Tansel Yucelen, Anthony J. Calise, and Rajeev Chandramohan

Georgia Institute of Technology
School of Aerospace Engineering
Atlanta, GA 30332-0150

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Adaptive Control I
WeA03 – Harborside Ballroom D

June 30, 2010
Outline

1 Motivation
   - Adaptive Control
   - Modification Terms
   - Gradient Optimization

2 $H_2$ Adaptive Control
   - Preliminaries
   - Formulation
   - Equivalent Adaptation Laws

3 Illustrative Example
   - Wing Rock Dynamics
   - Results

4 Conclusion and Ongoing Research
Outline

1 Motivation
   - Adaptive Control
   - Modification Terms
   - Gradient Optimization

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4 Conclusion and Ongoing Research
Adaptive control is an attractive approach
- Address system uncertainties and nonlinearities
- Preserve stability w/o excessively reliance on models
Adaptive Control (cont'd)

- **Indirect** adaptive control and **direct** adaptive control
  - IAC: Parameter estimation of system & adapting gains
  - DAC: Adapting gains in response to system variations
Goals

- Achieve performance w/o excessively reliance on models
- If possible, minimize the effort on tuning of adaptation gains
## What is a modification term?

- \( \dot{x}(t) = Ax(t) + Bu(t), \quad u(t) = -\hat{W}(t)x(t) \)
- \( \dot{\hat{W}}(t) = \gamma F(\cdot) \)

- \( \dot{\hat{W}}(t) = \gamma (F(\cdot) + G_{\text{mod}}(\cdot)) \)

## Why do we need a modification term?

- Adaptive control performance and robustness can be improved by employing modification terms.

  - To ensure bounded weights:
    - \( c \)-modification (Ioannou and Kokotovic, 1984)
    - \( e \)-modification (Narendra and Annaswamy, 1987)

  - To improve transient response:
    - \( Q \)-modification (Volyanskyy, Calise, and Yang, 2006)
    - Optimal modification (Nguyen, Krishnakumar, and Boskovic, 2008)
    - ALR modification (Calise, Yucelen, Muse, and Yang, 2009)
    - Kalman filter modification (Yucelen and Calise, 2009)
    - \( K \)-modification (Kim, Yucelen, and Calise, 2010)
What is a modification term?

\[ \dot{x}(t) = Ax(t) + Bu(t), \quad u(t) = -\hat{W}(t)x(t) \]
\[ \dot{\hat{W}}(t) = \gamma F(\cdot) \quad \rightarrow \quad \dot{\hat{W}}(t) = \gamma [F(\cdot) + G_{\text{mod}}(\cdot)] \]

Why do we need a modification term?

- Adaptive control performance and robustness can be improved by employing modification terms
  - To ensure bounded weights:
    - \( \sigma \)-modification (Ioannou and Kokotovic, 1984)
    - \( e \)-modification (Narendra and Annaswamy, 1987)
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Gradient Optimization in Adaptive Control

Scalar case

\[ \dot{x}(t) = ax(t) + bu(t), \quad u(t) = -\hat{w}(t)x(t) \]
\[ \dot{\hat{w}}(t) = \gamma [f(\cdot) + \hat{g}_{\text{mod}}(\cdot)], \quad \hat{g}_{\text{mod}}(\cdot) = -\hat{\hat{g}}_{\text{mod}}(\cdot)\hat{w}(t), \quad \hat{\hat{g}}_{\text{mod}}(\cdot) \geq 0 \]

Claim: Adaptation laws with modification terms are viewed as the negative gradient of an associated linear constraint in the form \( \hat{w}\phi_1(\cdot) = \phi_2(\cdot) \).

Proof: Consider the cost \( J(\hat{w}(t)) = ||\hat{w}\phi_1(\cdot) - \phi_2(\cdot)||_F^2 \), where its negative gradient with respect to \( \hat{w}(t) \) can be given by

\[ -\frac{\partial J(\hat{w}(t))}{\partial \hat{w}(t)} = -\phi_1^2(\cdot)\hat{w}(t) + \phi_1(\cdot)\phi_2(\cdot). \]

Let \( \phi_1(\cdot) = \sqrt{\hat{\hat{g}}_{\text{mod}}(\cdot)} \) and \( \phi_2(\cdot) = f(\cdot)/\phi_1(\cdot) \). Then,

\[ \dot{\hat{w}}(t) = \gamma \left[ -\frac{\partial J(\hat{w}(t))}{\partial \hat{w}(t)} \right] = \gamma [f(\cdot) - \hat{\hat{g}}_{\text{mod}}(\cdot)\hat{w}(t)]. \]
Scalar case

\[ \begin{align*}
\dot{x}(t) &= ax(t) + bu(t), \quad u(t) = -\hat{w}(t)x(t) \\
\dot{\hat{w}}(t) &= \gamma [f(\cdot) + g_{\text{mod}}(\cdot)], \quad g_{\text{mod}}(\cdot) = -\hat{g}_{\text{mod}}(\cdot)\hat{w}(t), \quad \hat{g}_{\text{mod}}(\cdot) \geq 0
\end{align*} \]

Claim: Adaptation laws with modification terms are viewed as the negative gradient of an associated linear constraint in the form \( \hat{w}\phi_1(\cdot) = \phi_2(\cdot) \).

Proof: Consider the cost \( J(\hat{w}(t)) = \|\hat{w}\phi_1(\cdot) - \phi_2(\cdot)\|^2 \), where its negative gradient with respect to \( \hat{w}(t) \) can be given by

\[ -\frac{\partial J(\hat{w}(t))}{\partial \hat{w}(t)} = -\phi_1^2(\cdot)\hat{w}(t) + \phi_1(\cdot)\phi_2(\cdot). \]

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Gradient Optimization in Adaptive Control

**Scalar case**

- \( \dot{x}(t) = ax(t) + bu(t), \quad u(t) = -\hat{w}(t)x(t) \)
- \( \dot{\hat{w}}(t) = \gamma[f(\cdot) + g_{\text{mod}}(\cdot)], \quad g_{\text{mod}}(\cdot) = -\hat{g}_{\text{mod}}(\cdot)\hat{w}(t), \quad \hat{g}_{\text{mod}}(\cdot) \geq 0 \)

**Claim:** Adaptation laws with modification terms are viewed as the negative gradient of an associated linear constraint in the form \( \hat{w}\phi_1(\cdot) = \phi_2(\cdot) \).

**Proof:** Consider the cost \( J(\hat{w}(t)) = ||\hat{w}\phi_1(\cdot) - \phi_2(\cdot)||_p^2 \), where its negative gradient with respect to \( \hat{w}(t) \) can be given by

\[
- \frac{\partial J(\hat{w}(t))}{\partial \hat{w}(t)} = -\phi_1^2(\cdot)\hat{w}(t) + \phi_1(\cdot)\phi_2(\cdot).
\]

Let \( \phi_1(\cdot) = \sqrt{\hat{g}_{\text{mod}}(\cdot)} \) and \( \phi_2(\cdot) = f(\cdot)/\phi_1(\cdot) \). Then,

\[
\dot{\hat{w}}(t) = \gamma - \frac{\partial J(\hat{w}(t))}{\partial \hat{w}(t)} = \gamma[f(\cdot) - \hat{g}_{\text{mod}}(\cdot)\hat{w}(t)].
\]
Both gradient and $H_2$ optimization approaches can be used to enforce a linear constraint to weight update law. **Difference?**

### Standard (gradient optimization based) adaptive control
- Improved performance
- Slow parameter convergence towards a local minimum
- Fixed adaptation gain (often needs to be chosen high)

### $H_2$ adaptive control
- Improved performance
- Fast convergence
- Time-varying adaptation gain
  - $H_2$ gain (optimal)
  - Does not require excessive tuning
- More effective in developing adaptation laws
Both gradient and $\mathcal{H}_2$ optimization approaches can be used to enforce a linear constraint to weight update law. **Difference?**

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Nonlinear Uncertain Dynamical System

- \( (A, B) \) and \( x(t) \) are known
- \( \Delta(x) \) is unknown matched uncertainty
- Linear parametrization: \( \Delta(x(t)) = W^T \beta(x(t)) + \varepsilon(x(t)) \)
**Adaptive Control**

**Preliminaries**

**Reference System**

\[
\dot{x}_m(t) = A_m x(t) + B_m u(t)
\]

**Adaptation Law**

\[
x(t) = A x(t) + B [u(t) + \Delta x(t)]
\]

- \( A_m \) is Hurwitz
- Lyapunov equation: \( 0 = A_m^T P + P A_m + L \)
- Matching conditions: \( A_m = A - B K_1 \) and \( B_m = B K_2 \)
Nominal (Baseline) Control

Control: \( u(t) = u_n(t) - u_{ad}(t) \)
Nominal Control: \( u_n(t) = -K_1x(t) + K_2r(t) \)
Adaptive Control

**Preliminaries**

Adaptive Control

\[ x(t) = x_m(t) + \Delta x(t) \]

Control:

\[ u(t) = u_a(t) - u_{ad}(t) \]

Adaptive Control:

\[ u_{ad}(t) = \hat{W}^T(t)\beta(x(t)) \]
Adaptation law with $\sigma$-modification

\[
\dot{W}(t) = \gamma \left[ \beta(x(t))e^T(t)PB - \sigma \hat{W} \right], \quad \gamma > 0, \quad \sigma > 0
\]

Linear constraint for adaptation law with $\sigma$-modification

The gradient based $\sigma$-modification adaptive control law given above can be obtained by using a gradient approach to enforcing the linear constraint

\[W^T \phi_1 = \phi_2(x(t)),\]

on the weights by minimizing the cost

\[
\sigma(W) = \frac{1}{2} ||W^T \phi_1 - \phi_2(x(t))||^2, \quad \text{where} \quad \phi_1 = \sqrt{\sigma} I \quad \text{and} \quad \phi_2(x(t)) = \frac{1}{\sqrt{\sigma}} \beta(x(t))e^T(t)PB.
\]
Adaptation law with $\sigma$-modification

$$\dot{\hat{W}}(t) = \gamma [\beta(x(t))e^T(t)PB - \sigma \hat{W}], \quad \gamma > 0, \quad \sigma > 0$$

Linear constraint for adaptation law with $\sigma$-modification

The gradient based $\sigma$-modification adaptive control law given above can be obtained by using a gradient approach to enforcing the linear constraint

$$W^T \phi_1 = \phi_2(x(t)),$$

on the weights by minimizing the cost

$$\mathcal{J}_\sigma(W) = \frac{1}{2} ||W^T \phi_1 - \phi_2(x(t))||^2,$$

where $\phi_1 = \sqrt{\sigma}I$ and $\phi_2(x(t)) = \frac{1}{\sqrt{\sigma}} \beta(x(t))e^T(t)PB.$
Adaptation law with $e$-modification

$$
\dot{W}(t) = \gamma [\beta(x(t)) e^T(t) PB - \sigma |e(t)| \hat{W}], \quad \gamma > 0, \quad \sigma > 0
$$

Linear constraint for adaptation law with $e$-modification

The gradient based $e$-modification adaptive control law given above can be obtained by using a gradient approach to enforcing the linear constraint

$$W^T \phi_1 = \phi_2(x(t)),$$

on the weights by minimizing the cost $J_\sigma(W) = \frac{1}{2} ||W^T \phi_1 - \phi_2(x(t))||^2$, where $\phi_1(x(t)) = \sqrt{\sigma |e(t)| I}$ and $\phi_2(x(t)) = \frac{1}{\sqrt{\sigma |e(t)|}} \beta(x(t)) e^T(t) PB$. 
Adaptation law with $e$-modification

$$
\dot{\hat{W}}(t) = \gamma [\beta(x(t))e^T(t)PB - \sigma |e(t)| \hat{\hat{W}}], \quad \gamma > 0, \quad \sigma > 0
$$

Linear constraint for adaptation law with $e$-modification

The gradient based $e$-modification adaptive control law given above can be obtained by using a gradient approach to enforcing the linear constraint

$$W^T \phi_1 = \phi_2(x(t)),$$

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$$J_\sigma(W) = \frac{1}{2} ||W^T \phi_1 - \phi_2(x(t))||^2,$$

where

$$\phi_1(x(t)) = \sqrt{\sigma |e(t)| I}$$

and

$$\phi_2(x(t)) = \frac{1}{\sqrt{\sigma |e(t)|}} \beta(x(t))e^T(t)PB.$$
Assumption

The constraint on the ideal weight matrix in an adaptive control design has the linear form:

\[ W^T \phi_1(t, x(t), u(t)) = \phi_2(t, x(t), u(t)) \]

Stochastic process

The problem of estimating \( W \) while enforcing the above linear constraint can be viewed as a standard problem in estimation theory by defining:

\[
\dot{W} = D_{w1} q(t) \\
\gamma(t) = \phi_1(t, x(t), u(t))^T W + D_{w2} q(t)
\]
Assumption

The constraint on the ideal weight matrix in an adaptive control design has the linear form:

$$W^T \phi_1(t, x(t), u(t)) = \phi_2(t, x(t), u(t))$$

Stochastic process

The problem of estimating \( W \) while enforcing the above linear constraint can be viewed as a standard problem in estimation theory by defining:

$$\dot{W} = D_{w1} q(t)$$
$$y(t) = \phi_1(t, x(t), u(t))^T W + D_{w2} q(t)$$
Construct an estimator for $W$ as

$$
\dot{\hat{W}}(t) = B_e [y(t) - \phi_1(t, x(t), u(t)) \dot{W}(t)]
$$

where logical choice for $y(t)$ is $\phi_2(t, x(t), u(t))$.

**Performance variable**

Let $z(t)$ be the performance variable given by

$$
z(t) = E_w(W - \hat{W}(t)) = E_w e_w(t),
$$

where the goal is to maintain $z(t)$ small under $q(t)$.

**Error dynamics**

$$
\dot{e}_w(t) = A_e e_w + (D_w1 - B_w D_w2) q(t), \quad A_e \triangleq -B_e \phi_1(t, x(t), u(t))^T
$$
Construct an estimator for $W$ as

$$\dot{\hat{W}}(t) = B_e [y(t) - \phi_1(t, x(t), u(t))^T \hat{W}(t)]$$

where logical choice for $y(t)$ is $\phi_2(t, x(t), u(t))$.

Let $z(t)$ be the performance variable given by

$$z(t) = E_w(W - \hat{W}(t)) = E_w e_w(t),$$

where the goal is to maintain $z(t)$ small under $q(t)$.

Error dynamics

$$\dot{e}_w(t) = A_e e_w + (D_{w1} - B_e D_{w2})q(t), \quad A_e \triangleq -B_e \phi_1(t, x(t), u(t))^T$$
Construct an estimator for $W$ as

$$\dot{\hat{W}}(t) = B_e [y(t) - \phi_1(t, x(t), u(t))^T \hat{W}(t)]$$

where logical choice for $y(t)$ is $\phi_2(t, x(t), u(t))$.

**Performance variable**

Let $z(t)$ be the performance variable given by

$$z(t) = E_w (W - \hat{W}(t)) = E_w e_w(t),$$

where the goal is to maintain $z(t)$ small under $q(t)$.

**Error dynamics**

$$\dot{e}_w(t) = A_e e_w + (D_{w1} - B_e D_{w2}) q(t), \quad A_e \triangleq -B_e \phi_1(t, x(t), u(t))^T$$
Error system from $q(t)$ to $z(t)$

$$G_e(s) = E_w(sI - A_e)^{-1} D, \quad \tilde{D} \triangleq D_{w1} - B_e D_{w2}$$

$\mathcal{H}_2$ optimization

Now, one needs to determine Kalman gain $B_e$ by minimizing the $\mathcal{H}_2$ norm of $G_e(s)$,

$$J(B_e) = ||G_e(s)||_2^2 = \text{tr}[E_w S(t) E_w^T],$$

subject to $S(t) = A_e S(t) + S(t) A_e^T + D D^T$. Minimization using Lagrange multipliers gives the Kalman gain:

$$B_e = S(t) \phi_1(t,x(t),u(t)) R^{-1}, \quad R \triangleq D_{w2} D_{w2}^T > 0.$$
Error system from $q(t)$ to $z(t)$

$$\mathcal{G}_e(s) = E_w(sI - A_e)^{-1} \tilde{D}, \quad \tilde{D} \triangleq D_{w1} - B_eD_{w2}$$

$\mathcal{H}_2$ optimization

Now, one needs to determine Kalman gain $B_e$ by minimizing the $\mathcal{L}_2$ norm of $\mathcal{G}_e(s)$,

$$J(B_e) = ||\mathcal{G}_e(s)||^2_2 = \text{tr}[E_wS(t)E^T_w],$$

subject to $\dot{S}(t) = A_sS(t) + S(t)A^T_s + D\tilde{D}^T$. Minimization using Lagrange multipliers gives the Kalman gain:

$$B_e = S(t)\phi_1(t, x(t), u(t))R^{-1}, \quad R \triangleq D_{w2}D^T_{w2} > 0$$
\[ \dot{W}(t) = -S(t)\phi_1(t, x, u)R(t)^{-1}\left[\phi_1(t, x, u)^T W(t) - \phi_2(t, x, u)^T\right] \]
\[ \dot{S}(t) = -S(t)\phi_1(t, x, u)R^{-1}(t)\phi_1(t, x, u)^T S(t) + Q, \quad Q = D_{w1}^T D_{w1} > 0 \]

\( \phi_1(t, x, u) \) and \( \phi_2(t, x, u) \)?

- Adaptation law with \( \sigma \)-modification:
  - \( \phi_1(t, x, u) = \sqrt{\sigma}(t) \)
  - \( \phi_2(t, x, u) = \frac{1}{\sqrt{\sigma}}\beta(x(t))e^T(t)PB \)

- Adaptation law with \( e \)-modification:
  - \( \phi_1(t, x, u) = \sqrt{\sigma|x(t)|}I \)
  - \( \phi_2(t, x, u) = \frac{1}{\sqrt{\sigma|x(t)|}}\beta(x(t))e^T(t)PB \)
Adaptive Control

Formulation

H₂ adaptation law

\[ \dot{W}(t) = -S(t)\phi_1(t,x,u)R(t)^{-1}\left[\phi_1(t,x,u)^T\dot{W}(t) - \phi_2(t,x,u)^T\right] \]
\[ \dot{S}(t) = -S(t)\phi_1(t,x,u)R^{-1}(t)\phi_1(t,x,u)^TS(t) + Q, \quad Q = D_w^T D_w > 0 \]

ϕ₁(t,x,u) and ϕ₂(t,x,u)?

- Adaptation law with σ - modification:
  - \( \phi_1(t,x,u) = \sqrt{\sigma}I \)
  - \( \phi_2(t,x,u) = \frac{1}{\sqrt{\sigma}}\beta(x(t))e^T(t)PB \)

- Adaptation law with e - modification:
  - \( \phi_1(t,x,u) = \sqrt{\sigma|e(t)|}I \)
  - \( \phi_2(t,x,u) = \frac{1}{\sqrt{\sigma|e(t)|}}\beta(x(t))e^T(t)PB \)
Equivalent $H_2$ Adaptation Laws

$H_2$ adaptation law with $\sigma$- modification ($H_2^\sigma$-AC)

\[
\dot{\hat{W}}(t) = S(t)R^{-1}\left[\beta(x(t))e^T(t)PB - \sigma\hat{W}(t)\right] \\
\dot{\hat{S}}(t) = -\sigma\hat{S}(t)R^{-1}\hat{S}(t) + Q
\]

$H_2$ adaptation law with $e$- modification ($H_2^e$-AC)

\[
\dot{\hat{W}}(t) = S(t)R^{-1}\left[\beta(x(t))e^T(t)PB - \sigma|e(t)|\hat{W}(t)\right] \\
\dot{\hat{S}}(t) = -\sigma|e(t)|S(t)R^{-1}S(t) + Q
\]
$\mathcal{H}_2$ adaptation law with $\sigma$- modification ($\mathcal{H}_2^\sigma$-AC)

$$\dot{W}(t) = S(t)R^{-1}\left[\beta(x(t))e^T(t)PB - \sigma\dot{W}(t)\right]$$

$$\dot{S}(t) = -\sigma S(t)R^{-1}S(t) + Q$$

$\mathcal{H}_2$ adaptation law with $e$- modification ($\mathcal{H}_2^e$-AC)

$$\dot{W}(t) = S(t)R^{-1}\left[\beta(x(t))e^T(t)PB - \sigma |e(t)|\dot{W}(t)\right]$$

$$\dot{S}(t) = -\sigma |e(t)|S(t)R^{-1}S(t) + Q$$
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A model of wing rock dynamics:

\[
\begin{bmatrix}
\dot{x}_1(t) \\
\dot{x}_2(t)
\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [u(t) + \Delta(x(t))]
\]

- Standard (gradient) adaptive controller versus $H_2$-adaptive controller
- Control objective is to minimize the oscillations
- $L = I$ for both adaptive architectures
- $Q = 0.1I$ and $R = 0.001I$ for $H_2$-adaptive controller
- Reference system is second order with a natural frequency of 1.2 rad/s, and a damping of 0.5
Illustrative Example

Nominal (Baseline) Control Response

Nominal control response
**Standard $\epsilon$- Modification Adapt. Contr.**

Standard $\epsilon$- modification adaptive controller: $\gamma = 1$ and $\gamma \times \sigma = 10$
Standard $\epsilon$- modification adaptive controller: $\gamma = 25$ and $\gamma \times \sigma = 10$
Standard $\varepsilon$- modification adaptive controller: $\gamma = 250$ and $\gamma \times \sigma = 10$
Standard $\epsilon$-Modification Adaptive Control (cont'd)

Standard $\epsilon$-modification adaptive controller: $\gamma = 1500$ and $\gamma \times \sigma = 10$
$\mathcal{H}_2^* -$ adaptive controller

$\mathcal{H}_2^* -$ adaptive controller: $\sigma = 10$

Illustrative Example

Results

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Illustrative Example

$\mathcal{H}_2$-adaptive controller (cont'd)

$\mathcal{H}_2$-gain

$|| S(t) R^{-1} ||$

Time (sec)

$H_2$-gain

$H_2$-adaptive controller

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Conclusion and Ongoing Research

Conclusion

- Construction of adaptive controller with $H_2$ optimization
- Improved performance
- Time-varying adaptation gain
  - $H_2$ gain (optimal)
  - Does not require excessive tuning
- The proposed approach can be used in place of all direct adaptive control laws that can be equivalently viewed as the gradient of a norm of the error in the linear constraint
Conclusion

- **Construction** of adaptive controller with $\mathcal{H}_2$ optimization
- **Improved** performance
- **Time-varying adaptation gain**
  - $\mathcal{H}_2$ gain (optimal)
  - Does not require excessive tuning
- The proposed approach can be used in place of all direct adaptive control laws that can be equivalently viewed as the gradient of a norm of the error in the linear constraint
Ongoing Research

- Extensions to derivative-free adaptive control
- Flight test by Georgia Tech UAV Research Facility
  - Chowdhary, Johnson, Kimbrell, Chandramohan, and Calise, “Adaptive control of an airplane under severe structural damage,” AIAA GNC 2010
  - Chandramohan, Yucelen, Calise, and Johnson, “Flight test results for Kalman filter and $H_2$ modification in adaptive control,” AIAA GNC 2010

GT Twinstar, 50% Right Wing Off
Thank You

Conclusion and Ongoing Research

QUESTIONS?