecture [22,23] cannot guarantee that centroid of the system will remain bounded, and therefore, the multiagent system as a whole may not be stable. Then, by using the frequency-limited adaptive control architecture introduced in Ref. 24, it will be shown in Theorems 3.3 and 3.4 that not only the centroid of the system will remain bounded, but also consensus will be achieved in the presence of nonlinear interagent disturbances.

The class of uncertainties considered here can be characterized by an unknown weight vector and known basis functions and the agent dynamics are now given by

$$\dot{x}_i(t) = u_i(t) - W_i^{\mathrm{T}}\sigma_i(x), \quad t \geq 0, \quad x_i(t_0) = x_{i0}, \quad (19)$$

where $i \in \mathcal{V}(\mathcal{G})$, $\sigma_i = [1, \sigma_{i1}, \ldots, \sigma_{is_i}]^{\mathrm{T}} : \mathbb{R}^n \rightarrow \mathbb{R}^{s_i+1}$ and $\sigma_{ij} : \mathbb{R}^n \rightarrow \mathbb{R}$ are known basis functions $\forall i \in \{1, \ldots, n\}$ and $\forall j \in \{1, \ldots, s_i\}$ and $W_i \in \mathbb{R}^{s_i+1}$ is an unknown weight vector.

Assumption 3.1. The first and second order partial derivatives of σ_{ij}, $\frac{\partial\sigma_{ij}}{\partial x}$ and $\frac{\partial^2\sigma_{ij}}{\partial^2 x}$, are bounded $\forall i \in \mathcal{V}(\mathcal{G})$ and $\forall j \in \{1, \ldots, s_i\}$. In addition, $\sigma_i(x) = \sigma_i(y)$ if $x_k - c = y_k,\ \forall k = \{1, \ldots, n\}$ where $c \in \mathbb{R}$.

Note that Assumption 3.1 implies that the uncertainty is invariant to the location of the overall system's centroid. That is, the uncertainty will not change if the entire system is translated in any direction. Therefore, the uncertainty is only a function of the relative displacement of the agents. Note that this class of uncertainties can be used to represent unknown interaction between agents as well as exogenous disturbances considered in the previous subsection.

In order to achieve consensus in the presence of the aforementioned nonlinear interagent disturbances, we propose

$$u_i(t) = -\sum_{i\sim j}(x_i - x_j) + \hat{W}_i^{\mathrm{T}}(t)\sigma_i(x), \quad i \in \mathcal{V}(\mathcal{G}), \quad (20)$$

where $\hat{W}_i(t)$ is the estimate of the unknown weight matrix to be defined later. Using (20) in (19) yields

$$\dot{x}_i(t) = -\mathrm{d}_i x_i(t) + \sum_{i\sim j} x_j(t) + \tilde{W}_i^{\mathrm{T}}(t)\sigma_i(x), \quad (21)$$

where $\tilde{W}_i(t) \triangleq \hat{W}_i(t) - W$. Once again, we now consider the state emulator given by

$$\dot{\hat{x}}_i(t) = -\mathrm{d}_i\hat{x}_i(t) + \sum_{i\sim j} x_j(t), \quad t \geq 0, \quad \hat{x}_i(t_0) = \hat{x}_{i0}. \quad (22)$$

In addition, from (21) and (22) the difference between the state emulator and the disturbed system is given by $\dot{\tilde{x}}_i(t) = -\mathrm{d}_i\tilde{x}_i(t) + \tilde{W}_i^{\mathrm{T}}(t)\sigma_i(x)$, $t \geq 0$, $\tilde{x}_i(t_0) = \tilde{x}_{i0}$, where $\tilde{x}_i(t) \triangleq x_i(t) - \hat{x}_i(t)$, $i \in \mathcal{V}(\mathcal{G})$. The networked system can now be described as

$$\begin{aligned} \dot{x}(t) &= -\mathcal{L}(\mathcal{G})x(t) + \tilde{W}^{\mathrm{T}}(t)\sigma(x),\ x(t_0) = x_0, \quad (23)\\ \dot{\hat{x}}(t) &= -\Delta(\mathcal{G})\hat{x}(t) + \mathcal{A}(\mathcal{G})x(t),\ \hat{x}(t_0) = \hat{x}_0, \quad (24)\\ \dot{\tilde{x}}(t) &= -\Delta(\mathcal{G})\tilde{x}(t) + \tilde{W}^{\mathrm{T}}(t)\sigma(x),\ \tilde{x}(t_0) = \tilde{x}_0, \quad (25)\end{aligned}$$

where $x(t) = [x_1(t)^{\mathrm{T}}, \ldots, \hat{x}_n(t)^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^n$, $\hat{x}(t) = [\hat{x}_1(t)^{\mathrm{T}}, \ldots, \hat{x}_n(t)^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^n$, $\tilde{x}(t) = [\tilde{x}_1(t)^{\mathrm{T}}, \ldots, \tilde{x}_n(t)^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^n$, denote the aggregated state, emulator state, and emulator state error, respectively, $\tilde{W}(t) = [\bar{\tilde{W}}_1(t), \ldots, \bar{\tilde{W}}_n(t)] \in \mathbb{R}^{S\times n}$ such that $\bar{\tilde{W}}_i(t) = [0_{s_{0i}}, \tilde{W}_i(t)^{\mathrm{T}}, 0_{s_{in}}]^{\mathrm{T}}$, $S = \sum_{i\in\mathcal{V}(\mathcal{G})}(s_i+1)$, $s_{0i} = \sum_{j=1}^{i-1}(s_j+1)$, $s_{in} = \sum_{j=i+1}^{n}(s_j+1)$, and $\sigma(x) = [\sigma_i(x)^{\mathrm{T}}, \ldots, \sigma_n(x)^{\mathrm{T}}]^{\mathrm{T}} \in \mathbb{R}^S$.

The adaptive weight update law can now be introduced as

$$\dot{\tilde{W}}_i(t) = -\gamma_i\sigma_i(x)\tilde{x}_i P_i, \quad t \geq 0, \quad \tilde{W}_i(0) = \tilde{W}_{i0}, \quad (26)$$

$i \in \mathcal{V}(\mathcal{G})$, where $\gamma_i > 0$ and $P_i > 0$ satisfies the Lyapunov equation $-2\mathrm{d}_iP_i + Q_i = 0$, where $Q_i > 0$.

Remark 3.3. In the above characterization of the uncertainty, it could be assumed that the nonlinear disturbances are dependent on every state in the network and, as a result, each agent must have access to every other agent's state in order to implement the adaptive weight update law. However, notice that if the parameterization of a nonlinear disturbance is limited such that $\sigma_i(x) = \sigma_i(\xi_i)$ where $\xi_i = Gx$, $G \in \mathbb{R}^{n_i\times n}$ and $n_i < n$, then the amount of information agent i needs is reduced. For example, suppose the interagent disturbance on agent i can be given as $\alpha\sin(x_i - x_j)$ then only the relative distance between agents i and j will be needed in order to implement the adaptive weight update law, in addition to the information needed to implement the state emulator given by (22). Therefore, the parameterization of the nonlinear disturbance and the graph structure determine the information needed in order to implement the proposed framework.

Theorem 3.2. Consider the network multiagent system given by (23)-(25) and the adaptive weight update law (26) subject to Assumption 3.1. The solution $(\tilde{x}(t), \tilde{W}(t))$ is Lyapunov stable for all $(0, \tilde{W}_0) \in \mathbb{R}^n \times \mathbb{R}^{S\times n}$, $\tilde{x}(t) \to 0$ as $t \to \infty$, and $\tilde{x}(t) \in \mathcal{L}_2$.

Proof. The candidate Lyapunov function given as

$$V(\tilde{x}, \tilde{W}) = \tilde{x}^{\mathrm{T}}P\tilde{x} + \frac{1}{2}\sum_{i\in\mathcal{V}(\mathcal{G})}\tilde{W}_i^2/\gamma_i, \quad (27)$$

where $P = \mathrm{diag}([P_1, \ldots, P_n])$ can be used to show the desired result using standard MRAC analysis [25]. ■

Note that despite Theorem 3.2 the centroid of the multiagent system, $c_{\mathcal{G}}(t) = \sum_{i\in\mathcal{V}(\mathcal{G})} x_i(t)$, is still not guaranteed to remain bounded. By applying the state transformation given by (13) to (24), we note that $\hat{c}_{\mathcal{G}}(t) = \int_0^t \sum_{i\in\mathcal{V}(\mathcal{G})} \mathrm{d}_i\tilde{x}_i(t)\ \mathrm{d}t$. Note that $\tilde{x} \in \mathcal{L}_2$ does not imply that $\hat{c}_{\mathcal{G}}(t)$ will remain bounded. Therefore, even though Theorem 3.2 ensures that $\tilde{x}(t)$ asymptotically converges to 0 there is no guarantee that the system as a whole will remain bounded.

In the following analysis we consider a frequency-limited adaptive framework and show that boundedness of the system's centroid can be guaranteed under this framework. This framework has been introduced in Ref. 24. Through proper reference model modification it was shown that fast robust adaptation can be achieved since the high frequency content of the tracking error signal is effectively suppressed.

In order to apply the frequency-limited adaptive framework to the uncertain multiagent system studied here, we first introduce a modification to the state emulator (22) as

$$\dot{\hat{x}}(t) = -\Delta(\mathcal{G})\hat{x}(t) + \mathcal{A}(\mathcal{G})x(t) + \kappa(\tilde{x}(t) - \tilde{x}_L(t)), \quad (28)$$

where $t \geq 0$, $\hat{x}(t_0) = \hat{x}_0$, $\kappa > 0$ and $\tilde{x}_L(t)$ is the frequency-limited (filtered) emulator state error given by

$$\dot{\tilde{x}}_{\mathrm{L}}(t) = -\Delta(\mathcal{G})\tilde{x}_{\mathrm{L}}(t) + \eta(\tilde{x}(t) - \tilde{x}_L(t)), \quad (29)$$

where $\tilde{x}_L(t_0) = 0$ and $\eta > 0$. As a result the aggregated emulator state error is now given as

$$\dot{\tilde{x}}(t) = -\Delta(\mathcal{G})\tilde{x}(t) + \tilde{W}(t)^{\mathrm{T}}\sigma(x) - \kappa(\tilde{x}(t) - \tilde{x}_L(t)), \quad (30)$$

where $\tilde{x}(t_0) = \tilde{x}_0$. Furthermore, to prevent confusion in notation we defined the *ideal* state emulator as

$$\dot{\hat{x}}_{\mathrm{rd}}(t) = -\Delta(\mathcal{G})\hat{x}_{\mathrm{rd}}(t) + \mathcal{A}(\mathcal{G})x(t),\ \hat{x}_{\mathrm{rd}}(0) = \hat{x}_0. \quad (31)$$

Finally, we will define the emulator deviation from (31) and (28) as

$$\dot{\bar{x}}(t) = -\Delta(\mathcal{G})\bar{x}(t) + \kappa(\tilde{x}(t) - \tilde{x}_L(t)), \quad (32)$$

where $\bar{x}(0) = 0$ and $\bar{x}(t) \triangleq \hat{x}(t) - \hat{x}_{\mathrm{rd}}(t)$. The following theorem proves stability of the networked multiagent system when the modified state emulator is used. The same theorem also establishes steady-state and transient performance guarantees of the proposed framework.

Theorem 3.3. Consider the uncertain network multiagent system given by (23), the modified state emulator

(28), along with (29), and (30) and the adaptive weight update law (26) subject to Assumption 3.1. The solution $(\tilde{x}(t), \tilde{x}_L(t), \bar{x}(t), \tilde{W}(t))$ is Lyapunov stable for all $(\tilde{x}_0, 0, 0, \tilde{W}_0) \in \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^{S\times n}$ and $\lim_{t\to\infty}\big(x(t) - x_{\rm rd}(t)\big) = 0$. In addition for $t > 0$,

$$\|x(t) - x_{\rm rd}(t)\|_{\mathcal{L}_\infty} \le \sqrt{\frac{\epsilon_{\mathcal{V}}}{\underline{P}}}\left(1 + \sqrt{\frac{\kappa\bar{P}}{2\xi\underline{Q}}}\right), \quad (33)$$

where $\xi \in (0, 1)$, $\bar{P} = \max_{i\in\mathcal{V}_\mathcal{G}}(P_i)$, $\underline{P} = \min_{i\in\mathcal{V}_\mathcal{G}}(P_i)$, $\underline{Q} = \min_{i\in\mathcal{V}_\mathcal{G}}(Q_i)$, $\epsilon_\mathcal{V} = \underline{\gamma}^{-1}\|\tilde{W}_0\|_{\rm F}^2 + \bar{P}\|\tilde{x}_0\|_2^2$ and $\underline{\gamma} = \min_{i\in\mathcal{V}_\mathcal{G}}(\gamma_i)$.

Proof. First consider the Lyapunov function candidate

$$\begin{aligned} V^*(\tilde{x}, \tilde{x}_L, \bar{x}, \tilde{W}) &= V\big(\tilde{x}, \tilde{W}\big) + \eta^{-1}\kappa\tilde{x}_{\rm L}^{\rm T}P\tilde{x}_{\rm L} \\ &\quad + 2\xi\kappa^{-1}\underline{Q}\bar{x}^{\rm T}P\bar{x}/\bar{P}, \end{aligned} \quad (34)$$

where $V\big(\tilde{x}, \tilde{W}\big)$ is given by (27). Note that $V^*(0) = 0$, $V^*(\tilde{x}, \tilde{x}_L, \bar{x}, \tilde{W}) > 0$ for all $(\tilde{x}, \tilde{x}_L, \bar{x}, \tilde{W}) \neq 0$, and $V^*(\tilde{x}, \tilde{x}_L, \bar{x}, \tilde{W})$ is radially unbounded. Differentiating (34) along the trajectories of (26), (29), (30) and (32) and defining $\tilde{x}_{\rm H}(t) \triangleq \tilde{x}(t) - \tilde{x}_{\rm L}(t)$ yields

$$\begin{aligned} \dot{V}^*(\cdot) &= -\tilde{x}^{\rm T}(t)Q\tilde{x}(t) - \eta^{-1}\kappa\tilde{x}_{\rm L}^{\rm T}(t)Q\tilde{x}_{\rm L}(t) \\ &\quad -2\xi\kappa^{-1}\underline{Q}\bar{x}^{\rm T}(t)Q\bar{x}(t)/\bar{P} - 2\kappa\tilde{x}_{\rm H}^{\rm T}(t)P\tilde{x}_{\rm H}(t) \\ &\quad +2\xi\underline{Q}\Big[2\bar{x}^{\rm T}(t)P^{\frac{1}{2}}P^{\frac{1}{2}}\tilde{x}_{\rm H}(t)\Big]/\bar{P}. \end{aligned} \quad (35)$$

Note that by Young's inequality [26, Fact 1.4.7] $2\bar{x}^{\rm T}P^{\frac{1}{2}}P^{\frac{1}{2}}\tilde{x}_{\rm H} \le \mu\bar{x}^{\rm T}P\bar{x} + \frac{1}{\mu}\tilde{x}_{\rm H}^{\rm T}P\tilde{x}_{\rm H}$, where $\mu > 0$. Therefore, if $\mu = \xi\underline{Q}/\bar{P}\kappa$ then (35) can be given as

$$\begin{aligned} \dot{V}^*(\cdot) &\le -\underline{Q}\|\tilde{x}(t)\|_2^2 - \eta^{-1}\kappa\underline{Q}\|\tilde{x}_{\rm L}(t)\|_2^2 \\ &\quad -2\xi\kappa^{-1}\underline{Q}^2\big[1-\xi\big]\|\bar{x}(t)\|_2^2/\bar{P}, \end{aligned} \quad (36)$$

and, hence, $\dot{V}^*(\cdot) \le 0$. Therefore, the solution $\big(\tilde{x}, \tilde{x}_L, \bar{x}, \tilde{W}\big)$ is Lyapunov stable. To show $\lim_{t\to\infty}\big(x(t) - x_{\rm rd}(t)\big) = 0$, note that $\sigma_i\big(x(t)\big)$ is bounded for all $t > 0$ and all $i \in \mathcal{L}(\mathcal{G})$, and hence, $\dot{\tilde{x}}(t)$ is bounded. Furthermore, since $\dot{\tilde{x}}_{\rm L}(t)$ and $\dot{\bar{x}}(t)$ are also bounded, then $\ddot{V}^*(\tilde{x}, \tilde{x}_L, \bar{x}, \tilde{W})$ is bounded for all $t > 0$. Now, it follows from Barbalat's lemma [27, Lemma 8.2] that $\lim_{t\to\infty}\dot{V}^*(\tilde{x}, \tilde{x}_L, \bar{x}, \tilde{W}) = 0$, which consequently shows that $\lim_{t\to\infty}\tilde{x}(t) = 0$, $\lim_{t\to\infty}\tilde{x}_{\rm L}(t) = 0$, and $\lim_{t\to\infty}\bar{x}(t) = 0$. Note that $x(t) - x_{\rm rd}(t) = \tilde{x}(t) + \bar{x}(t)$ and, thus, $\lim_{t\to\infty}\big(x(t) - x_{\rm rd}(t)\big) = 0$. Finally, since $\dot{V}^*(\tilde{x}(t), \tilde{x}_L(t), \bar{x}(t), \tilde{W}(t)) \le 0$ for all $t > 0$, this implies that $V^*(\tilde{x}(t), \tilde{x}_L(t), \bar{x}(t), \tilde{W}(t)) \le V^*\big(\tilde{x}_0, 0, 0, \tilde{W}_0\big)$. Using $V^*\big(\tilde{x}_0, 0, 0, \tilde{W}_0\big) \le \epsilon_V$ and

$$\underline{P}\|\tilde{x}(t)\|_2^2 \le V^*(\tilde{x}(t), \tilde{x}_L(t), \bar{x}(t), \tilde{W}(t)), \quad (37)$$

yields $\|\tilde{x}(t)\|_2 \le \sqrt{\epsilon_V/\underline{P}}$, and, hence, $\|\tilde{x}(t)\|_{\mathcal{L}_\infty} \le \sqrt{\epsilon_V/\underline{P}}$. Similarly, using $V^*(\tilde{x}_0, 0, 0, \tilde{W}_0) \le \epsilon_V$ and

$$\frac{2\xi\underline{P}\ \underline{Q}}{\kappa\bar{P}}\|\bar{x}(t)\|_2^2 \quad \le \quad V^*(\tilde{x}(t), \tilde{x}_L(t), \bar{x}(t), \tilde{W}(t)), \quad (38)$$

yields

$$\|\bar{x}(t)\|_{\mathcal{L}_\infty} \le \sqrt{\frac{\kappa\bar{P}}{2\xi\underline{P}\ \underline{Q}}\epsilon_V}. \quad (39)$$

It now follows that

$$\|x(t) - x_{\rm r_i}(t)\|_{\mathcal{L}_\infty} \le \|\tilde{x}(t)\|_{\mathcal{L}_\infty} + \|\bar{x}(t)\|_{\mathcal{L}_\infty}, \quad (40)$$

and hence, (33) is a direct consequence of using (37) and (39) in (40). This completes the proof. ■

Remark 3.4. As shown in Ref. 24, fast and robust adaptation can be achieved with the presented framework since the high-frequency content of the error signal is quickly suppressed. We refer to Ref. 24 for further details and results concerning the frequency-limited adaptive controller.

In order to analyze how the modified state emulator affects the overall stability of the system we apply the state transformation in (13) to (28). As before the transformed system can be partitioned as

$$\begin{aligned} \dot{\hat{z}}_1(t) &= A_1\hat{z}_1(t) + A_2\tilde{x}(t) - \kappa\hat{y}_1(t),\ \hat{z}_1(t_0) = \hat{z}_{10}, \quad (41) \\ \dot{\hat{c}}_\mathcal{G}(t) &= \sum_{i\in\mathcal{V}(\mathcal{G})}\big((\mathrm{d}_i + \kappa)\tilde{x}_i(t) - \kappa\tilde{x}_{\rm Li}(t)\big), \quad (42) \end{aligned}$$

where $\hat{c}_\mathcal{G}(t_0) = \hat{c}_{\mathcal{G}0}$, $\hat{y}_1 = [\tilde{x}_1 - \tilde{x}_{\rm L_1} - \tilde{x}_2 + \tilde{x}_{\rm L_2}, \ldots, \tilde{x}_1 - \tilde{x}_{\rm L_1} - \tilde{x}_n + \tilde{x}_{\rm L_n}]^{\rm T}$. Furthermore, we also partition the estimate of the adaptive weight, $\hat{W}_i$, given by (26) such that

$$\begin{aligned} \dot{\tilde{W}}_{i1}(t) &= -\gamma_i\tilde{x}_i(t)P_i,\ t \ge 0,\ \tilde{W}_{i1}(0) = \tilde{W}_{i10}, \quad (43) \\ \dot{\tilde{W}}_{is}(t) &= -\gamma_i\bar{\sigma}_i(x)\tilde{x}_i(t)P_i,\ t \ge 0,\ \tilde{W}_{is}(0) = \tilde{W}_{is0}, \quad (44) \end{aligned}$$

where $\bar{\sigma}_i = [\sigma_{i1}, \ldots, \sigma_{is_i}]^{\rm T} : \mathbb{R}^n \to \mathbb{R}^{s_i}$. Note that $\hat{W}_{i1}(t)$ is the estimate of the adaptive weight associated with the uncertainty characterized by a bias and $\dot{\tilde{W}}_{is}(t)$ is the estimate of the adaptive weight associated with the uncertainty parameterized by basis functions.

Theorem 3.4. Consider the uncertain network multiagent system given by (23), the partitioned state emulator (41) along with (29) and (30), and the partitioned adaptive weight update law (43) subject to Assumption 3.1. Then there exists $\mu^\star > 0$ such that for all $\kappa, \underline{\gamma} > \mu^\star$, where $\underline{\gamma} = \min_{i\in\mathcal{V}_\mathcal{G}}(\gamma_i)$, the solution $(\hat{z}_1(t), \tilde{x}_L(t), \bar{x}(t), \tilde{W}_1(t))$ is exponentially stable for all $(\hat{z}_{10}, 0, 0, \tilde{W}_{10}) \in \mathbb{R}^{n-1} \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^n$.

Proof. Note that the partitioned adaptive update law (43), the modified and partitioned state emulator (41), the frequency-limited (filtered) emulator state error dynamics (29), and the modified emulator states error dynamics (30) can be represented as a singularly perturbed system given by

$$\begin{aligned} \dot{\hat{z}}_1(t) &= A_1\hat{z}_1(t) + A_2\tilde{x}(t) + \kappa\hat{y}_1 \quad (45) \\ \dot{\tilde{x}}_L(t) &= -\Delta\tilde{x}_L(t) + \eta(\tilde{x}(t) - \tilde{x}_L(t)), \quad (46) \\ \epsilon\dot{\tilde{x}}(t) &= \epsilon(-\Delta\tilde{x}(t) + \tilde{W}(t)^{\rm T}\sigma(x)) - \tilde{x}(t) + \tilde{x}_L(t). \quad (47) \\ \epsilon\dot{\tilde{W}}_1(t) &= -\bar{P}\tilde{x}(t), \quad (48) \end{aligned}$$

where $\tilde{W}_1(t) = [\tilde{W}_{11}(t), \ldots, \tilde{W}_{1n}(t)]^{\rm T}$, $\epsilon = \frac{1}{\kappa} = \frac{1}{\underline{\gamma}}, \bar{P} = \mathrm{diag}([\bar{P}_1 \ldots, \bar{P}_n])$, and $\bar{P}_i = \gamma_i/\underline{\gamma}$. First we note that if $\tilde{W}_1(t) = \hat{z}_1(t) = \tilde{x}_L(t) = \tilde{x}(t) = 0$ then $\hat{y}_1 = 0$ and $\tilde{W}(t)^{\rm T}\sigma(x) = 0$ since $\tilde{W}_{i1} = 0$, $\bar{\sigma}_i = 0$, $\forall i \in \mathcal{V}(\mathcal{G})$ due to Assumption 3.1. Therefore, $\dot{\tilde{W}}_1(t) = \dot{\hat{z}}_1(t) = \dot{\tilde{x}}_L(t) = \dot{\tilde{x}}(t) = 0$. The proof now follows directly from Theorem 2.1. ■

Remark 3.5. Note that as a direct consequence of Theorem

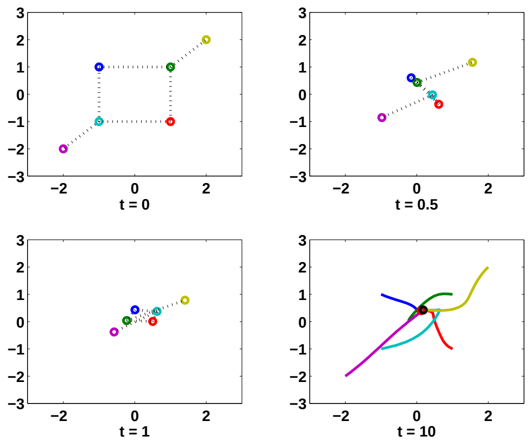

Fig. 1: The system response with interagent uncertainties when the proposed consensus protocol (20) is used. Dotted lines indicate neighbors and solids lines indicate trajectory history.

3.4, $\hat{c}_{\mathcal{G}}(t) = \int_0^t \sum_{i\in\mathcal{V}(\mathcal{G})} \big((\mathrm{d}_i + \kappa)\tilde{x}_i(t) - \kappa\tilde{x}_{\mathrm{L}i}(t)\big)\ \mathrm{d}t < \infty$ since $\tilde{x}(t)$ and $\tilde{x}_{\mathrm{L}}(t)$ can be bounded by a decaying exponential function. Therefore, unlike the standard adaptive control framework in Theorem 3.2, the frequency-limited adaptive control framework can guarantee that the centroid of the system remains bounded.

IV. Illustrative Numerical Example

Consider a networked multiagent system consisting of 6 agents whose neighboring relations are given by the adjacency matrix

$$\mathcal{A}(\mathcal{G}) = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix}. \tag{49}$$

For this example, the nonlinear interagent disturbances are parameterized such that $\sigma_i = [1, g_i(x_1), 1, g_i(x_2)]$ where $g_i(x_k) = (\frac{1}{2}\sum_{i\sim j}(x_{ik} - x_{jk}))^2$ and $k \in \{1, 2\}$. The unknown weight matrices, $W_i \in \mathbb{R}^{2\times 4}$, are given as $W_i = \begin{bmatrix} w_{i1} & w_{i2} & 0 & 0 \\ 0 & 0 & w_{i3} & w_{i4} \end{bmatrix}^{\mathrm{T}}$, where $w_1 = [-0.2, 1.1, 0.3, 0.1]$, $w_2 = [-0.7, 0.5, 0.2, 0.8]$, $w_3 = [0.6, -0.6, 0.1, -0.5]$, $w_4 = [0.2, 0.5, 0.1, -0.4]$, and $w_5 = [0.3, 0.2, 0.3, 0.1]$, $w_6 = [0.7, 0, -0.1, 0.2]$. Figure 1 shows the system response when interagent uncertainties are present and when the consensus protocol in (20), the adaptive weight update law in (26), and the modified state emulator system in (28) are used ($\gamma_i, P_i, \kappa, \eta = 1$). As expected from Theorem 3.4, the proposed framework achieves consensus. ▲

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