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A Kalman Filter Optimization Approach to Direct Adaptive Control

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This paper presents a novel Kalman filter based approach for approximately enforcing a linear constraint in adaptive control. One application is that this leads to an alternative forms for well known modification terms such as e- modification. It is shown that employing this approach does not increase the theoretical guaranteed ultimate bounds for the closed loop error signals of an existing adaptive design. In addition, it leads to smaller tracking errors without incurring significant oscillations in the system response, and without requiring high modification gain that can excite the unmodeled dynamics. Three novel modifications to classical adaptive laws are illustrated and tested on a model of wing rock dynamics.

I. Introduction

Adaptive control is an attractive approach in nonlinear systems theory due to its ability to cope with system uncertainties and failures. Adaptive controllers can be generally classified as either indirect and direct. Indirect adaptive controllers employ an estimation algorithm to estimate the unknown system parameters and adaptive controller gains. Direct adaptive controllers adapt feedback gains in response to system variations without requiring an estimation algorithm. This paper focuses on improving an already existing direct adaptive controller by modifying the adaptive law on the basis of approximately enforcing a linear constraint using a Kalman filter (KF) based algorithm[1−4].

In the literature, adaptation laws that impose constraints on the weights to improve an existing adaptive law are commonly referred as composite adaptation[5]. In general, these modification terms are found by taking the gradient of the enforced linear constraints. However, using a gradient method can result in slow parameter convergence towards a local minimum[6]. In addition, modification terms that are gradient based have a fixed adaptation gain, that often have to be chosen large to obtain satisfactory results, which can interact negatively with unmodeled dynamics, and amplify the effect of sensor noise. Here, our aim is to highlight the efficacy of using a KF optimization method to find the associated modification terms to be employed in an adaptive control setting. This approach is shown to give smaller tracking errors without incurring significant oscillations in the system response, and without requiring high adaptation gain that can excite the unmodeled dynamics. This approach also results in a variable gain that can give better results than a fixed gain based adaptive control law, especially under system uncertainties and failures.

Many modification terms are reported in the literature[7−16]. Included among these, σ- modification[7] adds a pure damping term to the adaptive law, while e- modification[8] adds a variable damping term depends on the training error signal. These terms are introduced to ensure that the adapted weights remain bounded. Background learning[9−11] uses current and past data concurrently in the adaptation process. It allows the adaptation law to continually train in the background based on past data while still being responsive to dynamic changes based on the current data. In this way, background learning incorporates long term learning. Q- modification[12−14] is similar in spirit to background learning in its intent to improve an adaptation

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performance by using a moving window of the integrated system uncertainty. An optimal control theory based modification term is also given in[15] to improve adaptation in the presence of large adaptive gain. More recently, an adaptive loop recovery (ALR) approach[16] has been introduced as a modification term in adaptive control with the objective of recovering the loop transfer properties of a reference model. All of these modification terms can alternatively be viewed as having been introduced with the desire to enforce a linear constraint on the weights in an adaptive control algorithm. With this perspective in mind, the proposed KF optimization method can be used as an alternative to all these listed modification terms to achieve the same objective, but with better conditioning.

In this paper, for illustrative purposes, we develop an improved version of the well known e- modification. We show how the standard e- modification term[7] can be interpreted as the gradient of a linear constraint, and we use this linear constraint to develop a Kalman filter based e- modification. As a second example, we treat the linear constraint imposed by ALR modification[16] using the KF method. The resulting KF-ALR modification is shown to improve upon standard ALR modification. Finally, we propose a solution to the problem of adaptation in the presence of input constraints. These KF-based modifications to standard adaptive laws are illustrated for an adaptive control problem associated with a model of wing rock dynamics.

In what follows, we will use the following notations: $tr[A]$ is the trace of matrix A, $vec[A]$ is an operator that stacks matrix A column by column, $|\cdot|$ denote vector norm, $||\cdot||$ denote matrix Frobenius norm, $\lambda(A)_{min}$ denote the smallest eigenvalue of matrix A, and $\lambda(A)_{max}$ denote the largest eigenvalue of matrix A.

The organization of the paper is as follows. Section II provides a brief background needed for the rest of the paper. Section III describes the main concept for enforcing a linear constraint by employing a KF optimization approach, including a theorem regarding a stability property that can be proven for this approach. The three examples of KF-based modification are given in Section IV. Conclusions are summarized in Section V.

II. Model Reference Adaptive Control

We begin by presenting a formulation of the model reference adaptive control problem. For this purpose, consider the following uncertain system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(u(t) + \Delta(x(t))), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the output, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{p\times n}$ are known matrices, and $\Delta(\cdot) : \mathbb{R}^n \to \mathbb{R}^m$ is the *unknown* matched uncertainty. Furthermore, we assume that $x(t),\ t \geq 0$, is available for feedback. In nonlinear control theory, the matrices $A,\ B,\ C$ in Eq. (1) are usually obtained by linearizing the nonlinear dynamics at selected equilibrium conditions, and the resulting set of linear models are used to design a gain scheduled controller. It is assumed that a baseline controller for the system in Eq. (1) exists for a neighborhood of each equilibrium point, and can be written in the form

$$u_n(t) = -K_x x(t) + K_r r(t), \tag{2}$$

where $r(t) \in \mathbb{R}^m$, is the bounded reference command, $K_x \in \mathbb{R}^{m\times n}$ is the state gain matrix, and $K_r \in \mathbb{R}^{m\times r}$ is the input gain matrix. It should be noted that existing controller designs commonly contain dynamics, in which case one can augment the controller dynamics with the dynamics in Eq. (1), and consider an expanded state made up of the system states and the controller states, and rewrite the dynamics and controller in the form of Eqs. (1) and (2). So there is no loss in generality with respect to dynamic controllers in assuming these forms. However, to further simplify the discussion we introduce the following assumption.

Assumption 1. The uncertainty in Eq. (1) can be linearly parameterized using a set of basis functions in the form

$$\Delta(x(t)) = W^{\mathrm{T}}\beta(x(t)), \tag{3}$$

where $W \in \mathbb{R}^{s\times m}$ is the unknown weight matrix, and $\beta(\cdot) : \mathbb{R}^n \to \mathbb{R}^s$ is the basis functions that are assumed to be known.

Given the above one can construct a reference model for the desired response of the closed loop system

$$\begin{aligned} \dot{x}_m(t) &= A_m x_m(t) + B_m r(t), \\ y_m(t) &= C x_m(t), \end{aligned} \tag{4}$$

where $x_m(t) \in \mathbb{R}^n$, is the desired state vector with $|x_m(t)| \leq \chi_m$, $y_m(t) \in \mathbb{R}^p$, $t \geq 0$, is the desired output, $A_m \in \mathbb{R}^{n\times n}$, and $B_m \in \mathbb{R}^{n\times m}$ are known matrices, and A_m is Hurwitz. Consider the augmenting adaptive controller given by

$$u(t) = u_n(t) - u_a(t), \tag{5}$$
$$u_a(t) = \hat{W}(t)^{\mathrm{T}}\beta(x(t)). \tag{6}$$

Defining $e(t) = x(t) - x_m(t)$, it is well known[5] that the following adaptive law

$$\dot{\hat{W}}(t) = \gamma\beta(x(t))e^{\mathrm{T}}(t)PB, \tag{7}$$

where γ is a positive learning rate, and $P \in \mathbb{R}^{n\times n}$ is a positive definite solution of the Lyapunov equation

$$A_m^{\mathrm{T}}P + PA_m + L = 0, \tag{8}$$

for any $L > 0$, ensures that $\hat{W}(t)$ remains bounded, and that $e(t) \to 0$ as $t \to \infty$. The resulting adaptive control system is illustrated in Figure 1.

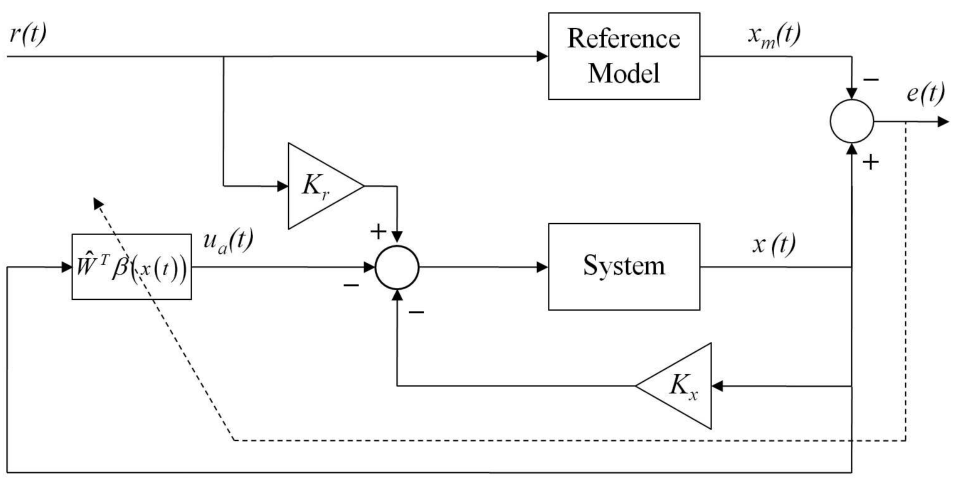

Figure 1. Augmenting adaptive control of a baseline control system.

In the case where there are bounded external disturbances or when *Assumption 1* is relaxed to

$$|\Delta(x(t)) - W^{\mathrm{T}}\beta(x(t))| < |\varepsilon(x(t))| < \epsilon,\ \forall x \in \mathcal{B}_\chi, \tag{9}$$

where $\mathcal{B}_\chi = \{x(t) : |x(t)| \leq \chi\}$ with $\chi = \chi_m + r$ is a constant, and r is defined later. Then, it can be shown that $e(t)$ is uniformly ultimately bounded (UUB)[17]. Furthermore, in this case the adaptive law in Eq. (7) can be modified to guarantee that $\hat{W}(t)$ remains bounded, using either σ- modification[7], e- modification[8], and/or parameter projection[18]. For example with e- modification the adaptation law becomes

$$\dot{\hat{W}}(t) = \gamma(\beta(x(t))e^{\mathrm{T}}(t)PB - \sigma|e(t)|\hat{W}(t)), \tag{10}$$

where σ is a positive learning rate.

Let $\tilde{W}(t) = \hat{W}(t) - W$, and define $\zeta(t) = [e(t)^{\mathrm{T}}\ \mathrm{vec}(\tilde{W}(t))^{\mathrm{T}}]^{\mathrm{T}}$ and $\mathcal{B}_r = \{\zeta(t) : |\zeta(t)| \leq r\}$. Let Ω_a be defined by $\Omega_a = \{\zeta(t) \in \mathcal{B}_r : \zeta(t)^{\mathrm{T}}\tilde{P}\zeta(t) \leq \alpha\}$, where $\tilde{P} = \mathrm{block}-\mathrm{diag}[P\ \gamma^{-1}I]$ and $\alpha = \min_{|\zeta(t)|=r}(\zeta(t)^{\mathrm{T}}\tilde{P}\zeta(t)) = r^2\lambda(\tilde{P})_{min}$. The next theorem highlights the UUB property of Eq. (10).

Theorem 1. If $\zeta(t_0) \in \Omega_\alpha$ and $r^2 > \frac{\lambda(P)_{max}\Theta_e^2 + \frac{1}{\gamma}\Theta_{\tilde{W}}^2}{\lambda(\tilde{P})_{min}}$, then the system defined in Eq. (1), with the control law in Eq. (5), together with the adaptive law in Eq. (10), is UUB.

Proof. Consider the Lyapunov-like function candidate

$$V(e(t), \tilde{W}(t)) = \frac{1}{2}e^{\mathrm{T}}(t)Pe(t) + \frac{1}{2\gamma}\mathrm{tr}[\tilde{W}^{\mathrm{T}}(t)\tilde{W}(t)]. \tag{11}$$

Substitute the adaptive law in Eq. (10) with $|\varepsilon(x(t))| \leq \epsilon$. Then, the derivative of Eq. (11) with respect to time can be expressed as

$$\begin{aligned}\dot{V}(e(t), \tilde{W}(t)) &= e^{\mathrm{T}}(t)P[A_m e(t) - B\tilde{W}^{\mathrm{T}}(t)\beta(x(t)) + B\varepsilon(x(t))] + \frac{1}{\gamma}tr[\tilde{W}(t)\dot{\hat{W}}(t)] \\ &= -\frac{1}{2}e^{\mathrm{T}}(t)Le(t) + e^{\mathrm{T}}(t)PB\varepsilon(x(t)) - \sigma\mathrm{tr}[\tilde{W}^{\mathrm{T}}(t)|e(t)|(W + \tilde{W}(t))].\end{aligned}$$

Let $c_1 = \frac{\lambda(L)_{min}}{2}$, $c_2 = \frac{\sigma}{2}$, and $c_3 = \frac{\sigma}{2}||W||^2 + ||PB||\epsilon$. Then, we have

$$\dot{V}(e(t), \tilde{W}(t)) \leq -|e(t)|\big(c_1|e(t)| + c_2||\tilde{W}||^2 - c_3\big), \tag{12}$$

where $\dot{V}(e(t), \tilde{W}(t))$ is negative as long as the term in braces is positive. Consequently, either $|e(t)| \geq \Theta_e$, or $||\tilde{W}(t)|| \geq \Theta_{\tilde{W}}$ renders $\dot{V}(e(t), \tilde{W}(t)) < 0$, where $\Theta_e = \frac{c_3}{c_1}$, and $\Theta_{\tilde{W}} = \sqrt{\frac{c_2}{c_1}}$. Therefore, $e(t)$ and $\tilde{W}(t)$ are uniformly bounded. Similar to [19], using $e(t)^{\mathrm{T}}Pe(t) \leq \lambda(P)_{max}|e(t)|^2$, where $\lambda(\cdot)_{max}$ denotes the maximum eigenvalue of a matrix, Eq. (12) can be written as

$$\dot{V}(e(t), \tilde{W}(t)) \leq -\frac{\sqrt{e(t)^{\mathrm{T}}Pe(t)}}{\sqrt{\lambda(P)_{max}}}\Big(c_1\frac{\sqrt{e(t)^{\mathrm{T}}Pe(t)}}{\sqrt{\lambda(P)_{max}}} + c_2||\tilde{W}||^2 - c_3\Big),$$

which is strictly negative when $\sqrt{e(t)^{\mathrm{T}}Pe(t)} > \sqrt{\lambda(P)_{max}}\Theta_e$ or $||\tilde{W}(t)|| > \Theta_{\tilde{W}}$. Then, $\Omega_\alpha \subset \mathcal{B}_r$ is a positively invariant set of error dynamics, $\dot{e}(t)$. Furthermore, define $\Omega_\beta = \{\zeta(t) \in \mathcal{B}_r : \zeta(t)^{\mathrm{T}}\tilde{P}\zeta(t) \leq \lambda(P)_{max}\Theta_e^2 + \frac{1}{\gamma}\Theta_{\tilde{W}}^2\}$. If $\Omega_\beta \subset \Omega_\alpha$, this requires that $\lambda(P)_{max}\Theta_e^2 + \frac{1}{\gamma}\Theta_{\tilde{W}}^2 < \alpha$. Then, the minimum size of $\mathcal{B}_r$ can be quantified by $r^2 > \frac{\lambda(P)_{max}\Theta_e^2 + \frac{1}{\gamma}\Theta_{\tilde{W}}^2}{\lambda(\tilde{P})_{min}}$. Therefore, if $\zeta(t_0) \in \Omega_\alpha$, then the closed loop system trajectories are UUB. □

Remark 1. In the case when the uncertainty is parameterized exactly, we don't require to include e-modification term. Hence, $c_2 = 0$ and $c_3 = 0$ in Eq. (12), and by LaSalle-Yoshizawa theorem[20], $e(t) \to 0$ as $t \to \infty$.

In the next section, we introduce a novel treatment for enforcing linear constraints.

III. Enforcing a Linear Constraint by KF Optimization

The idea behind our approach is to approximately enforce a linear constraint among the weights in an adaptive control design using a KF approach, such that the resulting modification term leads to improved tracking performance, at lower adaptation gains. To simplify the presentation, we restrict the form of the constraint of interest.

Assumption 2. The constraint on the ideal weight matrix in an adaptive control design has the linear form

$$W^{\mathrm{T}}\phi(t,x(t),u(t)) = 0, \tag{13}$$

where $W \in \mathbb{R}^{s\times m}$ is an unknown weight matrix, and $\phi(\cdot) : [0,\infty) \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{s\times l}$ is a known regressor vector for $l = 1$ or regressor matrix for $l > 1$.

The problem of estimating W while enforcing the linear constraint in Eq. (13) can be viewed as a standard problem in estimation theory by defining the stochastic process

$$\begin{aligned} \dot{W} &= D_{w1}q(t), \\ y(t) &= \phi(t,x(t),u(t))^{\mathrm{T}}W + D_{w2}q(t), \end{aligned} \tag{14}$$

where $q(t)$ is a zero-mean, Gaussian, white noise process, and it is assumed that $D_{w1}^{\mathrm{T}}D_{w2} = 0$. Note that $\phi(t,x(t),u(t))^{\mathrm{T}}W = 0$ by Eq. (13). One can construct an estimator for W as

$$\begin{aligned} \dot{\hat{W}}(t) &= A_e\hat{W}(t) + B_e y(t), \\ \hat{y}(t) &= \hat{W}(t). \end{aligned} \tag{15}$$

Now, let $z(t)$ be the performance variable given by

$$z(t) = E(W - \hat{W}(t)) = Ee_w(t), \tag{16}$$

where E is a weighting matrix and $e_w(t) = W - \hat{W}(t)$. The goal is to keep $z(t)$ small under $q(t)$. To satisfy this goal let

$$\begin{aligned} \dot{e}_w &= -B_e\phi(t,x(t),u(t))^{\mathrm{T}}W - A_e\hat{W}(t) + (D_{w1} - B_eD_{w2})q(t) \\ &= -B_e\phi(t,x(t),u(t))^{\mathrm{T}}e_w + (D_{w1} - B_eD_{w2})q(t), \end{aligned} \tag{17}$$

where $A_e = -B_e\phi(t,x(t),u(t))^{\mathrm{T}}$. Define the error system $\tilde{\mathcal{G}}$ from $q(t)$ to $z(t)$ as

$$\tilde{\mathcal{G}} \sim \left[\begin{array}{c|c} -B_e\phi(t,x(t),u(t))^{\mathrm{T}} & D_{w1} - B_eD_{w2} \\ \hline E & 0 \end{array}\right] = \left[\begin{array}{c|c} \tilde{A} & \tilde{D} \\ \hline E & 0 \end{array}\right], \tag{18}$$

where it is an equivalent representation of the transfer function $\tilde{\mathcal{G}}(s) = E(sI - \tilde{A})^{-1}\tilde{D}$ with s is the Laplace variable. Now, one needs to determine Kalman gain B_e by minimizing the $\mathcal{L}_2$ norm of $\tilde{G}$, i.e.,

$$J(B_e) = \|\tilde{G}\|_2^2 = tr[ES(t)E^{\mathrm{T}}] = tr[S(t)\Omega] \tag{19}$$

or

$$J(B_e) = \|\tilde{G}\|_2^2 = tr[\tilde{D}^{\mathrm{T}}\bar{S}(t)\tilde{D}] = tr[\bar{\Omega}\bar{S}(t)],$$

subject to $\tilde{A}$ is asymptotically stable, where $\Omega = E^{\mathrm{T}}E$, $\bar{\Omega} = \tilde{D}\tilde{D}^{\mathrm{T}}$, and $S(t)$ satisfies

$$\dot{S}(t) = \tilde{A}S(t) + S(t)\tilde{A}^{\mathrm{T}} + \bar{\Omega}, \tag{20}$$

and $\bar{S}(t)$ satisfies

$$\dot{\bar{S}}(t) = \tilde{A}^{\mathrm{T}}\bar{S}(t) + \bar{S}(t)\tilde{A} + \Omega.$$

To optimize Eq. (19) subject to the constraint in Eq. (20), form the Lagrangian [21]

$$\mathcal{L}(B_e, \mu(t)) \quad = \quad tr[S(t)\Omega] + \mu(t)^{\mathrm{T}} vec\ [\tilde{A}S(t) + S(t)\tilde{A}^{\mathrm{T}} + \bar{\Omega} - \dot{S}(t)],$$

and let $\mu(t) = vec[\bar{S}(t)^{\mathrm{T}}]$. Then, $\mathcal{L}(B_e, \mu(t))$ can be rearranged as

$$\begin{aligned}
\mathcal{L}(B_e, \mu(t)) &= tr[S(t)\Omega] + [vec\ \bar{S}(t)^{\mathrm{T}}]^{\mathrm{T}} vec\ [\tilde{A}S(t) + S(t)\tilde{A}^{\mathrm{T}} + \bar{\Omega} - \dot{S}(t)] \\
&= tr[S(t)\Omega] + tr[\bar{S}(t)(\tilde{A}S(t) + S(t)\tilde{A}^{\mathrm{T}} + \bar{\Omega} - \dot{S}(t))], \qquad (21)
\end{aligned}$$

by the fact $tr(AB) = (vec\ A^{\mathrm{T}})^{\mathrm{T}}(vec\ B)$ for some A and B. Now, it is trivial to find the covariance update law and the Kalman gain from

$$\frac{\partial \mathcal{L}(B_e, \mu(t))}{\partial \bar{S}(t)} = 0 \quad \Rightarrow \quad \dot{S}(t) = \tilde{A}S(t) + S(t)\tilde{A}^{\mathrm{T}} + \bar{\Omega},$$
$$\frac{\partial \mathcal{L}(B_e, \mu(t))}{\partial B_e} = 0 \quad \Rightarrow \quad B_e = S(t)\phi(t, x(t), u(t))(D_{w2}D_{w2}^{\mathrm{T}})^{-1}.$$

The normalized Kalman gain can be further given as [22]

$$B_e = S(t)\phi(t, x(t), u(t))R(t)^{-1},$$

where $R_0 = D_{w2}D_{w2}^{\mathrm{T}} > 0$ is replaced with $R(t) = R_0 + \phi(t, x(t), u(t))^{\mathrm{T}} S(t)\phi(t, x(t), u(t)) > 0$. Because, in this case $\phi(t, x(t), u(t))R(t)^{-1} \in \mathcal{L}_\infty$. Hence, we have

$$\begin{aligned}
\dot{\hat{W}}(t) &= B_e\phi(t, x(t), u(t))^{\mathrm{T}}\hat{W}(t) \\
&= -S(t)\phi(t, x(t), u(t))R(t)^{-1}\phi(t, x(t), u(t))^{\mathrm{T}}\hat{W}(t), \qquad (22)
\end{aligned}$$

by *Assumption 2*, and

$$\dot{S}(t) \quad = \quad -S(t)\phi(t, x(t), u(t))R^{-1}(t)\phi(t, x(t), u(t))^{\mathrm{T}}S(t) + Q, \qquad (23)$$

where $Q = D_{w1}^{\mathrm{T}}D_{w1} > 0$.

Remark 2. The solution of Eq. (23), $S(t)$, is symmetric, positive definite, and uniformly bounded (see *Theorem A.1* and *Theorem A.2* in Appendix).

Now, using Eq. (22) as a modification term in the adaptive law given in Eq. (10), we arrive at

$$\dot{\hat{W}}(t) = \gamma(\beta(x(t))e^{\mathrm{T}}(t)PB - \sigma|e(t)|\hat{W}(t) - kS(t)\phi(t, x(t), u(t))R^{-1}(t)\phi(t, x(t), u(t))^{\mathrm{T}}\hat{W}(t)), \qquad (24)$$

where k is a positive modification gain, and $kS(t)$ can be interpreted as its variable gain. The next theorem highlights that enforcing a linear constraint by using the KF optimization approach does not increase the theoretical guaranteed ultimate bounds given in *Theorem 1.*

Theorem 2. If $\zeta(t_0) \in \Omega_{\hat{\alpha}} = \{\zeta(t) \in \mathcal{B}_{\hat{r}} : \zeta(t)^{\mathrm{T}}\tilde{P}\zeta(t) \leq \hat{\alpha}\}$ where $\mathcal{B}_{\hat{r}} = \{\zeta(t) : |\zeta(t)| \leq \hat{r}\}$ and $\hat{\alpha} = \min_{|\zeta(t)|=\hat{r}}(\zeta(t)^{\mathrm{T}}\tilde{P}\zeta(t)) = \hat{r}^2\lambda(\tilde{P})_{min}$, and $\hat{r}^2 > \frac{\lambda(P)_{max}\Theta_e^2 + \frac{1}{\gamma}\hat{\Theta}_{\tilde{W}}^2}{\lambda(\tilde{P})_{min}}$, then the system defined in Eq. (1), with the control law in Eq. (5), and with the adaptive law in Eq. (24).

Proof. Consider the Lyapunov-like function candidate in Eq. (11), and substitute the adaptive law in

Eq. (24) with $|\varepsilon(x(t))| \leq \epsilon$. Then, the derivative of Eq. (11) with respect to time can be expressed as

$$\begin{aligned}\dot{V}(e(t), \tilde{W}(t)) &= -\frac{1}{2}e^{\mathrm{T}}(t)Le(t) + e^{\mathrm{T}}(t)PB\varepsilon(x(t)) - \sigma\mathrm{tr}[\tilde{W}^{\mathrm{T}}(t)|e(t)|(W+\tilde{W}(t))] \\ &\quad -k\mathrm{tr}[\tilde{W}^{\mathrm{T}}(t)S(t)\phi(t,x(t),u(t))R(t)^{-1}\phi(t,x(t),u(t))^{\mathrm{T}}(W+\tilde{W}(t))],\end{aligned}$$

where the above equation can be further arranged by using *Assumption 2* and *Lemma A.1* in Appendix as

$$\begin{aligned}\dot{V}(e(t), \tilde{W}(t)) &= -\frac{1}{2}e^{\mathrm{T}}(t)Le(t) + e^{\mathrm{T}}(t)PB\varepsilon(x(t)) - \sigma\mathrm{tr}[\tilde{W}^{\mathrm{T}}(t)|e(t)|(W+\tilde{W}(t))] \\ &\quad -k\mathrm{tr}[S(t)\phi(t,x(t),u(t))R(t)^{-1}\phi(t,x(t),u(t))^{\mathrm{T}}\tilde{W}(t)\tilde{W}^{\mathrm{T}}(t)] \\ &\leq -\frac{1}{2}e^{\mathrm{T}}(t)Le(t) + e^{\mathrm{T}}(t)PB\varepsilon(x(t)) - \sigma\mathrm{tr}[\tilde{W}^{\mathrm{T}}(t)|e(t)|(W+\tilde{W}(t))] - \lambda_\eta\mathrm{tr}[\tilde{W}(t)\tilde{W}^{\mathrm{T}}(t)],\end{aligned}$$

where $\lambda_\eta = k\lambda\Big(\frac{S(t)\phi(t,x(t),u(t))R(t)^{-1}\phi(t,x(t),u(t))^{\mathrm{T}} + (S(t)\phi(t,x(t),u(t))R(t)^{-1}\phi(t,x(t),u(t))^{\mathrm{T}})^{\mathrm{T}}}{2}\Big)_{min}$ is nonnegative definite as a result of *Remark 2*. Let $c_1 = \frac{\lambda(L)_{min}}{2}$, $\hat{c}_2 = \frac{\sigma}{2} + \frac{\lambda_\eta}{|e(t)|}$, and $c_3 = \frac{\sigma}{2}||W||^2 + ||PB||\epsilon$. Then, we have

$$\dot{V}(e(t), \tilde{W}(t)) \leq -|e(t)|\big(c_1|e(t)| + \hat{c}_2||\tilde{W}||^2 - c_3\big), \tag{25}$$

where $\dot{V}(e(t), \tilde{W}(t))$ is negative as long as the term in braces is positive. Consequently, either $|e(t)| \geq \Theta_e$, or $||\tilde{W}(t)|| \geq \hat{\Theta}_{\tilde{W}}$ renders $\dot{V}(e(t), \tilde{W}(t)) < 0$, where $\Theta_e = \frac{c_3}{c_1}$, and $\hat{\Theta}_{\tilde{W}} = \sqrt{\frac{\hat{c}_2}{c_1}}$. It follows from a similar analysis as given in the proof of *Theorem 1* to show that $e(t)$ and $\tilde{W}(t)$ are UUB. □

Remark 3. One can conclude from this result that imposing a linear constraint in an adaptive law using the proposed KF optimization approach preserves the same boundedness property of the adaptive law without the constraint imposed when λ_η in Eq. (25) becomes zero. In other words, when λ_η in Eq. (25) is not zero, i.e., when it is positive definite, then this implies an improvement of the results of *Theorem 1*.

IV. Application to a Model of Wing Rock Dynamics

In this section we first show how the KF approach for enforcing a linear constraint can be used to derive an alternative for the standard e- modification term in adaptive control. Next we illustrate an alternative way to implement ALR modification[16]. As a third example, we introduce a novel solution to the problem of input constraints imposed by an actuator, and compare it with the method of hedging[23,24]. All of the examples are treated using an adaptive control approach to the problem of stabilizing uncertain wing rock dynamics[25].

A. A Novel Alternative to e- Modification Architecture

In Section III, we introduce a KF modification term to an adaptive control law. In this subsection, we will replace standard e-modification term with KF e-modification term. *Corollary A.1* in Appendix shows that this replacement is valid. Specifically, the standard e- modification term in Eq. (10) can be viewed as having been introduced with the desire to enforce the following linear constraint

$$|e(t)|^{\frac{1}{2}}W = 0, \tag{26}$$

by adding a component to $\dot{\hat{W}}(t)$ along the gradient of $\||e(t)|^{\frac{1}{2}}\hat{W}(t)\|_F^2$ with respect to $\hat{W}(t)$. The constraint in Eq. (26) satisfies *Assumption 2*. Applying Eqs. (22) and (23), we propose the following KF based e-modification term

$$\dot{\hat{W}}(t) = -|e(t)|S(t)R^{-1}(t)\hat{W}(t), \tag{27}$$

$$\dot{S}(t) = -|e(t)|S(t)R^{-1}(t)S(t) + Q. \tag{28}$$

The conventional adaptation law in Eq. (7) becomes

$$\begin{aligned}\dot{\hat{W}}(t) &= \gamma(\beta(x(t))e^{\mathrm{T}}(t)PB - \sigma_{kf}|e(t)|S(t)R^{-1}(t)\hat{W}), \\ \dot{S}(t) &= -|e(t)|S(t)R^{-1}(t)S(t) + Q,\end{aligned} \tag{29}$$

where σ_{kf} is a positive learning rate. We compare the standard e- modification based adaptation law in Eq. (10) with the new e- modification based adaptation law in Eq. (29) on a model of wing rock dynamics to illustrate the advantages of the proposed KF approach to constraint enforcement.

For this purpose, consider the following dynamics

$$\begin{bmatrix}\dot{x}_1(t) \\ \dot{x}_2(t)\end{bmatrix} = \begin{bmatrix}0 & 1 \\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t) \\ x_2(t)\end{bmatrix} + \begin{bmatrix}0 \\ 1\end{bmatrix}\big[u(t) + \Psi(x(t))\big], \tag{30}$$

where $\Psi(x(t)) = c_0 + c_1x_1(t) + c_2x_2(t) + c_3|x_1(t)|x_2(t) + c_4|x_2(t)|x_2(t) + c_5x_1^3(t)$ with $c_0 = 0$, $c_1 = 0.2314$, $c_2 = 0.6918$, $c_3 = -0.6245$, $c_4 = 0.0095$, and $c_5 = 0.0214$ (see[12,25] for details). Note that we used the formulation in Eq. (3) to estimate $\Psi(x(t))$, and we used sigmoid type basis functions. In Eq. (30), $x_1(t)$ represents the roll angle, and $x_2(t)$ represents the roll rate. In this example, the control objective is to minimize the oscillations of the wing rock dynamics in order to stabilize the system at the zero trim condition. Therefore, the roll command is considered to be zero. We selected the initial state values as $x(0) = [6^o, 3^o/s]^{\mathrm{T}}$ and $L = I$ for both Eq. (10) and Eq. (29), and $Q = I$ and $R_0 = 10^{-3}I$ for the KF design procedure for Eq. (29). The reference model is selected to be second order with a natural frequency of $0.5rad/s$, and a damping of 0.707. Figures 2-6 present the results.

Figure 2 shows the baseline control response without uncertainty and with uncertainty. This figure shows that the uncertainty significantly degrades system performance. Figures 3-6 compares the standard e-modification based adaptive control law with the KF based e- modification adaptive control law for different gains. It is obvious from these figures that the achieved system performance is good only when we employ the KF based e- modification. The standard e- modification based adaptive control law does not produce a reasonable result for any of these cases. In addition, the KF based e- modification requires significantly less control effort. Note that the gain in the KF approach to e- modification is now time varying due to the presence of $S(t)$.

B. A Novel Alternative to ALR Modification Architecture

In Ref.[16], an ALR modification term is proposed for use in adaptive control to preserve the stability margins of the reference model defined in Eq. (4). This is done by approximately imposing a linear constraint on the weights so that the loop properties of the reference model are asymptotically recovered as the gain on the modification term is increased. In this subsection, the proposed KF optimization approach is applied to the linear ALR constraint $W^{\mathrm{T}}\beta_x(x(t)) = 0$ as a second example, where $\beta_x(x(t)) \triangleq \frac{d\beta(x(t))}{dx(t)} \in \mathbb{R}^{s\times n}$ is the derivative of the basis function with respect to $x(t) \in \mathbb{R}^n$. The standard ALR modification term[16] is found by taking the negative gradient of the following quadratic function with respect to weights, W, as

$$\nu = \frac{1}{2}\mathrm{tr}[(W^{\mathrm{T}}\beta_x(x(t)))^{\mathrm{T}}(W^{\mathrm{T}}\beta_x(x(t)))], \tag{31}$$

$$\frac{\partial\nu}{\partial W} = \beta_x(x(t))\beta_x^{\mathrm{T}}(x(t))W. \tag{32}$$

Combining the modification term in Eq. (32) with the adaptive law in Eq. (10) becomes

$$\dot{\hat{W}}(t) = \gamma(\beta(x(t))e^{\mathrm{T}}(t)PB - \sigma|e(t)|\hat{W}(t) - k_{grad}\beta_x(x(t))\beta_x^{\mathrm{T}}(x(t))\hat{W}(t)), \tag{33}$$

where k_{grad} is a positive learning rate of this gradient derivation of ALR term. Since $\phi(t, x(t), u(t)) = \beta_x(x(t))$ for the ALR problem, applying Eqs. (22) and (23) leads to the following adaptive law

$$\begin{aligned} \dot{\hat{W}}(t) &= \gamma(\beta(x(t))e^{\mathrm{T}}(t)PB - \sigma|e(t)|\hat{W}(t) - k_{kf}S(t)\beta_x(x(t))R^{-1}(t)\beta_x^{\mathrm{T}}(x(t))\hat{W}(t)), \\ \dot{S}(t) &= -S(t)\beta_x(x(t))R^{-1}\beta_x(x(t))^{\mathrm{T}}S(t) + Q, \end{aligned} \tag{34}$$

where k_{kf} is a positive learning rate of this KF based derivation of ALR term, and $S(t)$ can be found from Eq. (23).

We compare the performance of the adaptive law in Eq. (33) with the adaptive law in Eq. (34), using the same wing rock dynamics as presented in the previous subsection. The design parameters are $\sigma = 0.001$, $L = I$, $Q = I$, and $R_0 = I$. Figures 7-9 show the standard ALR adaptive control responses using Eq. (33) with several different gains. It is clear from these figures that the system response is reasonable only for the highest gains case when $\gamma = 100$, $k_{grad} = 100$ as depicted in Figure 9. Figures 10-12 show the KF based ALR (KF-ALR) adaptive control responses using Eq. (34) for the same gain settings. It is obvious from these figures that for all cases the adaptive controller is able to suppress the uncertainty successfully. However, employing KF optimization in the ALR approach stabilizes the system without incurring oscillations in the system response, and without requiring high adaptation gain.

C. A New u_e- Modification Architecture for Input Constraints

A new treatment of actuator dynamics and limits is proposed using the KF based optimization procedure. Note that, we present the following results for illustration purposes without proof. For this purpose, let $u(t)$ be the desired control applied to a nonlinear uncertain system through an actuator, and let $u_s(t)$ be the output of the actuator. Defining $u_e(t) = u_s(t) - u(t)$, our aim is to apply the following linear constraint

$$|u_e(t)|^{\frac{1}{2}}W = 0, \tag{35}$$

to the adaptive law given by

$$\dot{\hat{W}}(t) = \gamma(\beta(x(t))e^{\mathrm{T}}(t)PB - \sigma\hat{W}(t)), \tag{36}$$

where σ is a positive learning rate. Applying Eqs. (22) and (23), we get

$$\dot{\hat{W}}(t) = -|u_e(t)|S(t)R^{-1}(t)\hat{W}(t), \tag{37}$$

$$\dot{S}(t) = -|u_e(t)|S(t)R^{-1}(t)S(t) + Q, \tag{38}$$

where the adaptive law in Eq. (36) with u_e modification term in Eq. (37) becomes

$$\begin{aligned} \dot{\hat{W}}(t) &= \gamma(\beta(x(t))e^{\mathrm{T}}(t)PB - \sigma\hat{W}(t) - \sigma_{u_e}|u_e(t)|S(t)R^{-1}(t)\hat{W}(t)), \\ \dot{S}(t) &= -|u_e(t)|S(t)R^{-1}(t)S(t) + Q, \end{aligned} \tag{39}$$

where σ_{u_e} is a positive learning rate. This framework can also deal with time delay. The main assumption is that the actuator output is known or can be estimated to sufficient accuracy.

We compare u_e- modification with hedging[23,24], that uses the adaptive law in Eq. (36) and modifies the reference model in Eq. (4) with the hedge signal. The same wing rock adaptive control problem is used, with the design parameters $\gamma = 1$, $\sigma = 3$, $\sigma_{u_e} = 50$, $Q = I$ and $R_0 = I$. Furthermore, the following actuator dynamics are included

$$\dot{u}_s(t) = -\tau u_s(t) + \tau u(t), \tag{40}$$

where $\tau = 1/3$.

Figure 13 shows that the presence of actuator dynamics significantly degrades performance when using the adaptive law in Eq. (36). In Figures 14 and 15, hedging was used to modify the reference model dynamics. Figure 14 shows that the response with hedging becomes less oscillatory. However, Figure 15 shows that hedging modifies the reference model into a response that is more oscillatory. Figure 16 shows the response when u_e- modification is employed. The response is improved without modifying the reference model. Figure 17 compares the results of hedging and u_e- modification for the case where an amplitude limit of ± 0.25 is applied. Both methods improve the system performance significantly when subjected to a hard limit, however u_e- modification achieves this without changing the reference model. Finally, Figure 18 shows the performance of the adaptive controller with u_e- modification, for the case where the actuator dynamics include an input time delay of 0.1 seconds. The system response is unstable without u_e- modification, and this response is significantly improved when u_e- modification term is employed.

V. Conclusion

The intent of this paper has been to present a procedure that more effectively enforces linear constraints to modify an existing adaptive control design by using a Kalman filter optimization approach. Employing this approach does not increase the ultimate bounds guaranteed by existing modification terms. Furthermore, stability analysis shows that the Kalman filter based modification term adds a non-positive term to the time derivative of a Lyapunov function. The results using a model of wing rock dynamics illustrates the presented theory and shows significant improvement over other gradient based modification approaches. One key difference in the Kalman filter based approach to modification is that the resulting gain on the modification term is time varying. The proposed approach to deriving a modification term for an adaptive controller can be used in place of all modification terms that can be equivalently viewed as the gradient of a linear constraint on the adaptation gains.

Appendix

In this section, we present necessary results that were used throughout the paper. We first state the following lemma that is used in the proof of *Theorem 2*, *Theorem A.2*, and *Corollary A.1.*

Lemma A.1. For any square matrix A, and for any square nonnegative definite matrix B,

$$\lambda\left(\frac{A+A^{\mathrm{T}}}{2}\right)_{min} trB \ \leq \ trAB \ \leq \ \lambda\left(\frac{A+A^{\mathrm{T}}}{2}\right)_{max} trB$$

holds.

Proof. See *Lemma 2.1* in [26] . □

Now, the following result presents the necessary condition for the solution of Eq. (23), $S(t)$, to be positive definite.

Theorem A.1. The solution of Eq. (23), $S(t)$, exists, and is symmetric and nonnegative definite for all $t \geq 0$. In addition, if $S(\tau)$ is positive definite for some $\tau \geq 0$, then $S(t)$ is positive definite for all $t > \tau$, or if Q is positive definite, then $S(t)$ is positive definite for all $t \geq 0$.

Proof. See *Proposition 1.1* in [27] . □

Since Q is defined to be a symmetric and positive definite matrix in Eq. (23), then the solution of Eq. (23), $S(t)$, is positive definite for all $t \geq 0$ by *Theorem A.1.* Next, we present the boundedness property for the solution of Eq. (23), $S(t)$.

Theorem A.2. The solution of Eq. (23), $S(t)$, is uniformly bounded.

Proof. Since $S(t)$ is positive definite for all $t \geq 0$ by *Theorem A.1*, then consider the Lyapunov-like function candidate

$$V(S(t)) = tr[S(t)]$$

where its time derivative can be expressed as

$$\begin{aligned} \dot{V}(S(t)) &= -tr[S(t)\phi(t,x(t),u(t))[R_0 + \phi(t,x(t),u(t))^{\mathrm{T}} S(t)\phi(t,x(t),u(t))]^{-1}\phi(t,x(t),u(t))^{\mathrm{T}} S(t) + Q] \\ &= -tr[S(t)\phi(t,x(t),u(t))R_0^{-1}\phi(t,x(t),u(t))^{\mathrm{T}} S(t)] - tr[S(t)] + tr[Q] \\ &= -tr[\phi(t,x(t),u(t))R_0^{-1}\phi(t,x(t),u(t))^{\mathrm{T}} S^2(t)] - tr[S(t)] + tr[Q] \end{aligned}$$

Now, it follows from *Lemma A.1* that

$$\begin{aligned} \dot{V}(S(t)) &\leq -\lambda\big(\phi(t,x(t),u(t))R_0^{-1}\phi(t,x(t),u(t))^{\mathrm{T}}\big)_{min} ||S(t)||^2 - ||S(t)|| + \lambda\big(Q\big)_{max} \\ &= -\Big[\lambda\big(\phi(\cdot)R_0^{-1}\phi(\cdot)^{\mathrm{T}}\big)_{min}^{0.5} ||S(t)|| + \lambda\big(\phi(\cdot)R_0^{-1}\phi(\cdot)^{\mathrm{T}}\big)_{min}^{-0.5}\Big]^2 \\ &\quad + \Big[\lambda\big(Q\big)_{max} + \lambda\big(\phi(\cdot)R_0^{-1}\phi(\cdot)^{\mathrm{T}}\big)_{min}^{-1}\Big] \end{aligned}$$

Let $d_1 = \lambda\big(\phi(t,x(t),u(t))R_0^{-1}\phi(t,x(t),u(t))^{\mathrm{T}}\big)_{min}$ and $d_2 = \lambda\big(Q\big)_{max}$, and $\Theta_S = \frac{\sqrt{d_1^{-2} + d_2} - d_1^{-1}}{\sqrt{d_1}}$. Then, $||S(t)|| > \Theta_S$ renders $\dot{V}(S(t)) < 0$. Hence, $S(t)$ is uniformly bounded. □

Notice that Θ_S can be made arbitrarily small by selecting Q such that $\lambda\big(Q\big)_{max}$ is sufficiently small. Finally, the next corollary results from *Theorem 2* that assures that KF e-modification can be replaced with standard e-modification.

Corollary A.1. If $\zeta(t_0) \in \Omega_{\bar{\alpha}} = \{\zeta(t) \in \mathcal{B}_{\bar{r}} : \zeta(t)^{\mathrm{T}}\tilde{P}\zeta(t) \leq \bar{\alpha}\}$ where $\mathcal{B}_{\bar{r}} = \{\zeta(t) : |\zeta(t)| \leq \bar{r}\}$ and $\bar{\alpha} = \min_{|\zeta(t)|=\bar{r}} (\zeta(t)^{\mathrm{T}}\tilde{P}\zeta(t)) = \bar{r}^2\lambda(\tilde{P})_{min}$, and $\bar{r}^2 > \frac{\lambda(P)_{max}\bar{\Theta}_e^2 + \frac{1}{\gamma}\bar{\Theta}_{\tilde{W}}^2}{\lambda(\tilde{P})_{min}}$, then the system defined in Eq. (1), with the control law in Eq. (5), together with the adaptive law in Eq. (29), is UUB.

Proof. Consider the Lyapunov-like function candidate in Eq. (11), and substitute the adaptive law in Eq. (29) with $|\varepsilon(x(t))| \leq \epsilon$. Then, the derivative of Eq. (11) with respect to time can be expressed as

$$\begin{aligned} \dot{V}(e(t),\tilde{W}(t)) &= -\frac{1}{2}e^{\mathrm{T}}(t)Le(t) + e^{\mathrm{T}}(t)PB\varepsilon(x(t)) - \sigma_{kf} tr[\tilde{W}^{\mathrm{T}}|e(t)|S(t)R(t)^{-1}\hat{W}(t)] \\ &= -\frac{1}{2}e^{\mathrm{T}}(t)Le(t) + e^{\mathrm{T}}(t)PB\varepsilon(x(t)) - \sigma_{kf}|e(t)|tr[S(t)R(t)^{-1}\tilde{W}(t)\tilde{W}^{\mathrm{T}}(t)] \\ &\quad -\sigma_{kf} tr[S(t)R(t)^{-1}|e(t)|^{\frac{1}{2}}(|e(t)|^{\frac{1}{2}}W)\tilde{W}^{\mathrm{T}}(t)] \\ &= -\frac{1}{2}e^{\mathrm{T}}(t)Le(t) + e^{\mathrm{T}}(t)PB\varepsilon(x(t)) - \sigma_{kf}|e(t)|tr[S(t)R(t)^{-1}\tilde{W}(t)\tilde{W}^{\mathrm{T}}(t)] \end{aligned}$$

by the constraint in Eq. (26). Now, it follows from *Lemma A.1* that

$$\dot{V}(e(t),\tilde{W}(t)) \leq -|e(t)|\Big(\frac{\lambda(L)_{min}}{2}|e(t)| + \frac{\lambda_\eta}{2}\|\tilde{W}(t)\|^2 - \|PB\|\epsilon\Big)$$

where $\lambda_\eta = \sigma_{kf}\lambda\Big(\frac{S(t)R(t)^{-1} + (S(t)R(t)^{-1})^{\mathrm{T}}}{2}\Big)_{min}$ is positive definite as a result of *Theorem A.1*, and $\dot{V}(e(t),\tilde{W}(t))$ is negative as long as the term in the braces is positive. Let $c_1 = \frac{\lambda(L)_{min}}{2}$, $\bar{c}_2 = \frac{\lambda_\eta}{2}$, $\bar{c}_3 = \|PB\|\epsilon$, $\bar{\Theta}_e = \frac{\bar{c}_3}{c_1}$, and $\bar{\Theta}_{\tilde{W}} = \sqrt{\frac{\bar{c}_2}{c_1}}$. Consequently, either $|e(t)| \geq \bar{\Theta}_e$, or $\|\tilde{W}(t)\| \geq \bar{\Theta}_{\tilde{W}}$ renders $\dot{V}(e(t),\tilde{W}(t)) < 0$. It follows from a similar analysis as given in the proof of *Theorem 1* to show that $e(t)$ and $\tilde{W}(t)$ are UUB. □

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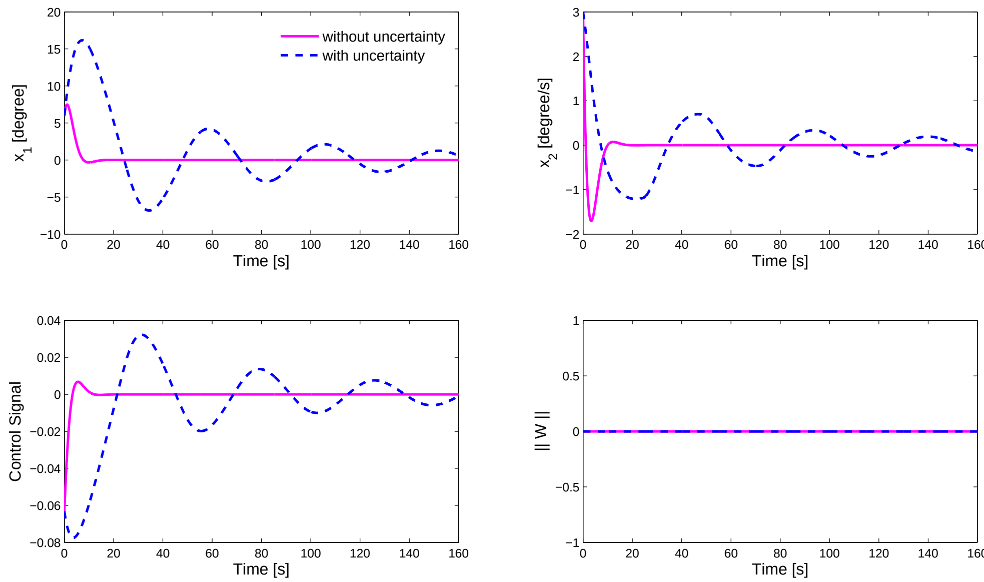

Figure 2. Baseline control response without and with uncertainty.

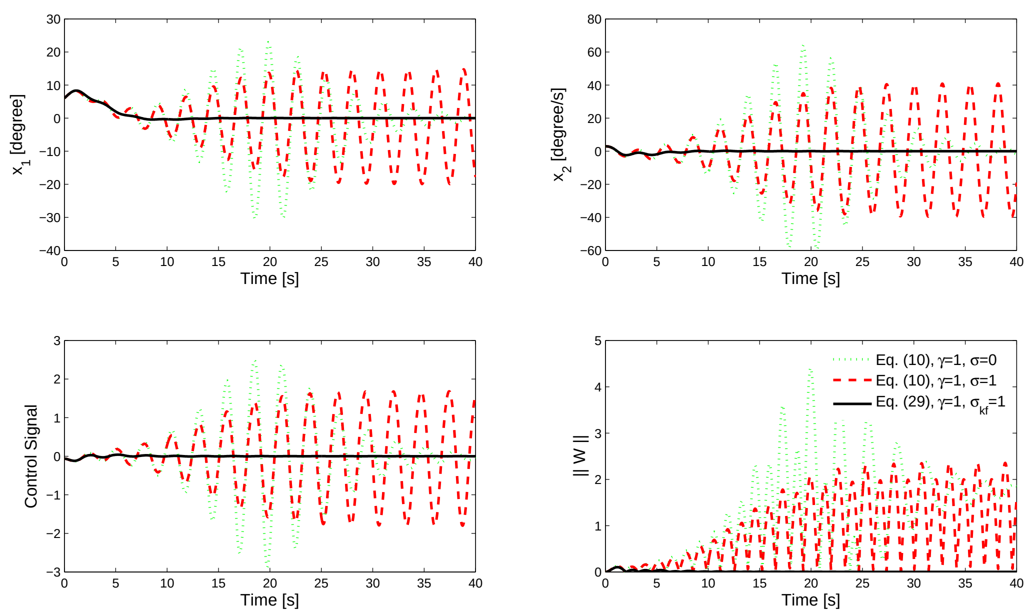

Figure 3. Comparison of adaptive controller responses without modification, with e-modification, and with KF based e- modification with all gains set to 1.

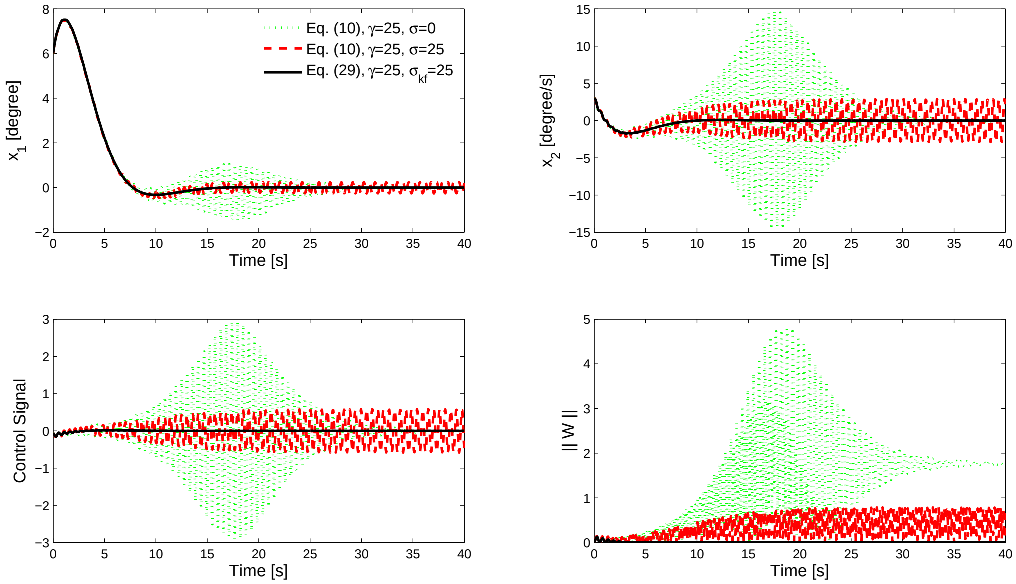

Figure 4. Comparison of adaptive controller responses without modification, with e-modification, and with KF based *e*- modification with all gains set to 25.

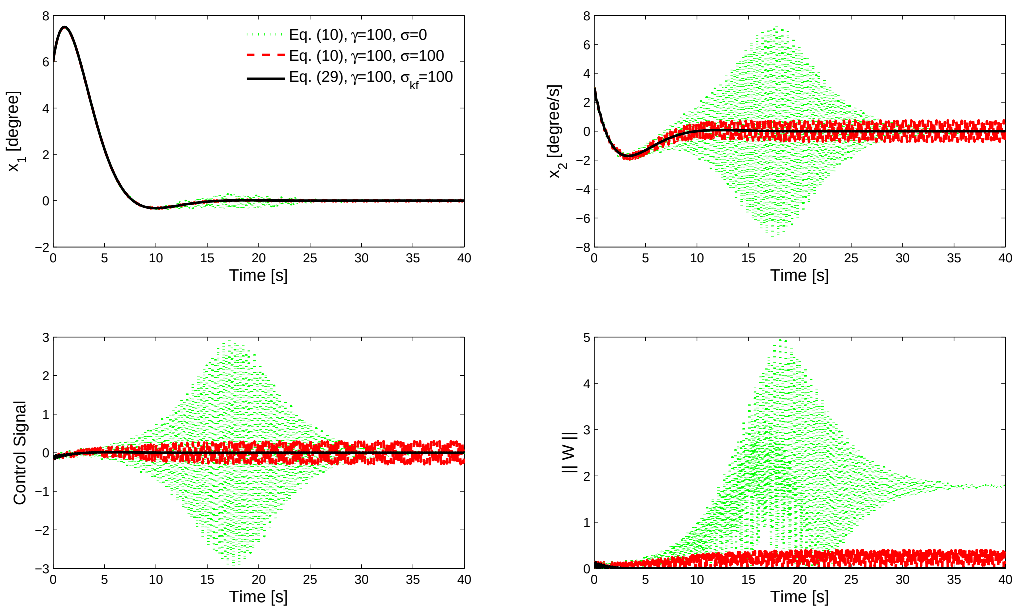

Figure 5. Comparison of adaptive controller responses without modification, with e-modification, and with KF based *e*- modification with all gains set to 100.

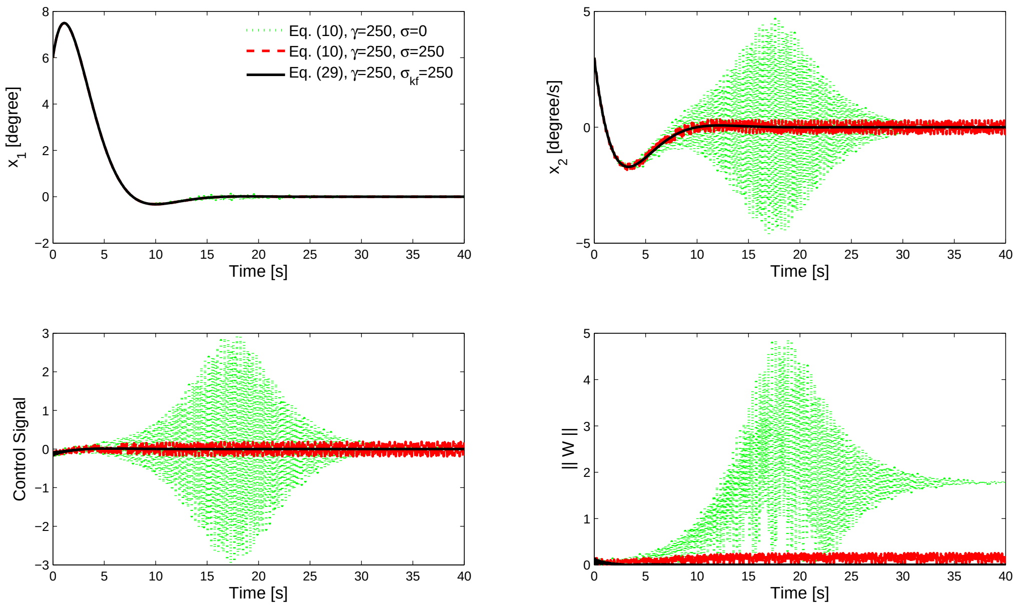

Figure 6. Comparison of adaptive controller responses without modification, with e-modification, and with KF based e- modification with all gains set to 250.

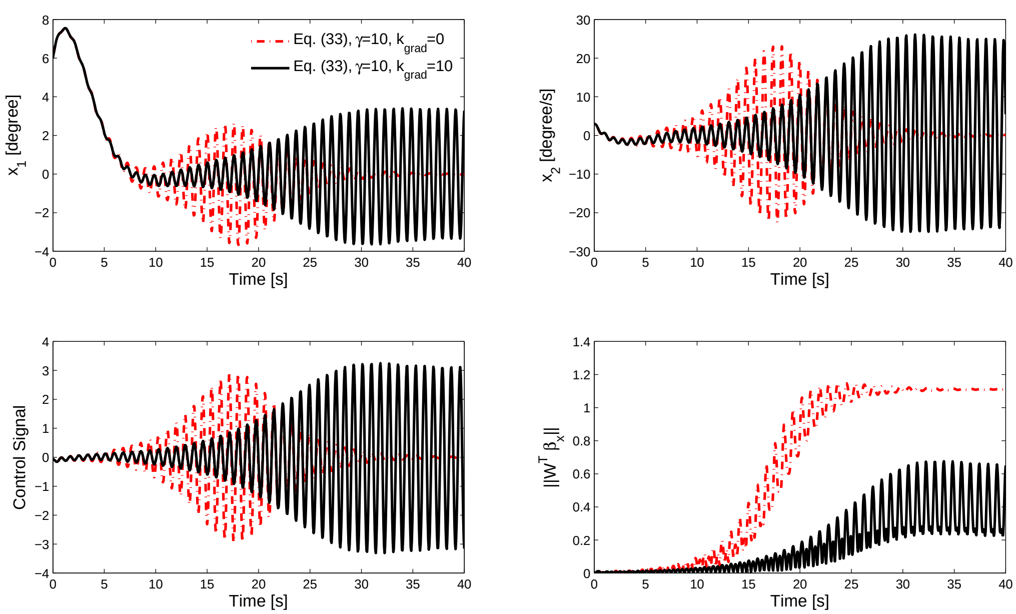

Figure 7. Standard ALR based adaptive control response with uncertainty using Eq. (33)

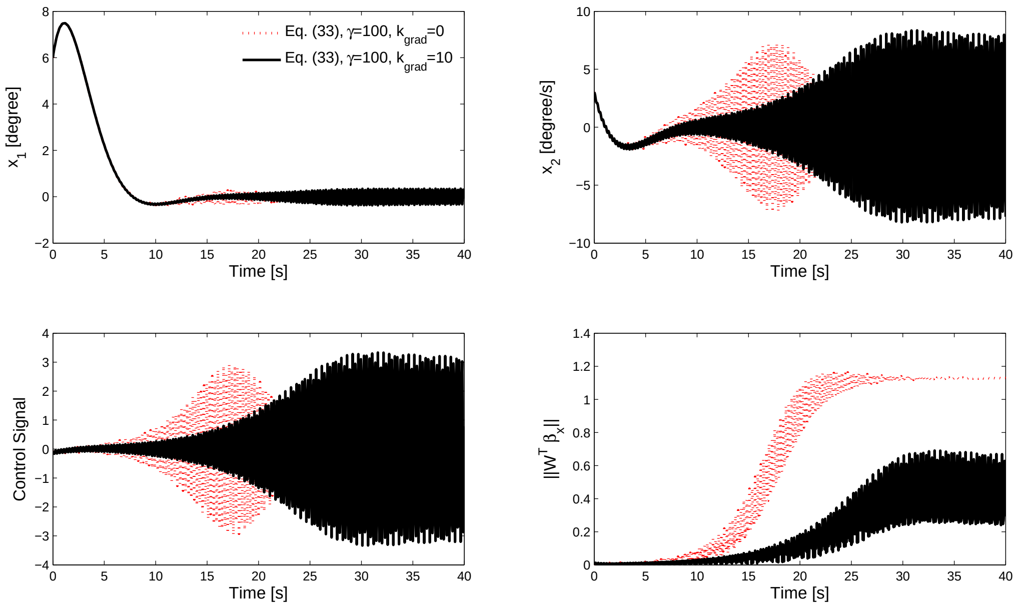

Figure 8. Standard ALR based adaptive control response with uncertainty using Eq. (33)

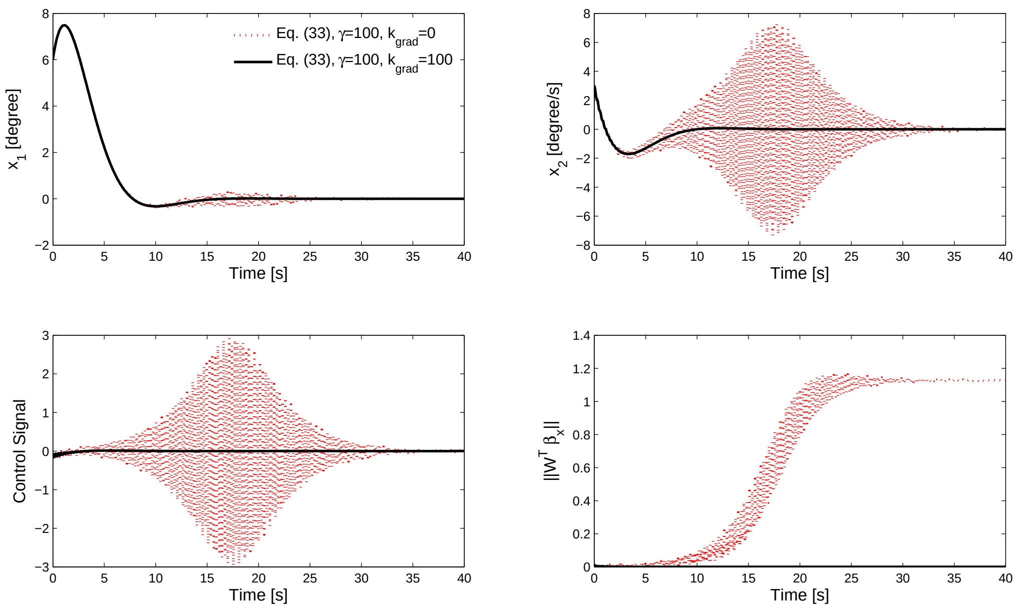

Figure 9. Standard ALR based adaptive control response with uncertainty using Eq. (33)

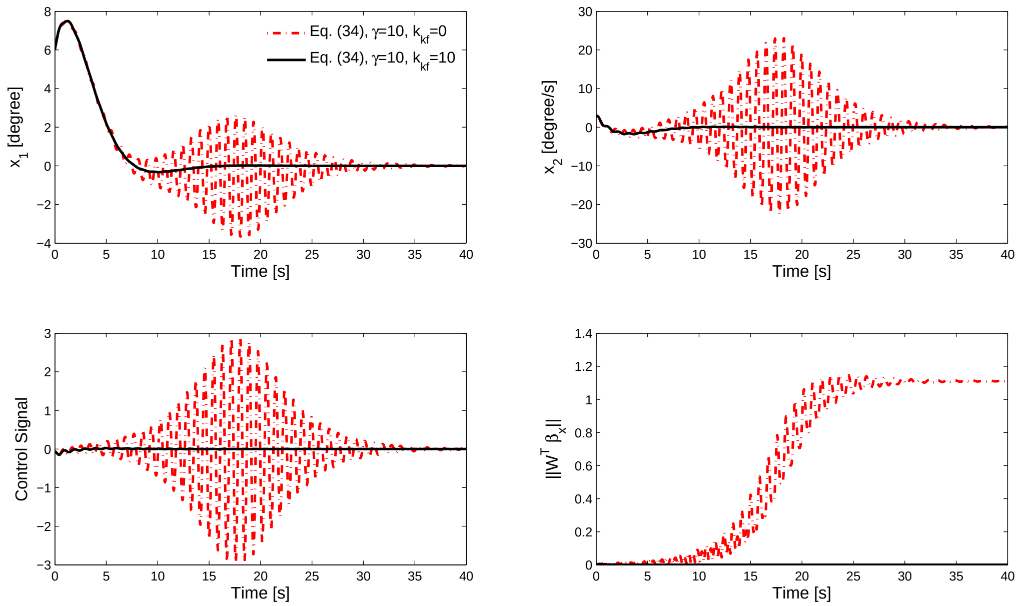

Figure 10. KF-ALR based adaptive control response with uncertainty using Eq. (34)

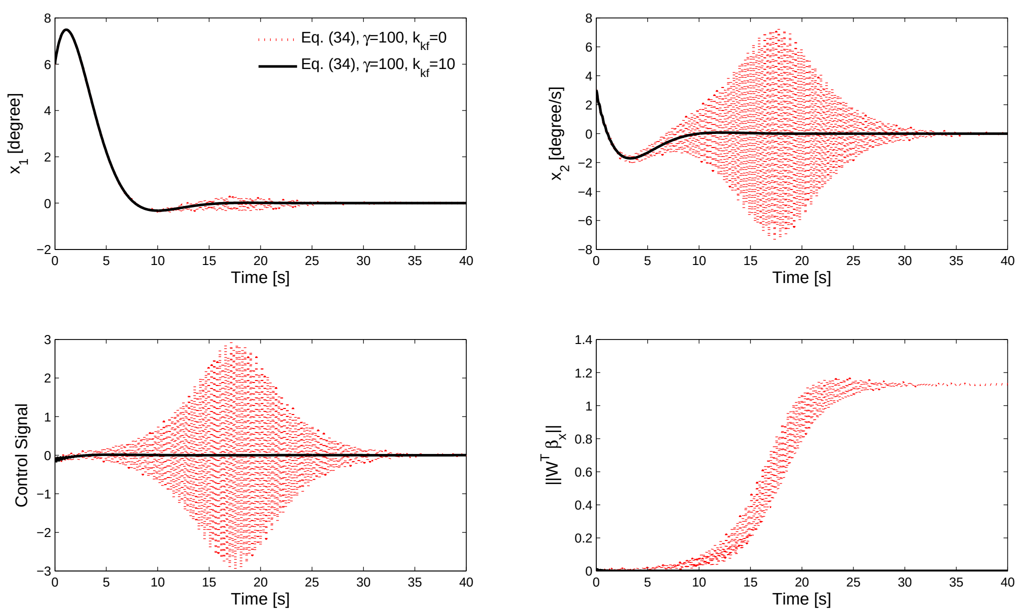

Figure 11. KF-ALR based adaptive control response with uncertainty using Eq. (34)

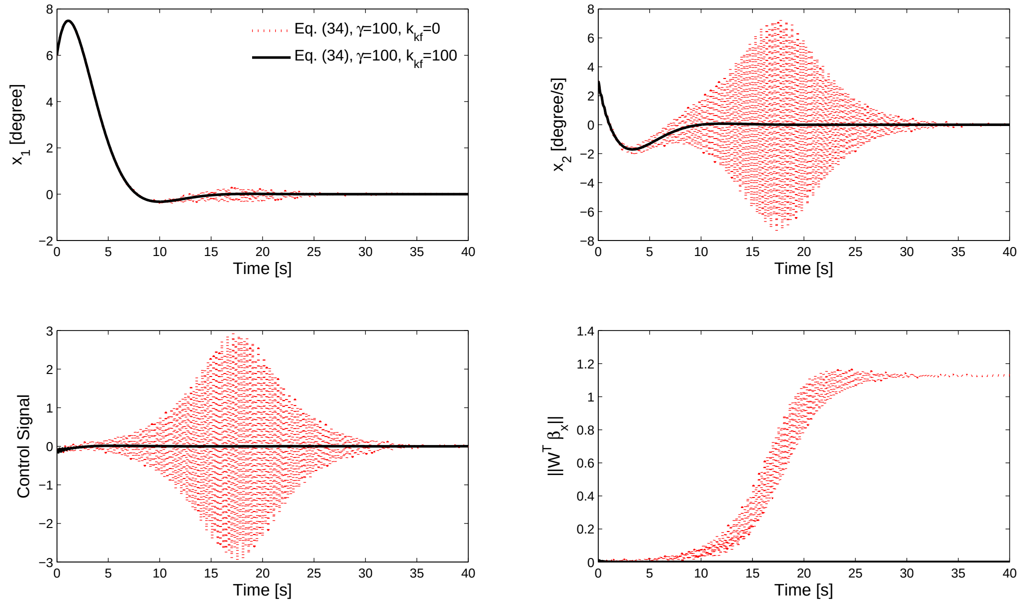

Figure 12. KF-ALR based adaptive control response with uncertainty using Eq. (34)

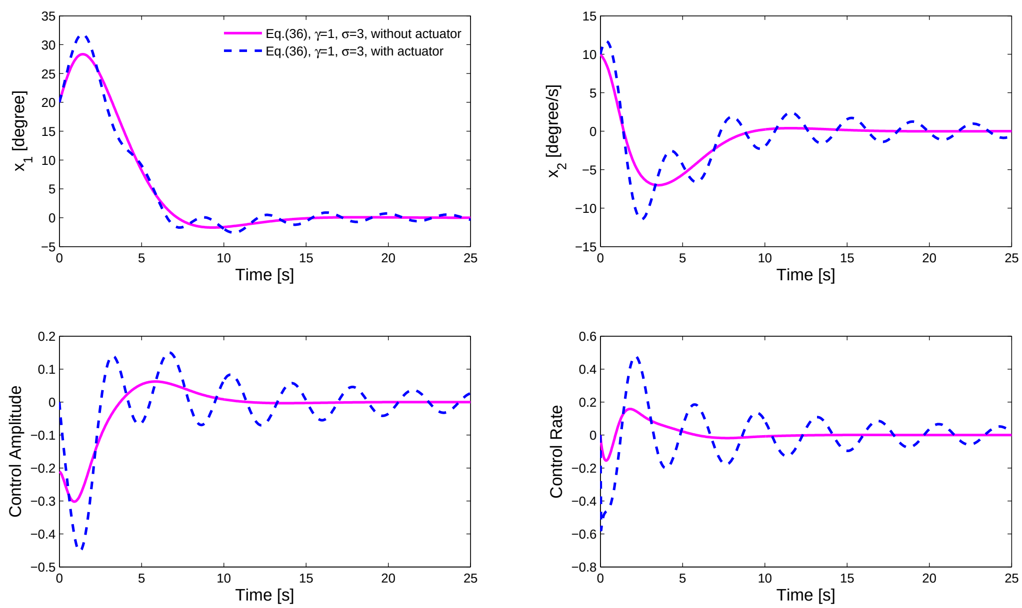

Figure 13. Adaptive control response without and with actuator dynamics.

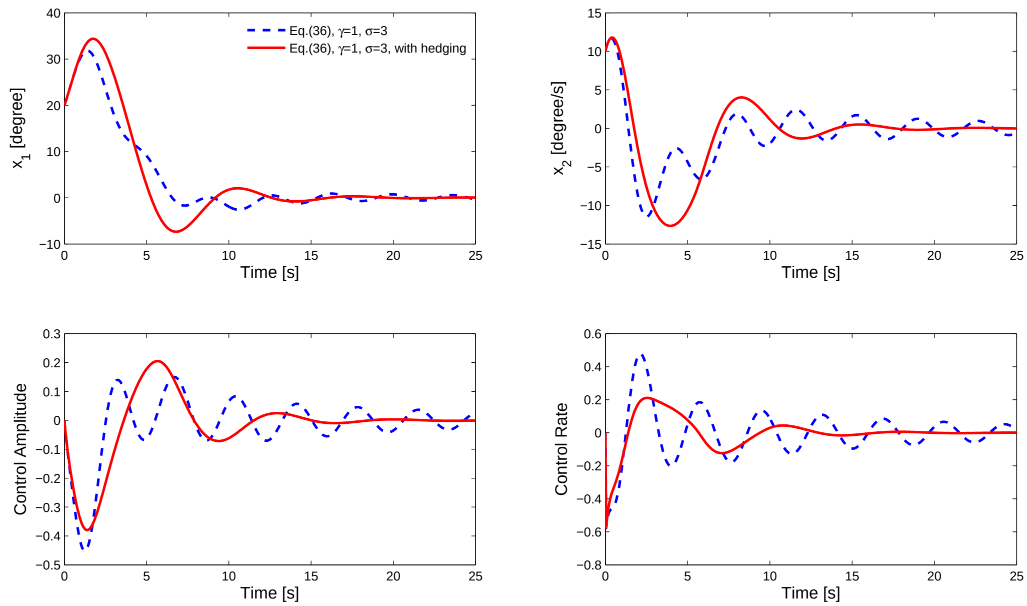

Figure 14. Performance of adaptive controller with hedging, with actuator dynamics.

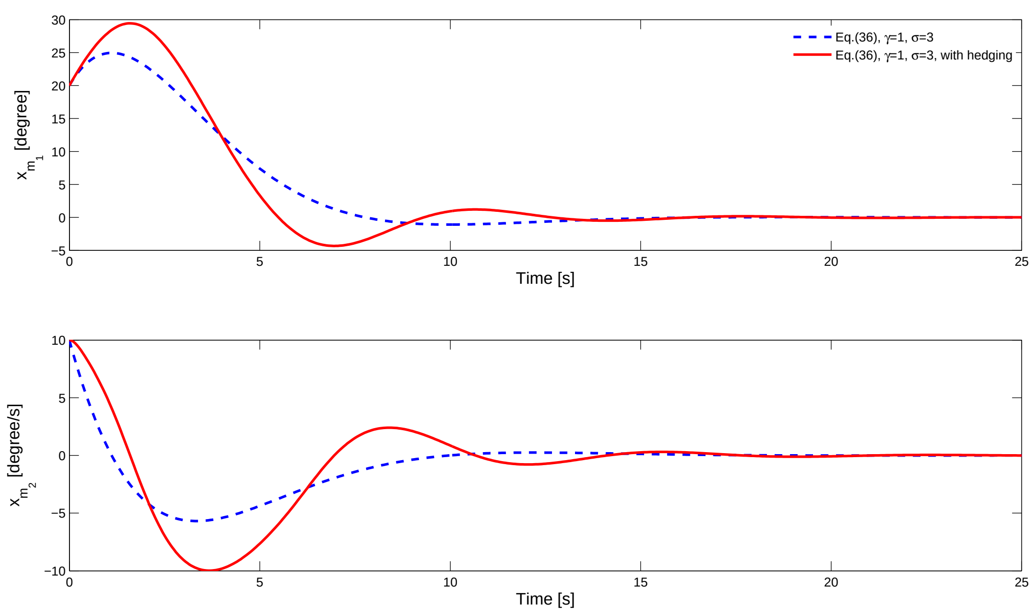

Figure 15. Response of reference model without and with hedging, with actuator dynamics.

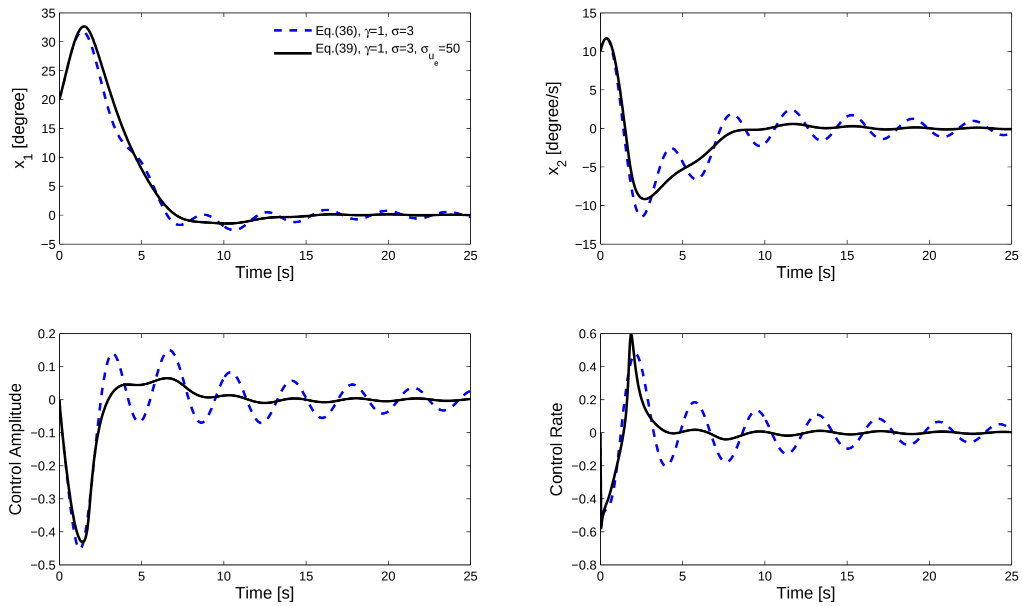

Figure 16. Performance of adaptive controller with u_e- modification, with actuator dynamics.

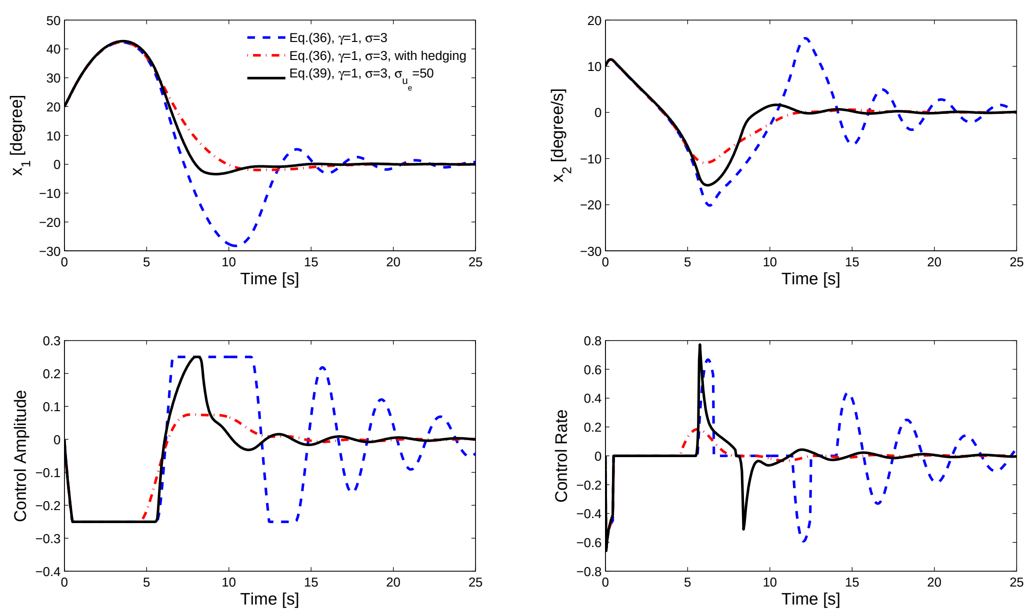

Figure 17. Comparison of adaptive control responses with hedging versus with u_e- modification, with actuator dynamics and limits.

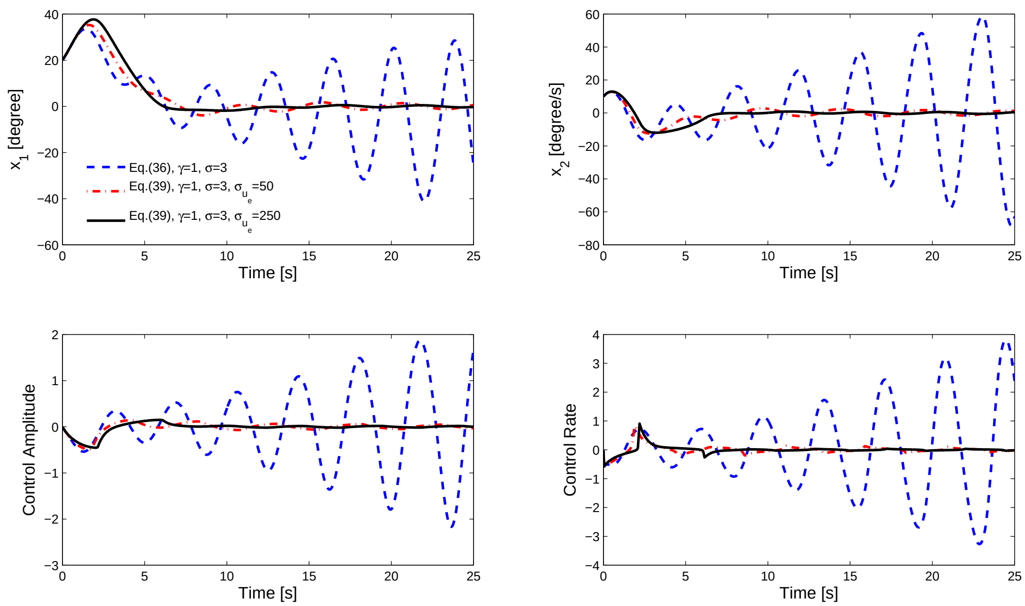

Figure 18. Performance of adaptive controller with u_e- modification, with actuator dynamics and time delay.