\(K\)-modification in Adaptive Control

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Overview

- Introduction
- $\mathcal{K}$-modification
- Example with Wing-rock
- Conclusion
**Introduction**

- **The basic idea** is to estimate the uncertain plant and/or controller parameters on-line based on the measured system signal and use the estimated parameters in control input computation.

- An adaptive controller can be regarded as an inherently nonlinear dynamic system with on-line parameter estimation.

- **The basic objective** is to maintain consistent performance of a system in the presence of uncertainty or unknown variation in plant parameters.
Two main approaches for constructing adaptive controllers:

- **Indirect Adaptive Control**: Parameter estimation of the system and adapting controller gains.
- **Direct Adaptive Control**: Adapting controller gains in response to system variations.

Goal here is to improve performance with Direct Adaptive Control.
Modification term in adaptive control:

Adding damping to adaptive law to keep adaptive parameters bounded

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<th>Modification term</th>
<th>Why use?</th>
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<tr>
<td>$\sigma$-mod</td>
<td>ensures bounded weights</td>
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<tr>
<td>e-mod</td>
<td>ensures bounded weights</td>
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<td>$Q$-mod</td>
<td>improved transient response</td>
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<td>Optimal-mod</td>
<td>guaranteed GM/PM</td>
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<td>ALR-mod</td>
<td>recovers loop transfer</td>
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<td>Kalman-filter optimization</td>
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A new modification for adaptive control by adding a tunable stiffness term to the weight update law.

It is easily combined with well known $\sigma$- and $e$-modifications that act as damping terms.

It provides a filtering effect that is frequency dependent to improve adaptive system performance so that smooth transient response can be obtained.
Consider the system dynamics expressed as

\[ \dot{x}(t) = Ax(t) + B[u(t) + \Delta(x(t))] \]

with assuming that

\[ \Delta(x(t)) = W^T \beta(x(t)) + \varepsilon(x(t)) \]

where

\[ x \in \mathbb{R}^n, \quad A \in \mathbb{R}^{n \times n}, \quad B \in \mathbb{R}^{n \times m}, \quad W \in \mathbb{R}^{q \times m}, \quad \beta \in \mathbb{R}^q \]

Consider the control law given by

\[ u(t) = -K_x x(t) + K_r r(t) - \hat{W}^T(t) \beta(x(t)) \]

that has a weight update law in the form of
\[ \dot{\hat{W}}(t) = \gamma \beta(x(t)) e^T(t) PB - \dot{\hat{W}}_m(t) - k \int_{t-T}^t \dot{\hat{W}}(s) ds \]

where \( \dot{\hat{W}}_m(t) \) is \( \sigma \) or e-modification term and \( T \) is an updating time interval.

Overall Control System Architecture
Second order weight update law

\[ \ddot{W}(t) + C_w \dot{W}(t) + K_w \dot{W}(t) = F_w \]

where \( C_w, K_w, F_w \) are damping, stiffness, forcing terms that are expressed as \((m=1)\)

\[ C_w = \sigma \]
\[ K_w = \gamma \left[ \beta_x(x(t)) B e^T(t) + \beta(x(t)) B^T \right] P B \beta^T(x(t)) + k \]
\[ F_w = \gamma \left[ \beta_x(x(t)) \left( A_m x(t) B_m r(t) + B \Delta(x(t)) \right) e^T(t) + \beta(x(t)) \left( A_m e(t) + B \Delta(x(t)) \right) \right] P B \]
\[ + k \hat{W}(t - T) \]

\[ C_w = \lambda \| e(t) \| \]
\[ K_w = \gamma \left[ \beta_x(x(t)) B e^T(t) + \beta(x(t)) B^T \right] P B \beta^T(x(t)) + \lambda \frac{e^T(t) \dot{e}(t)}{\| e(t) \|} + k \]
\[ F_w = \gamma \left[ \beta_x(x(t)) \left( A_m x(t) + B_m r(t) + B \Delta(x(t)) \right) e^T(t) + \beta_x(x(t)) \left( A_m x(t) + B \Delta(x(t)) \right) \right] P B \]
\[ + k \hat{W}(t - T) \]
Damping ratio and Natural frequency

\[ \zeta(t) = \frac{C_w(t)}{2 \sqrt{K_w(t)}} = \frac{\sigma}{2 \sqrt{\gamma \left[ \beta_x(x(t))B e^T(t) + \beta(x(t))B^T \right] PB\beta^T(x(t)) + k}} \]

\[ \omega_n(t) = \sqrt{K_w(t)} = \sqrt{\gamma \left[ \beta_x(x(t))B e^T(t) + \beta(x(t))B^T \right] PB\beta^T(x(t)) + k} \]

\[ \zeta(t) = \frac{C_w(t)}{2 \sqrt{K_w(t)}} = \frac{\lambda \|e(t)\|}{2 \sqrt{\gamma \left[ \beta_x(x(t))B e^T(t) + \beta(x(t))B^T \right] PB\beta^T(x(t)) + \lambda \frac{e^T(t)\dot{e}(t)}{\|e(t)\|} + k}} \]

\[ \omega_n(t) = \sqrt{K_w(t)} = \sqrt{\gamma \left[ \beta_x(x(t))B e^T(t) + \beta(x(t))B^T \right] PB\beta^T(x(t)) + \lambda \frac{e^T(t)\dot{e}(t)}{\|e(t)\|} + k} \]
Consider the following wing-rock dynamics

\[
\begin{align*}
\dot{x}_1(t) &= x_2(t) \\
\dot{x}_2(t) &= u(t) + \Delta(x(t))
\end{align*}
\]

where

\[
\Delta(x(t)) = b_0 + b_1x_1(t) + b_2x_2(t) + b_3|x_1(t)|x_2(t) + b_4|x_2(t)|x_2(t) + b_5|x_1|^3(t)
\]

\[
b_{0,1,2,3,4,5} = [0, 0.2314, 0.6918, -0.6245, 0.0095, 0.0214]
\]

Initial conditions are \( x_1(0) = \phi(0) = 6^\circ, x_2(0) = \dot{\phi}(0) = 3^\circ / \text{sec} \)

The reference model is selected to be second order with a natural frequency of 0.5 \( \text{rad/sec} \) and damping ratio of 0.707.

Sigmoidal basis functions \( \beta_i(x(t)) = \frac{1}{1 + e^{-x_i(t)}}, i = 1,2 \) are used.

Reference model, aircraft, and baseline control responses
Adaptation parameters setting

\[
\dot{\hat{W}}(t) = \gamma \beta(x(t))e^T(t)PB - \lambda \|e(t)\|\dot{\hat{W}}(t) - k \int_{t-T}^{t} \hat{W}(s) ds
\]

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<th>Case 1</th>
<th>Adaptation gain</th>
<th>e-modification gain</th>
<th>Stiffness gain</th>
<th>Updating time interval</th>
<th>Time delay</th>
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<tbody>
<tr>
<td></td>
<td>$\gamma$</td>
<td>$\lambda$</td>
<td>$k$</td>
<td>$T$ [sec]</td>
<td>[sec]</td>
</tr>
<tr>
<td>Case 1</td>
<td>25</td>
<td>100</td>
<td>0, 250</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>Case 2</td>
<td>25</td>
<td>100</td>
<td>250</td>
<td>0.15</td>
<td>0.15</td>
</tr>
</tbody>
</table>
Case 1: No Time Delay

$\gamma=25, \lambda=100, k=250, T=0.1$

$\gamma=25, \lambda=100, k=0$
\[ \gamma = 25, \quad \lambda = 100, \quad k=0 \]

Case 2:
Time Delay of 0.15 sec

\[ \gamma = 100, \quad \lambda = 100, \quad k=250, \quad T=0.15 \]
Conclusion

◆ $K$-modification is used to achieve an update law with a prescribed natural frequency and damping ratio in the error transients.

◆ $K$-modification provides two additional tuning parameters ($k$, $T$) to improve transient response.

◆ $K$-modification improves robustness of adaptive design to time delay.

◆ Closed loop system error is uniformly ultimately bounded (UUB).
Questions ?