Nonlinear Quadratic Optimal Control for Cascaded Multilevel Static Compensators

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Abstract — This paper presents a novel nonlinear quadratic optimal control technique for multiple-input, multiple-output (MIMO) cascaded multilevel static compensators (STATCOM), based on state-dependent Riccati equation (SDRE) technique. The proposed approach is applied online, using a gradient-type neural network algorithm for the computation of SDRE based nonlinear control. The algorithm is fast, computationally simple, and independent of initial conditions. This new control strategy is applied, in a detailed simulation, to the average d-q model of STATCOM to track a desired DC voltage and a reactive current, that demonstrates a relatively fast control ability and good dynamic performance.

Index Terms — Static compensator (STATCOM), state-dependent Riccati equation (SDRE), online neural network computation, nonlinear quadratic optimal control

I. INTRODUCTION

Power compensation is an important aspect in the control of power systems, because it decreases transmission losses, prevents large amplitude variations in the receiving end voltages, and increases power transmission capabilities. Reactive power is compensated traditionally by connecting/disconnecting capacitor or inductor banks through mechanical switches that are slow and imprecise. Over the last two decades, the reactive power compensator, based on force commutated solid state power electronic devices, such as thyristor-controlled reactors and capacitors, have gained popularity. However, precision of these devices depend on a power system line condition at the point of common coupling voltage. To avoid this problem, static compensator (STATCOM) is developed in recent years. A STATCOM is a power electronic device which is able to emulate electronically both inductive and capacitive loads. In order to reduce higher harmonics while keeping the lower switching fundamental frequency, multilevel converter is an attractive configuration. Cascade multilevel STATCOM, that is based on the connection of H-bridges, seems to be the most feasible topology for many reasons [1]–[3].

STATCOM control is playing an important role in the reactive power provision. The traditional STATCOM control design methods are based on PI-type controllers, that are easy to work with. However, the system response provided by a PI controller is slow, because of the required calculation time for active and reactive powers. Many advanced control strategies are proposed in the recent years [3]–[5]. An important approach is proposed by D. Soto et. al. [3] that is based on input-output linearization of the STATCOM nonlinear system. However, the use of linearization technique in control design provides relatively good controllers in specific operating points. Therefore, it is more reliable to use direct nonlinear control strategies to control such systems.

In this paper, the solution to the control of cascaded multilevel STATCOMs, is given using state-dependent Riccati equation (SDRE) based nonlinear quadratic optimal control. The SDRE solution, however, is determined using gradient-type neural networks, that allow one to implement this approach online for real-time control purposes. This new approach for STATCOM systems provides relatively fast control ability, a wide operating range, a good dynamic performance, and allows for tradeoff between the control accuracy and control effort, which is a rare property for other control design methods.

The paper is organized as follows. Section II presents a simplified STATCOM modeling in a d-q synchronous reference frame, while section III presents nonlinear quadratic control strategy, based on state dependent Riccati equation technique with its online neural network computation approach. Then, simulation results are given in section IV in detail. Finally, conclusions are summarized in section V.

II. STATCOM MODELING

The STATCOM converter model, in a d-q synchronous reference frame, rotating at the supply frequency \( w_s \), with the d-axis aligned with the supply voltage is shown in Fig.1 [see 3], where \( L, R \) and \( C \) represent the equivalent interface inductance, the AC line resistance, and DC side capacitance. This figure shows a generator that is connected to STATCOM via a transmission line. In addition, Fig.2 shows the per phase equivalent circuit of STATCOM. The state model of the considered STATCOM model is given in as

...
\[
\begin{align*}
\frac{di_d}{dt} &= -\frac{R}{L}i_d + \omega_s i_q - \frac{1}{L}v_{dc}u_1 + \frac{1}{L}v_{ds} \\
\frac{di_q}{dt} &= -\frac{R}{L}i_q - \omega_s i_d - \frac{1}{L}v_{dc}u_2 + \frac{1}{L}v_{qs} \\
\frac{dv_{dc}}{dt} &= \frac{3}{2}v_{ds}\frac{i_d}{C_{eq}}
\end{align*}
\]

(2.1)

where \(i_d, i_q\) are the equivalent two phase \(d-q\) axis currents of the transformed three-phase AC line/STATCOM system. \(v_{dc}\) is the DC voltage of the equivalent STATCOM capacitor, \(C_{eq}\), and \(v_{ds}\) and \(v_{qs}\) are the \(d-q\) axis components of the AC line source voltage. Also, \(u_1\) and \(u_2\) are the modulation indexes in the \(d-q\) axis, respectively, and are given as

\[
\begin{align*}
u_1 &= k(\alpha)v_{dc}\cos \delta \\
u_2 &= k(\alpha)v_{dc}\sin \delta
\end{align*}
\]

(2.2)

where \(\delta\) is the angle of the STATCOM voltage vector with respect to the \(d\)-axis, and \(k(\alpha)\) is the ratio of the peak voltage of STATCOM to \(v_{dc}\). The objective of the nonlinear quadratic optimal control is to force reactive current \(i_q\) and DC voltage \(v_{dc}\) to track their corresponding reference values. In equation (2.1), the equilibrium values are not all zero and are given by

\[
\begin{align*}
u_1 &= \frac{1}{v_{dc}}(-Ri_d + L\omega_s i_q + v_{ds}) \\
u_2 &= \frac{1}{v_{dc}}(-Ri_q - L\omega_s i_d + v_{qs}) \\
v_{dc}i_q &= 0
\end{align*}
\]

(2.3)

Let us define new state and control variables, as

\[
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} i_d \\ i_q - i_{qr} \\ v_{dc} - v_{dc_{ref}} \end{bmatrix}
\quad \text{and} \quad
\begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = \begin{bmatrix} u_1 - u_{1e} \\ u_2 - u_{2e} \end{bmatrix}
\]

(2.4)

where \(i_{qr}, v_{dc_{ref}}\) are the reference values for reactive current \(i_q\) and DC voltage \(v_{dc}\). Also \(u_{1e}\) and \(u_{2e}\) are the necessary constant values for the STATCOM system to keep the equilibrium state \(x=[x_1, x_2, x_3]^T\) at zero, and are given by

\[
\begin{align*}
u_{1e} &= \frac{1}{v_{dc}}(L\omega_s i_{qr} + v_{dc}) \\
u_{2e} &= -\frac{R}{v_{dc_{ref}}}i_{qr}
\end{align*}
\]

(2.5)

Plugging (2.4) and (2.5) in equation (2.1) gives the following steady state error equation for STATCOM model

\[
\begin{align*}
\dot{x}_1 &= -\frac{R}{L}x_1 + \omega_s x_2 - \frac{1}{L}u_{1e}x_3 - \frac{1}{L}(x_3 + v_{dc_{ref}})u_1 \\
\dot{x}_2 &= -\omega_s x_1 - \frac{R}{L}x_2 + \frac{1}{L}u_{2e}x_3 - \frac{1}{L}(x_3 + v_{dc_{ref}})u_2 \\
\dot{x}_3 &= \frac{1.5v_{dc}}{C_{eq}}(x_3 + v_{dc_{ref}})
\end{align*}
\]

(2.6)

\[
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}
\]

where the control problem is to find \(\bar{u} = [\bar{u}_1, \bar{u}_2]^T\) so that \(x=[x_1, x_2, x_3]^T\) goes to zero.

The nonlinear quadratic optimal control design for the system given in (2.6) is explained in the following section.

III. NONLINEAR QUADRATIC OPTIMAL CONTROL

In this section, we develop a nonlinear quadratic optimal control strategy for cascaded multilevel static compensators, based on state-dependent Riccati equation (SDRE). For this purpose, dynamic model (2.6) can be rewritten in a state-dependent linear form, as

\[
\dot{x} = A(x)x + B(x)u
\]

(3.1)

where \(A(x)\) and \(B(x)\) are given as

\[
A(x) = \begin{bmatrix} \frac{R}{L} & \omega_s & \frac{u_{1e}}{L} \\ -\omega_s & \frac{R}{L} & \frac{u_{2e}}{L} \\ \frac{1.5v_{dc}}{C_{eq}} & 0 & 0 \end{bmatrix}, \quad B(x) = \begin{bmatrix} -x_3 - \frac{V_{dc_{ref}}}{L} \\ 0 \\ 0 \end{bmatrix}
\]

(3.2)

Considering the above system model, in the following subsections, we present a state feedback nonlinear quadratic regulator (NQR) design for optimal control of the STATCOM system, based on SDRE. In addition, we describe an online
computation methodology, for real-time implementation of the proposed techniques.

A. Nonlinear Quadratic Regulator

It is obvious from the STATCOM model (2.6) and also given in [3] that all the state variables are available for the synthesis a nonlinear quadratic control. Here, the aim is to find a state feedback control of the form $u=K(x)x$ that minimizes a cost function given by the integral

$$J(x_0,u) = \frac{1}{2} \int f(x)+u^TRu \, dt$$

(3.3)

where $Q \in \mathbb{R}^{n \times n}$ is a positive symmetric semi-definite matrix, and $R \in \mathbb{R}^{n \times n}$ is a symmetric positive-definite matrix. Moreover, $x^TQx$ is a measure of control accuracy and $u^TRu$ is a measure of control effort [11]. It should be mentioned here that SDRE formulation allows one to tradeoff between the control accuracy and control effort.

Then, a state-feedback control law can be given as

$$u(x) = -K(x)x = -R^{-1}B(x)P(x)x$$

(3.4)

where $P(x)$ is the unique, symmetric, positive-definite solution of the state-dependent Riccati equation of the form

$$A^T(x)P(x) + P(x)A(x) + Q - P(x)B(x)R^{-1}B(x)^T(x)P(x) = 0$$

(3.5)

Then, the resulting closed loop system can be written as

$$\dot{x} = (A(x) - R^{-1}B(x)B^T(x)P(x))x$$

(3.6)

where the closed loop system matrix $A_{CL}(x)$ satisfies the pointwise Hurwitz condition $\text{Re}(A_{CL}(x))<0$.

Moreover, the selection of $A(x)$ and $B(x)$ is not unique [8] and has to satisfy the conditions in the following remark.

**Remark 1.** Although the selection of $A(x)$ and $B(x)$ has infinitely many parameterizations, one has to consider a suitable selection of these state-dependent matrices so that the resulting state-dependent linear system is controllable or stabilizable. The state-dependent controllability matrix

$$\psi_c = \begin{bmatrix} B(x) & A(x)B(x) & \cdots & A^{(n-1)}(x)B(x) \end{bmatrix}$$

(3.7)

is controllable if and only if it has full rank, which requires that we have

$$x^T\psi_{c,\text{cont}}>0$$

(3.8)

Also, the system is said to be stabilizable if the uncontrollable states are stable. This indicates that the choice for $A(x)$ and $B(x)$ given in (3.3) is a suitable selection, since it makes (3.2) controllable. Then, there always exists a control law (3.4) that makes the compensated system locally asymptotically stable by the following theorem.

**Theorem 1.** Assume the system in (2.6), such that $f(x)$ and $\partial f(x)/\partial x_i$ $(i=1,\ldots,n)$ are continuous in $x$ for all $|x| \leq \tilde{r}$, $\tilde{r} > 0$, and that $f(x)$ can be written as $f(x)=A(x)x$, where $g(x)=B(x)$. Assume further that $A(x)$ and $B(x)$ are continuous. If $A(x)$, and $B(x)$ are chosen such that the pair $(A(x),B(x))$ is stabilizable for all $x \in M \subset \mathbb{R}^n$, then the system (2.6) is said to be locally stabilizable.

**Proof.** Refer to proof of Theorem 5.1 in Banks et al. [8].

Therefore, there is a need to solve the state-dependent Riccati equation (3.5) to realize an SDRE-based NQR law. The real-time computation method for the required SDRE is given in the following subsection.

B. Real-Time Gradient Type Neural Network Solution of State-Dependent Riccati Equations

It is well known that the solution of the SDRE could not be found analytically [8]–[12], but only for very limited number of nonlinear systems. Therefore, one should require an online computation method for SDRE.

In the literature, a few number of computational methods have been proposed to solve SDRE, in real-time [9]–[12]. In addition, most of these methods are numerically complex, relatively not fast enough to achieve nonlinear system stabilization, and/or depend on correct initial conditions. Here, we present a new fast, computationally simple, and efficient real-time method for state-dependent Riccati equation based nonlinear control synthesis using gradient-type neural networks. This technique is extended from the solution of algebraic Riccati equation in [12] and [13] to SDRE’s solution.

For this purpose, consider (3.5) as a SDRE that one needs to solve. As it is given, the solution of $P(x)$ must be positive definite and symmetric. It is well known that (3.5) has a unique solution which is positive-definite and symmetric if there exists a Cholesky factor [12]. Therefore, a constraint for positive-definiteness and symmetricity of $V(x)$ is added by requiring,

$$G_i(P(x),L(x)) = [g_{i,j}] = L(x)L^T(x) - P(x) = 0, \quad j,k = 1,\ldots,n$$

(3.9)

where $g_{i,j}$ is the $jk^{th}$ element of the objective function $G_i$, and $L$ is a Cholesky factor. Let the following be (3.5) that we need to solve to obtain a robust stabilizing control law,

$$G_i(V(x)) = [g_{i,j}] = A^T(x)P(x) + P(x)A(x) + Q - P(x)N(x)P(x) = 0, \quad j,k = 1,\ldots,n$$

(3.10)

where $N(x)=B(x)R^{-1}B^T(x)$, and $g_{i,j}$ is the $jk^{th}$ element of the objective function $G_i$. To solve $P(x)$ from (3.9) and (3.10), the following Lyapunov energy function is first derived [13],

$$E[G_i(P(x),L(x)),G_i(P(x))]=\frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} [g_{i,j}^2 + g_{j,i}^2]$$

(3.11)

Then, a matrix-oriented gradient algorithm is developed to find the update rule for $P(x)$ by changing the variables in the direction of the negative gradient of the energy function $E$ to minimize (3.11), as
\[
\frac{dP(x)}{dt} = -n_P \frac{\partial E}{\partial P(x)} \quad (3.12)
\]
\[
\frac{dL(x)}{dt} = -n_L \frac{\partial E}{\partial L(x)} \quad (3.13)
\]

Therefore, the update law can be given as [12], [13],
\[
\frac{dP(x)}{dt} = -n_P \bigl[A(x)\Psi_1 + \Psi_1 A^T(x) + \Psi_2 \bigr] - N(x)P(x)\Psi_1 - \Psi_1 P(x)N^T(x)
\]
\[
\frac{dL(x)}{dt} = -n_L \bigl[\Psi_2 L(x)\bigr] \quad (3.14)
\]
\[
\frac{dL(x)}{dt} = -n_L \bigl[\Psi_2 L(x)\bigr] \quad (3.15)
\]

where \(n_P\) and \(n_L\) are positive scalar learning factors, and
\[
\Psi_1(V(x)) = \mathcal{S} \left[ A^T(x)P(x) + P(x)A(x) + O \right. \\
\left. -P(x)N(x)P(x) \right]
\]
\[
\Psi_2(V(x), L(x)) = \mathcal{S} \left[ L(x)L^T(x) - P(x) \right]
\]

where \(\mathcal{S}\) is a symmetric non-decreasing activation function. Typical examples of \(\mathcal{S}\) is given in [12]. Here, we selected the activation function as
\[
\mathcal{S}(f(x)) = f(x) \quad (3.16)
\]
\[
\mathcal{S}(f(x)) = f(x) \quad (3.17)
\]

where \(\mathcal{S}\) is a symmetric non-decreasing activation function. Typical examples of \(\mathcal{S}\) is given in [12]. Here, we selected the activation function as
\[
\mathcal{S}(f(x)) = f(x) \quad (3.18)
\]

The architecture of the gradient-type neural network seeking the positive definite and symmetric solution of (3.5) consists of two bidirectionally connected layers, where (3.16), (3.17) act as hidden layers, and (3.14), (3.15) act as output layers [12], [13]. A schematic of the SDRE neural system is shown in Fig. 3.

![Fig. 3 - Schematic of the SDRE neural system](image)

**IV. SIMULATION RESULTS**

In this section, the proposed approach has been carried out for multilevel cascaded STATCOM system in order to verify the applicability and the performance of the nonlinear quadratic optimal control strategy, using MATLAB/Simulink.

For this purpose, the parameters of the STATCOM model are given in Table I [see 3], while the nonlinear quadratic regulator (NQR) and gradient-type neural network parameters are given Table II. It should be mentioned here that additional white noise has been added to the STATCOM system model in order to make the simulation results similar to the results under real-time conditions. The results, are shown in figures Fig.4 – Fig.9.

Fig.4 – Fig.6 show the active current, reactive current, and total DC voltage variations, respectively. From Fig.5 and Fig.6, it is obvious that the quadrature current and DC voltage track their desired values with a settling time of 7 ms without overshoot. Reactive current change affects DC voltage, and DC voltage change affects reactive current as a disturbance with a small magnitude. However, the NQR technique recovers these disturbances in a short time, smaller than 7 ms. Moreover, the magnitude of the control signals, in Fig.7 and Fig.8, are small enough to be implemented for real-time control of STATCOM systems, which is a rare property for other nonlinear control methods in the literature. Finally, Fig.9 shows the efficiency of the gradient-type neural network in solving the state-dependent Riccati equation. This online computation approach is also fast enough to be implemented in real-time.

<table>
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<tr>
<th>TABLE I MODEL PARAMETERS</th>
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<tr>
<td>Model Symbols</td>
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<th>TABLE II CONTROLLER PARAMETERS</th>
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<tr>
<td>Controller Symbols</td>
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<td>(Q)</td>
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<tr>
<td>(R)</td>
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<tr>
<td>(n_P, n_L)</td>
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<td>(P(0), L(0))</td>
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![Fig. 4 – Active current](image)

![Fig. 5 - Reactive current (solid line) and its reference (dashed line)](image)
V. Conclusion

A new nonlinear quadratic optimal control approach for cascaded multilevel static compensators was reported, based on state-dependent Riccati equation (SDRE) and its online gradient-type neural network solution techniques. The proposed approach was successful in tracking the desired reactive-type current and DC voltage references in a relatively short time with an effective dynamic response. In addition, the gradient-type neural network was shown to be capable of accurately solving the SDRE computation, online, for optimal control implementation. Hence, the proposed control could potentially be used for real-time control of cascaded multilevel STATCOM systems.

References