A NeuroAdaptive Control Architecture
for Nonlinear Uncertain Dynamical Systems
with Input Actuator Constraints

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   ■ Neuroadaptive Control Architecture
3 Adaptive Tracking with Input Constraints
   ■ Neuroadaptive Control Architecture
4 Amplitude and Rate Saturation Constraints
5 Illustrative Numerical Examples
   ■ Van Der Pol Oscillator
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6 Conclusion and Ongoing Research
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Motivation and Goals

Motivation

- Many loops can be coupled (MIMO)
- Dynamics are nonlinear and uncertain
- Actuator dynamics
  - Amplitude and rate constraints
  - Quantization constraints
  - Time-delay constraints

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Motivation and Goals

**Goals**

- Develop a direct **neuroadaptive control** framework for **non-linear uncertain** dynamical systems
- Address system uncertainties and non-linearities
- Address input actuator constraints
- Achieve system stability & performance **without** (excessive) reliance on system models
Develop a direct neuroadaptive control framework for non-linear uncertain dynamical systems

Address system uncertainties and nonlinearities

Address input actuator constraints

Achieve system stability & performance without (excessive) reliance on system models
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Adaptive Control for Nonminimum Phase Systems
Motivation and Goals

Neuroadaptive Control Architecture

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**NNs** can approximate a large class of **continuous nonlinear** maps

- **Linear** parametrization of system uncertainty
- **Nonlinear** parametrization of system uncertainty

- Inherently parallel architecture
  - Parallel weight update laws
  - Update NN online

- These properties make **NNs** a viable paradigm for adaptive system identification and control

  - Complex highly nonlinear uncertain dynamical systems
**Motivation and Goals**

Neuroadaptive Control Architecture

**GEORGIA TECH**

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- These properties make **NNs** a viable paradigm for adaptive system **identification** and **control**
  - Complex highly **nonlinear uncertain** dynamical systems
Can tolerate large system uncertainty levels

- Generally implemented without any regard to actuator amplitude and rate saturation constraints
- Adaptive controllers continue to adapt when feedback loop has been severed due to presence of actuator saturation
  - Degrade system performance
  - Drive system to instability
- Adaptive gain updates need to be modified when input constraints are active
Adaptive Control Schemes

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Can we develop a new neuroadaptive control architecture that
- Ensures tracking in the face of actuator constraints
- Minimizes the effect of input actuator constraints
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$\dot{x}(t) = A x(t) + B [u(t) + \Delta(x(t))]$, $x(0) = x_0$, $t \geq 0$

- $x(t)$ is available for full state feedback
- $(A, B)$ is known, $\Delta(x)$ is unknown
- Linearly parameterized NN: $\Delta(x) = W^T \beta(x)$
Reference System

\[ 0 = A_m^T P + PA_m + R, \quad P > 0, \quad R > 0 \]

Matching conditions: \( \exists K_1, K_2 \) s.t. \( A_m = A - BK_1 \) & \( B_m = BK_2 \)
Adaptive Control for Nonminimum Phase Systems

\[ u(t) = -K_1 x(t) + K_2 r(t) - \hat{W}^T(t) \beta(x(t)) \]

- **Weight update**: \( \dot{\hat{W}}(t) = \Gamma \beta(x(t)) e_x^T(t) P B \), \( \Gamma > 0 \), \( \hat{W}(0) = \hat{W}_0 \)
- **Error states**: \( e_x(t) = x(t) - x_m(t) \)
- **Guarantees Lyapunov stability** and \( e_x(t) \to 0 \) as \( t \to \infty \)
Adaptive Tracking with Input Constraints

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Adaptive Tracking with Input Constraints

Neuroadaptive Control Architecture

Actuator Dynamics

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Uncertain System

$\dot{x}(t) = f(x(t), v(t)), \quad x(0) = x_0, \quad t \geq 0$

$u(t) = x(t)$

New Adaptation

New Adaptation

Actuator

Actuator

Uncertain System

Uncertain System

Modified Reference System

Modified Reference System

$\mathcal{A}_a: \begin{cases} \dot{x}_a(t) = f(x_a(t), v(t)), & x_a(0) = x_0, \quad t \geq 0 \\ u(t) = x_a(t) \end{cases}$

- Actuator dynamics $f(x_a, v)$ are known
- $\mathcal{A}_p$: $(A, B)$ is known, $\Delta(x)$ is unknown
Adaptive Tracking with Input Constraints

Neuroadaptive Control Architecture

Modified Reference System

$\mathcal{G}_m : \dot{x}_m(t) = A_m x_m(t) + B_m r(t) + \eta(t), \quad x_m(0) = x_{m0}, \quad t \geq 0$

- $\eta(t) = B v(t)$
- $A_m$ is Hurwitz, $0 = A_m^T P + P A_m + R$, $P > 0$, $R > 0$
- Matching conditions: $\exists K_1, K_2$ s.t. $A_m = A - B K_1$ and $B_m = B K_2$
Adaptive Tracking with Input Constraints

Neuroadaptive Control Architecture

Adaptation

\[ u_d(t) = -K_1x(t) + K_2r(t) - \hat{W}^T(t)\beta(x(t)) \]

- **Weight update:** \( \hat{W}(t) = \Gamma[B(x)\left(e_x^T(t)P - e_u^T(t)\tilde{K}(t)B\right)], \quad \Gamma > 0 \)
- **Control error:** \( e_u(t) = v(t) - u(t) + u_d(t) \)
- **Linearized loop gain:** \( \tilde{K}(t) \triangleq [K_1 + \hat{W}^T(t)\beta'(x(t))] \)
Adaptive Tracking with Input Constraints

Neuroadaptive Control Architecture

New Adaptation

\[ \dot{v}(t) = -\frac{1}{2} \tilde{Q} e_u(t) - g(t) + B^T P e_x(t), \quad v(0) = v_0, \quad t \geq 0 \]

- \( \tilde{Q} > 0 \)
- \[ g(t) \triangleq \left[ -\tilde{K}(t) [A_m x(t) + B_m r(t) + Bv(t) - Be_u(t)] - f(x(t), v(t)) + K_2 r(t) - \Gamma [\beta(x(t)) e_x^T(t) P B + \zeta(t)] \beta(x(t)) \right] \]
- Guarantees LS and \( e_x(t) \to 0 \) and \( e_u(t) \to 0 \) as \( t \to \infty \)

Guarantees

LS

e_x(t) \to 0

e_u(t) \to 0

as \( t \to \infty \)
Uncertain System $x(t)$

Actuator $u(t)$

New Adaptation $v(t)$

Adaptation $e_u(t)$

Adaptation $u_d(t)$

Modified Reference System $e_x(t)$

Modified Reference System $x_m(t)$

Adaptation $r(t)$

Actuator $g(t)$

New Adaptation $u(t)$

Uncertain System $x(t)$
Uncertain System $x(t)$

New Adaptation $v(t)$

Actuator $u(t)$

Uncertain System $x(t)$

Modified Reference System $x_m(t)$

Adaptation $u_d(t)$

$g(t)$

$e_u(t)$

$r(t)$

$e_x(t)$

$ex(t)$

$ud(t)$

$e(t)$

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Amplitude and Rate Saturation Constraints

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Amplitude and Rate Saturation Constraints

For amplitude saturation $G_a$ reduces to $G_{amp}$

$$\begin{align*}
  x_a(t) &= 0, \quad x_a(0) = x_{a0} \\
  u(t) &= x_a(t)
\end{align*}$$

with

$$u_i(t) = u_{i_{\text{max}}} \text{sat} \left( \frac{u_i(t)}{u_{i_{\text{max}}}} \right)$$

and $u_{i_{\text{max}}}$ is the maximum amplitude saturation limit of control input $u_i(t), i = 1, \ldots, m$

For rate saturation, $G_a$ reduces to $G_{\text{rate}}$ with

$$\dot{u}_i(t) = \dot{u}_{i_{\text{max}}} \text{sat} \left( \frac{\dot{u}_i(t)}{\dot{u}_{i_{\text{max}}}} \right)$$

and $\dot{u}_{i_{\text{max}}}$ is the maximum rate saturation limit of control input $u_i(t), i = 1, \ldots, m$
Illustrative Numerical Examples

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Van der Pol oscillator

\[ \ddot{z}(t) = -2(\dot{z}^2(t) - 1)\dot{z}(t) - z(t) + u(t), \quad \dot{z}(0) = \dot{z}_0, \quad z(0) = z_0 \]

Actuator with time constant \( \tau_a \)

\[ \begin{align*}
\dot{x}_a(t) & = -\tau_a x_a(t) + \tau_a v(t), \quad x_a(0) = 0, \quad t \geq 0 \\
u(t) & = x_a(t)
\end{align*} \]

Let \( x_1(t) = z(t) \) and \( x_2(t) = \dot{z}(t) \), then

\[ \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} (u(t) + \Delta(x(t))), \quad x_0 = \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} \]

\[ \Delta(x) = -2x_1^2 x_2 \text{ and } x = [x_1, x_2]^T \]
Van der Pol oscillator

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\( \Delta(x) = -2x_1^2 x_2 \) and \( x = [x_1, x_2]^T \)
- \( \mathcal{G}_{in} \) is second-order
  - Natural frequency \( \omega_n = 2.0 \)
  - Damping ratio \( \zeta = 1.5 \)

\[ \beta(x) = [1, \beta_1(x_1), \beta_2(x_2)]^T \text{ with} \]
\[ \beta_i(x_i) = \frac{1}{1 + e^{-a_i x_i}}, \quad a_i > 0, \quad i = 1, 2 \]

- We set \( \Gamma = 2.5I, \; Q = 15I_2, \) and \( \bar{Q} = 60 \)

- Comparison with a hedging method
  - Standard neuroadaptive control with a hedge signal
  - Same basis function, adaptive gain and \( Q \)
Illustrative Numerical Examples

- $\mathcal{G}_{in}$ is second-order
  - Natural frequency $\omega_n = 2.0$
  - Damping ratio $\zeta = 1.5$

- $\beta(x) = [1, \beta_1(x_1), \beta_2(x_2)]^T$ with
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- \( B_{r_0} \) is second-order
  - Natural frequency \( \omega_n = 2.0 \)
  - Damping ratio \( \zeta = 1.5 \)

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Design

- $G_{in}$ is second-order
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Illustrative Numerical Examples

**Example 1:** $\tau_a = 1/2.5$

- Control Amplitudes
- Control Rates

**Performance of the hedging based standard neuroadaptive control**
Example 1: $\tau_a = 1/2.5$
Example 1: $\tau_d = 1/2.5$

Error signals approach zero asymptotically
Illustrative Numerical Examples

Example 2: $\tau_a = 1/1.5$

- Performance of the hedging based standard neuroadaptive control

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**Performance of the hedging based standard neuroadaptive control**
Example 2: $\tau_0 = 1/1.5$

Performance of the proposed neuroadaptive control
Example 2: $\tau_a = 1/1.5$

- Error signals approach zero asymptotically
Example 3: $\tau_a = 1/1.5$, $u_{max} = \pm 2.5$, and $\dot{u}_{max} = \pm 2.5$
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Performance of the proposed neuroadaptive control
Illustrative Numerical Examples

Example 3: $\tau_a = 1/1.5$, $u_{\text{max}} = \pm 2.5$, and $\dot{u}_{\text{max}} = \pm 2.5$

- Error signals approach zero asymptotically
Example 3: $\tau_a = 1/1.5$, $u_{max} = \pm 2.5$, and $\dot{u}_{max} \pm 2.5$

Error signals approach zero asymptotically for $\Gamma = 2.5$ and $\Gamma = 250$
Example 4: \( \tau_a = 1/1, I_{\text{quant}} = 0.5, u_{\text{max}} = \pm 2, \) and \( \dot{u}_{\text{max}} = \pm 2 \)
Illustrative Numerical Examples

**Example 4:** $\tau_a = 1/1$, $I_{\text{quant}} = 0.5$, $\theta_{\text{max}} = \pm 2$, and $\dot{\theta}_{\text{max}} = \pm 2$

**Performance of the proposed neuroadaptive control**
Example 4: $\tau_a = 1/1$, $I_{\text{quant}} = 0.5$, $u_{\text{max}} = \pm 2$, and $\dot{u}_{\text{max}} = \pm 2$.

Error signals approach zero asymptotically.
Example 4: $\tau_a = 1/1$, $I_{\text{quant}} = 0.5$, $u_{\text{max}} = \pm 2$, and $\dot{u}_{\text{max}} = \pm 2$.

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Conclusion

- **Developed** an **neuroadaptive control** architecture
  - Nonlinear multivariable **uncertain** systems
  - Input **actuator constraints**
- Formulated the approach
  - To address the slow actuator dynamics
  - To address amplitude and rate saturation constraints
- Considered
  - Linear parametrization of the uncertainty
  - Nonlinear parametrization of the uncertainty (in paper)

Ongoing Research

- Extensions to derivative-free adaptive control
- Extensions to output feedback adaptive control
- Extensions to discrete-time systems
Conclusion and Ongoing Research

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Thank You

QUESTIONS?