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Flight Test Results for Kalman Filter and $\mathcal{H}_2$ Modification in Adaptive Control

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Flight control systems designed using linear control theory do not perform adequately over the entire operating range of aircraft. Furthermore flight control systems developed for unmanned areil vehicles are expected to meet performance specifications in the presence of nonlinearities and/or failures. Adaptive controllers are ideal candidates for flight control sytem design due to their inherent ability to adapt to compensate for nonlinearities and or failures. Modification terms like e-modification, σ-modification and others are added to standard adaptive laws to ensure boundedness of the adaptive parameters. In this paper flight test results for Kalman filter implementations of ALR-modification for a Nonlinear in parameters neural network adaptive controller are presented. In addition flight test results of an $\mathcal{H}_2$ adaptive control with e-modification are also presented.

I. Introduction

Design of robust flight control systems has recently become a necessity due to advances in the design of aircraft that are required to perform at extreme angles of attack and side slip. In addition mission requirements impose additional constriants on flight control systems. Adaptive controllers have shown to satisfy strigent performance requirements in the presence of uncertainities and/or failures. In literature adaptive controllers obtined by augmenting linear controllers with neural networks have been shown to exceed the performance of linear controllers[1−2]. In these controllers the network weights are updated online so as to cancel uncertainities and compensate for actuator and structural failures. In addition to standard network update laws, modification terms have been used to ensure that the adaptive parameters remain uniformly ultimately bounded[3−12]. The modification terms are obtained by taking the gradient of a norm on the constraint applied to a function of the weights. Unfortunately, gradient methods can result in slow parameter convergence. Most modification terms are appended to the standard adaptive law with a fixed gain. In order to improve performance of the standard adaptive law with modification, it becomes necessary to use high gains with modification terms. One of the draw backs of using high gains with modification terms is that they can interact negatively with the unmodelled dynamics leading to oscillatory response which is undesireable. Recently a Kalman filter (KF) method of imposing constraints has been developed that has shown to be more effective than standard modification terms[13−14]. Additionally the KF method has been extended recently for more general constraints and termed $\mathcal{H}_2$ adaptive control,[1920].

A vareity of modification terms that have been studied in literature[3−12]. Included among these, σ-modification[3] adds a pure damping term to the adaptive law, while e-modification[4] adds a variable damping term that depends on the training error signal. These terms are introduced to ensure that the adapted weights remain bounded. Background learning modification has been explored to incorporate past data to improve long term performance[5−7]. Q- modification[8−10] is similar to background learning and improves

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adaptation performance by using a moving window of the integrated system uncertainty. A new optimal control theory based modification that improves adaptation in the presence of large adaptive gain[11] has also been proposed. More recently, an adaptive loop recovery (ALR) approach[12] has been introduced as a modification term in adaptive control with the objective of recovering the loop transfer properties of a reference model.

All of these modification terms can be viewed as to reduce a linear constraint on the weights in an adaptive control algorithm. The proposed KF optimization method can be used as an alternative to all modification terms to achieve the same objective, but with better responses. Flight Test results for KF versions of ALR modifications are presented. In addition flight test results of $\mathcal{H}_2$ adaptive control for the same constraints given above are presented.

The organization of the paper is as follows. Section II provides a brief background on model reference adaptive controller. Section III describes the main concept for reducing a linear constraint violation by employing a KF optimization approach. In Section IV KF-based ALR modification is presented along with flight test results. Section V provides the flight test results of $\mathcal{H}_2$ adaptive control implementation of e-modification term. Conclusions are summarized in Section VI.

II. Model Reference Adaptive Control

A brief overview of model reference adaptive control problem is given next due to the fact that KF based modification and $\mathcal{H}_2$ adaptive control is developed in this context. Consider the following uncertain system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + B(u(t) + \Delta(x(t))), \\ y(t) &= Cx(t), \end{aligned} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the known state vector, $u(t) \in \mathbb{R}^m$ is the control input, $y(t) \in \mathbb{R}^p$ is the output, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{p\times n}$ are known matrices, and $\Delta(\cdot) : \mathbb{R}^n \to \mathbb{R}^m$ is the matched uncertainty. The matrices $A,\ B,\ C$ in Eq. (1) are obtained by linearizing the nonlinear dynamics at selected equilibrium points. A baseline controller for the system in Eq. (1) exists and is given as

$$u_n(t) = -K_x x(t) + K_r r(t), \tag{2}$$

where $r(t) \in \mathbb{R}^m$, is the bounded reference command, $K_x \in \mathbb{R}^{m\times n}$ is the state gain matrix, and $K_r \in \mathbb{R}^{m\times r}$ is the input gain matrix. For dynamic compenstors, one can augment the controller dynamics with the dynamics in Eq. (1), and consider an expanded state made up of the system states and the controller states, and rewrite the dynamics and controller in the form of Eqs. (1) and (2). The uncertainities in the system need to satisfy the following.

Assumption 1. The uncertainty in Eq. (1) can be parameterized by a nonlinear in the parameters neural network satisfying

$$|\Delta(x(t)) - W^{\mathrm{T}}\beta(V^{\mathrm{T}}(x(t)))| < |\varepsilon(x(t))| < \epsilon,\ \forall x \in \mathcal{B}_\chi, \tag{3}$$

where $\mathcal{B}_\chi = \{x(t) : |x(t)| \leq \chi\}$, $W \in \mathbb{R}^{s\times m}$ and $V \in \mathbb{R}^{l\times s-1}$ are the unknown weight matrices, $\beta(V^{\mathrm{T}}(x(t))) = [1, \beta_1(V_1^{\mathrm{T}}(x)), \beta_2(V_2^{\mathrm{T}}(x)), .., \beta_{s-1}(V_{s-1}^{\mathrm{T}}(x))]^{\mathrm{T}} \in \mathbb{R}^s$, $\beta_i(z) = 1/1 + e^{-a_i z}$, $a_i > 0$, $i = 1, 2, ..., s-1$ and V_i denotes the ith column of V

Knowing A,B,K_x and K_r we can construct a reference model for the the closed loop system as

$$\dot{x}_m(t) = A_m x_m(t) + B_m r(t), \tag{4}$$

where $Am = A - BK_x$, $Bm = BK_r$, and it is assumed that Am is Hurwitz by design. Now the nominal controller is augmented with an adaptive element

$$u(t) = u_n(t) - u_a(t), \tag{5}$$

$$u_a(t) = \hat{W}(t)^{\mathrm{T}}\beta(V^{\mathrm{T}}(x(t))), \tag{6}$$

The aim of the adaptive controller design is that error between $y(t)$ and $r(t)$ goes to zero assymptotically while ensuring that all internal signals in the system are bounded. Defining the error, $e(t) = x(t) - x_m(t)$, it has been shown[16] that the following adaptive law

$$\begin{aligned} \dot{\hat{W}}(t) &= -\gamma_w(\beta(\hat{t}) - \hat{\beta}'(t)\hat{V}^T(t)\mu(x(t)))e^{\mathrm{T}}(t)PB - \dot{\hat{W}}_m \\ \dot{\hat{V}}(t) &= -\gamma_v(\mu(x(t))e^{\mathrm{T}}(t)PB\hat{w}^{\mathrm{T}}(t)\hat{\beta}'(t)) - \dot{\hat{V}}_m, \end{aligned} \tag{7}$$

where γ_w and γ_v are positive learning rates for the weights, and $P \in \mathbb{R}^{n\times n}$ is a positive definite solution of the Lyapunov equation

$$A_m^{\mathrm{T}}P + PA_m + L = 0, \tag{8}$$

for any $L > 0$ and where $\dot{\hat{W}}_m$ and $\dot{\hat{V}}_m$ are the modification terms. The modification terms can be any of the following modifications σ- modification[3], e-modification[4], and/or parameter projection[15]. These modification terms ensure that $e(t)$ is uniformly ultimately bounded[17]. The adaptive law with e-modification is given by

$$\dot{\hat{W}}(t) = -\gamma_w(\beta(\hat{t}) - \hat{\beta}'(t)\hat{V}^T(t)\mu(x(t)))e^{\mathrm{T}}(t)PB - \sigma_w\|e^{\mathrm{T}}(t)PB\|\hat{W}, \tag{9}$$

$$\dot{\hat{V}}(t) = -\gamma_v(\mu(x(t))e^{\mathrm{T}}(t)PB\hat{w}^{\mathrm{T}}(t)\hat{\beta}'(t)) - \sigma_v\|e^{\mathrm{T}}(t)PB\|\hat{V}, \tag{10}$$

where σ_w and σ_v are positive learning rates.

The resulting adaptive control system is illustrated in Figure 1.

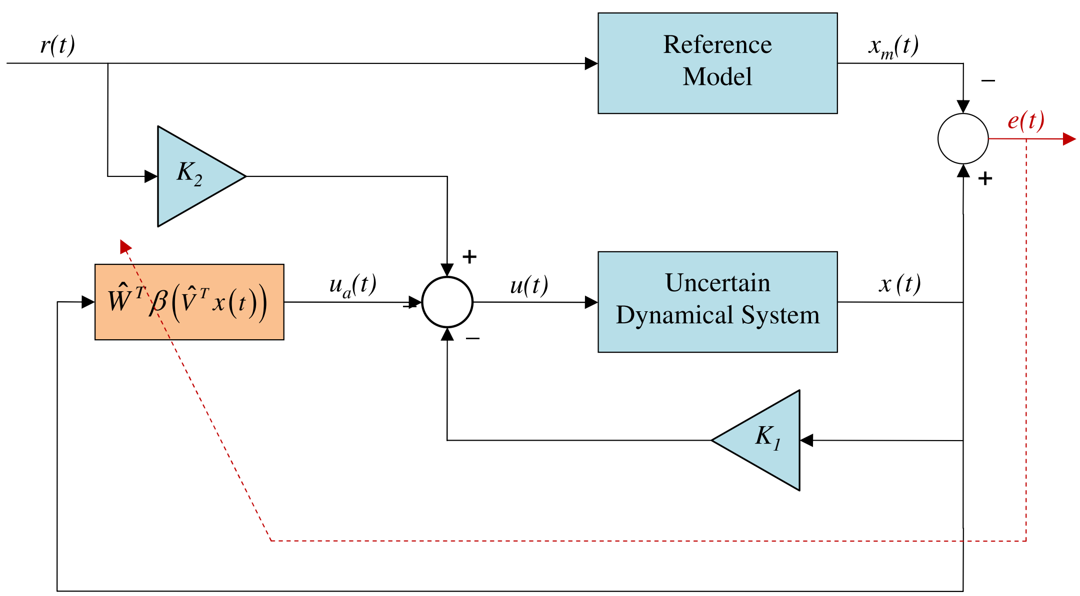

Figure 1. Augmenting adaptive control of a baseline control system.

III. Approximate Enforcement of a Linear Constraint by KF Optimization

This section gives an overview of the KF method for introducing a modification term in an adaptive control law[13−14]. KF method of implemnting modification are limited to the constraints on the weight having the form given in *Assumption 2.*

Assumption 2. The constraint on the weight estimate in an adaptive control design has the following linear form

$$\hat{W}^{\mathrm{T}}(t)\phi_1(t, x(t), u(t)) = 0, \tag{11}$$
$$\hat{V}^{\mathrm{T}}(t)\phi_2(t, x(t), u(t)) = 0, \tag{12}$$

where $\phi_1(\cdot) : [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{s \times l}$, $\phi_2(\cdot) : [0, \infty) \times \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^{l \times s}$ are known. The problem of estimating W and V while approximately enforcing the linear constraint in Eq. (11) and Eq. (12) is now formulated as an optimization problem. Eq. (11) and Eq. (12) can be expressed in the following equivalent vector form

$$vec(\phi_1^{\mathrm{T}}(t, x(t), u(t))\hat{W}(t)) = \Phi_1(t, x(t), u(t))^{\mathrm{T}}\omega_1 = 0 \tag{13}$$
$$vec(\phi_2^{\mathrm{T}}(t, x(t), u(t))\hat{V}(t)) = \Phi_2(t, x(t), u(t))^{\mathrm{T}}\omega_2 = 0 \tag{14}$$

where $\Phi_1(t, x(t), u(t)) = I_{m \times m} \otimes \phi(t, x(t), u(t)) \in \mathbb{R}^{ms \times ml}$ and $\omega_1 = vec(W) \in \mathbb{R}^{ms}$. and $\Phi_2(t, x(t), u(t)) = I_{l \times l} \otimes \phi(t, x(t), u(t)) \in \mathbb{R}^{ls-1 \times ml}$ and $\omega_2 = vec(V) \in \mathbb{R}^{l \times s-1}$.

Define the stochastic processes:

$$\dot{\omega}_1 = q_1(t), \tag{15}$$
$$z_1(t) = \Phi_1^{\mathrm{T}}(t, x(t), u(t))\omega_1 + r_1(t), \tag{16}$$

$$\dot{\omega}_2 = q_2(t), \tag{17}$$
$$z_2(t) = \Phi_2^{\mathrm{T}}(t, x(t), u(t))\omega_2 + r_2(t), \tag{18}$$

where $q_1(t)$ and $q_2(t)$ and $r_1(t)$,$r_2(t)$ are zero-mean, Gaussian, white noise processes with covariances

$$\mathbb{E}\{q_1(t)q_1^{\mathrm{T}}(\tau)\} = \bar{Q}_1\delta(t - \tau), \quad \bar{Q}_1 \in \mathbb{R}^{ms \times ms} > 0 \tag{19}$$
$$\mathbb{E}\{r_1(t)r^{\mathrm{T}}(\tau)\} = \bar{R}_1\delta(t - \tau), \quad \bar{R}_1 \in \mathbb{R}^{ml \times ml} > 0 \tag{20}$$

$$\mathbb{E}\{q_2(t)q_2^{\mathrm{T}}(\tau)\} = \bar{Q}_2\delta(t - \tau), \quad \bar{Q}_2 \in \mathbb{R}^{l(s-1) \times l(s-1)} > 0 \tag{21}$$
$$\mathbb{E}\{r_2(t)r_2^{\mathrm{T}}(\tau)\} = \bar{R}_2\delta(t - \tau), \quad \bar{R}_2 \in \mathbb{R}^{ml \times ml} > 0 \tag{22}$$

and $z_1(t)$ and $z_2(t)$ are regarded as measurements. The estimate of $z_1(t)$ and $z_2(t)$ are then given by

$$\hat{z}_1(t) = \Phi_1^{\mathrm{T}}(t, x(t), u(t))\hat{\omega_1}(t) \tag{23}$$
$$\hat{z}_2(t) = \Phi_2^{\mathrm{T}}(t, x(t), u(t))\hat{\omega_2}(t) \tag{24}$$

where $\hat{\omega}_1(t)$ and $\hat{\omega}_2(t)$ are estimates of ω_1 and ω_2 respectively . The Kalman filters associated with this problem formulations are given by:

$$\dot{\hat{\omega}}_1(t) = \bar{S}_1(t)\Phi_1(t, x(t), u(t))\bar{R}_1^{-1}(z_1(t) - \hat{z_1}(t)), \quad \hat{\omega_1}(0) = 0 \tag{25}$$
$$\dot{\bar{S}}_1(t) = -\bar{S}_1(t)\Phi_1(t, x(t), u(t))\bar{R}_1^{-1}\Phi_1^{\mathrm{T}}(t, x(t), u(t))\bar{S}_1(t) + \bar{Q}_1, \quad \bar{S}_1(0) = \bar{S}_{10} \tag{26}$$

$$\dot{\hat{\omega}}_2(t) = \bar{S}_2(t)\Phi_2(t, x(t), u(t))\bar{R}_2^{-1}(z_2(t) - \hat{z}_2(t)), \quad \hat{\omega}_2(0) = 0 \tag{27}$$
$$\dot{\bar{S}}_2(t) = -\bar{S}_2(t)\Phi_2(t, x(t), u(t))\bar{R}_2^{-1}\Phi_2^{\mathrm{T}}(t, x(t), u(t))\bar{S}_2(t) + \bar{Q}_2, \quad \bar{S}_2(0) = \bar{S}_{20} \tag{28}$$

where $\bar{S}_1(t) \in \mathbb{R}^{ms \times ms}$ and $\bar{S}_2(t) \in \mathbb{R}^{(s-1)l \times (s-1)l}$. Since the constraints in Eq. (11) and (12) are to be satisfied approximately, Choose $z_1(t) = 0$ and $z_2(t) = 0$ in Eq. (25) and (26). If in addition , let $\bar{R}_1 = I_{m \times m} \otimes R_1$, and $\bar{R}_2 = I_{(s-1)\times(s-1)} \otimes R_2$ with $R_1 \in \mathbb{R}^{s \times s} > 0$,$R_2 \in \mathbb{R}^{l \times l} > 0$ and $\bar{Q}_1 = I_{m \times m} \otimes Q1$ and

$\bar{Q}_2 = I_{(s-1)\times(s-1)} \otimes Q2$, with $Q1 \in \mathbb{R}^{s\times s} > 0$ and $Q2 \in \mathbb{R}^{l\times l} > 0$, as shown in[13], Eq.(25)−(28) reduce to:

$$\dot{\hat{W}}(t) = -S_1(t)\phi(t,x(t),u(t))R_1^{-1}\phi_1^{\mathrm{T}}(t,x(t),u(t))\hat{W}(t), \quad \hat{W}(0)=0 \tag{29}$$
$$\dot{S}_1(t) = -S_1(t)\phi_1(t,x(t),u(t))R_2^{-1}\phi_1^{\mathrm{T}}(t,x(t),u(t))S_1(t)+Q_1, \quad S_1(0)=S_{10} \tag{30}$$

$$\dot{\hat{V}}(t) = -S_2(t)\phi_2(t,x(t),u(t))R_2^{-1}\phi_2^{\mathrm{T}}(t,x(t),u(t))\hat{V}(t), \quad \hat{V}(0)=0 \tag{31}$$
$$\dot{S}_2(t) = -S_2(t)\phi_2(t,x(t),u(t))R_2^{-1}\phi_2^{\mathrm{T}}(t,x(t),u(t))S_2(t)+Q_2, \quad S_2(0)=S_{20} \tag{32}$$

where $S_1(t) \in \mathbb{R}^{s\times s}$ and $S_2(t) \in \mathbb{R}^{l\times l}$.

Incorporating Eq. (29) and (31) as the modification term in the adaptive law given in Eq. (7), This reduces to:

$$\dot{\hat{W}}(t) = -\gamma_w(\beta\hat{(t)} - \hat{\beta}'(t)\hat{V}^T(t)\mu(x(t)))e^{\mathrm{T}}(t)PB - k_w S_1(t)\phi_1(t,x(t),u(t))R_1^{-1}\phi_1(t,x(t),u(t))^{\mathrm{T}}W\hat{(t)}, \tag{33}$$
$$\dot{\hat{V}}(t) = -\gamma_v(\mu(x(t))e^{\mathrm{T}}(t)PB\hat{w}^{\mathrm{T}}(t)\hat{\beta}'(t)) - k_v S_2(t)\phi_2(t,x(t),u(t))R_2^{-1}\phi_2(t,x(t),u(t))^{\mathrm{T}}V\hat{(t)}, \tag{34}$$

where k_w and k_v are positive modification gains. Since $S_1(t)$ and $S_2(t)$ vary over time $k_wS_1(t)$and $k_vS_2(t)$ provide a gain that varies over time.

The ultimately uniformly bounded property of the adpative controller using the KF method of implementing linear constraints has been shown in Ref.21

IV. Flight Test Results

Flight test results using both the KF and $\mathcal{H}_2$ method of implementing mmodification terms are obtained on a small twin engine UAV called the Twinstar. The Twinstar is one of the many UAV platforms available with the UAV lab at the School of Aerospace Engineering at Georgia Tech. The Twinstar is equipped with FCS 20 which is a fully functional autopilot along with an Inertila Measuring Unit, Air data system and a GPS. Since the Twinstar is constructed of reinforced styrofoam, It is also possible to introduce structural failures. In addition to the various platforms to test flight control algorithms, the UAV lab also maintains a ground station to cocontrol UAV's as well as to tune controller parameters during flight. This enables the tuning of controller gains while the UAV is in flight which significantly reduces flight test time.

A. KF based ALR Modification for the Twinstar

In Ref.12 a new method of adaptation that improves the robustness of adaptive control has been proposed. This consists of a modification term that is added to the standard adaptive law. This modification termed as Adaptive Loop Recovery (ALR) is obtained by imposing a linear constraint on the weights such that the loop properties of the refrence model associated with the non-adaptive design are maintained. For the nonlinear in the parameters neural network, in order to prserve the loop properties of the reference model it is only necessary to impose a constraint on the output weights $W\hat{(t)}$. The linear constraint that is necessary to preserve the loop properties of the reference model is given by $W^{\mathrm{T}}\beta_x(x(t)) = 0$, where $\beta_x(x(t)) \triangleq \frac{d\beta(x(t))}{dx(t)} \in \mathbb{R}^{s\times n}$ is the derivative of the basis function with respect to $x(t) \in \mathbb{R}^n$. The gradient of the ALR modification term[12] is found by taking the negative gradient of the following quadratic function with respect to weights, W, as

$$\nu = \frac{1}{2}\mathrm{tr}[(W^{\mathrm{T}}\beta_x(x(t)))^{\mathrm{T}}(W^{\mathrm{T}}\beta_x(x(t)))], \tag{35}$$
$$\frac{\partial\nu}{\partial W} = \beta_x(x(t))\beta_x^{\mathrm{T}}(x(t))W. \tag{36}$$

Combining the modification term in Eq. (36) with the standard adaptive law Eq. (12) results in

$$\dot{\hat{W}}(t) = -\gamma_w(\beta\hat{(t)} - \hat{\beta}'(t)\hat{V}^T(t)\mu(x(t)))e^{\mathrm{T}}(t)PB - k_{grad}\beta_x(x(t))\beta_x^{\mathrm{T}}(x(t))\hat{W}(t)), \tag{37}$$

where k_{grad} is a positive learning rate of this gradient version of ALR modification. Since $\phi(t, x(t), u(t)) = \beta_x(x(t))$ for the ALR problem, applying Eqs. (30) and (29) leads to the following adaptive law

$$\begin{aligned} \dot{\hat{W}}(t) &= -\gamma_w(\hat{\beta(t)} - \hat{\beta}'(t)\hat{V}^T(t)\mu(x(t)))e^{\mathrm{T}}(t)PB - \sigma|e(t)|\hat{W}(t) - k_{kf}S(t)\beta_x(x(t))R^{-1}(t)\beta_x^{\mathrm{T}}(x(t))\hat{W}(t), \\ \dot{S}(t) &= -S(t)\beta_x(x(t))R^{-1}\beta_x(x(t))^{\mathrm{T}}S(t) + Q, \end{aligned} \quad (38)$$

where k_{kf} is a positive learning rate.
The effectiveness of the KF version of ALR modification given in Eq.(38) is evaluated in simulation. The simulation was carried out using the Ga Tech simulation environment which contains a high fidelity model of the Twinstar. The KF based ALR modification term was combined with a baseline adaptive controller develped for the Twinstar. The details of the baseline adaptive controller for the Twinstar is given in Ref 18. In the simulation, the baseline adaptive control and KF based ALR modification is turned on around 20 seconds with k_{kf} set to 10,$\sigma = 0.1$,s_1=1 and $\gamma_w = 1$. In order to test the effectiveness of the KF method of ALR modification to time delays,a time delay of 0.1 milli second was introduced into the elevator actuator at around forty seconds. Figure (2) shows the response for this case. In order to compare the performance of the KF based ALR modification with that of the baseline adaptive controller, another test was carried out during which a time delay of 0.1 milli seconds was introduced around 23 seconds with just the baseline adaptive controller. As seen in Figure (3), the pitch rate error increases. As observed from the plots from simulation, the KF method of implementing ALR improves performance as compared to the baseline adaptive controller.

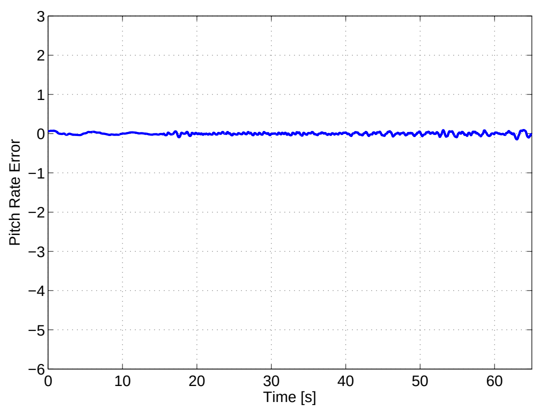

Figure 2. Pitch rate error plot with KF ALR modification

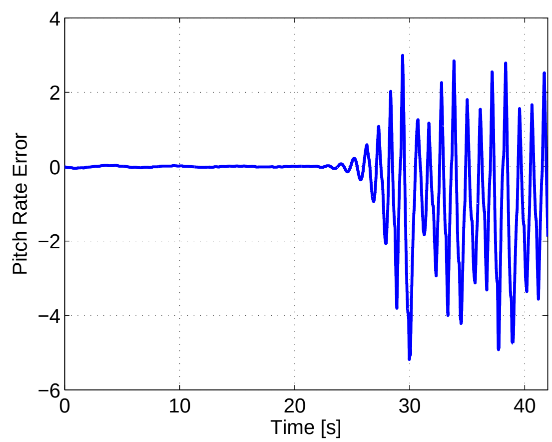

Figure 3. Pitch rate error plot with no ALR modification

To evaluate the KF based ALR modification in flight tests a time delay was injected into the elevator actuator and the pitch rate error was observed. As in the simulation, the baseline adaptive control and KF based ALR modification is turned on around at the beginning of the flightwith k_{kf} set to 1 ,$\sigma = 0.1$,s_1=0.1 and $\Gamma_w = 1$. Next a series of time delays from 0.08 seconds to 0.14 seconds were introduced into the elevator actuator through the ground station. Figure (4) shows the response of the system for this case. As seen from the plots, the KF based ALR modification prevents the aircraft from entering into oscillations in pitch. The oscillations in the pitch rate error peak to around 0.25 $rads^{-1}$. A similar procedure was not flown with the baseline adaptive controller, since the pitch oscillations grow quite large even for small time delays introduced into the elevator actuator channel.

V. $\mathcal{H}_2$ adaptive control for Twinstar

In Ref.19, a novel method for Model reference adaptive control laws that are derived using an H2 optimization approach has been developed. It has been shown in Ref.19 that the $\mathcal{H}_2$ adaptive control provides better tracking control without requiring high adaptation gain and provides an optimal time varying gain. The $\mathcal{H}_2$ adaptive control is a generalization of the KF method described in the above section. This method has been currently developed only for Linear in the parameter neural network. $\mathcal{H}_2$ based $e-$modification was

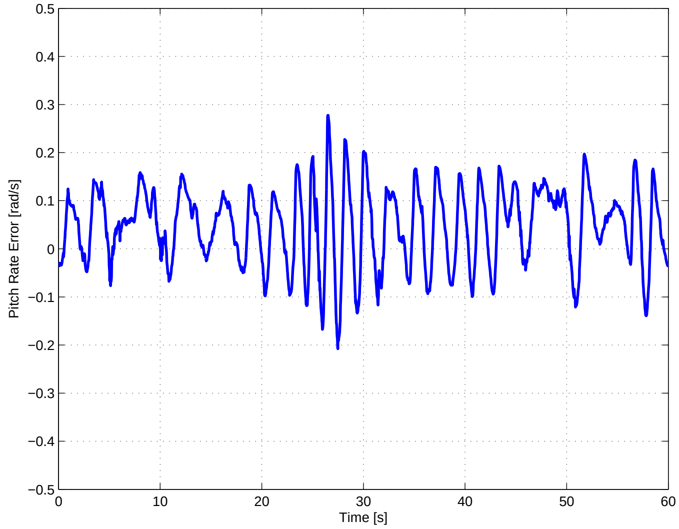

Figure 4. Flight Test Results of Pitch rate error with KF ALR modification

evaluated in flight tests on the Twinstar. Initially a simulation was carried out to determine the performance of the $\mathcal{H}_2$ adaptive control for tracking a commanded flight path. The baseline controller for the Twinstar was not modified from that used in the evaluation of the KF based ALR modification. The baseline controller was augmented with $\mathcal{H}_2$ linear in the parameter neural network. In simulation the aircraft was commanded to follow a race track pattern and performnce of the controller was evaluated. Figure 5 shows the flight trajectory. Also plotted are the innerloop angular errors Figure 6

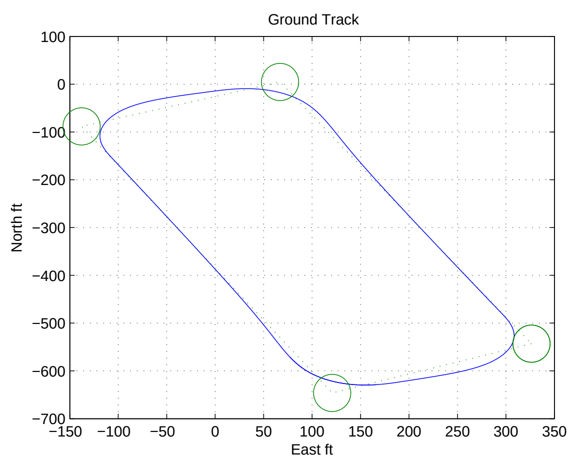

Figure 5. Twinstar Simulation Trajectory

To evaluate the $\mathcal{H}_2$ adaptive control in flight tests, as in the simulation, the aircraft is commanded to follow a race track pattern. As observed from the Figure 7, the aircraft tracks the commanded trajectory with a small error. The inner loop angular errors are also plotted in Figure 8. One of the main advantages of the $\mathcal{H}_2$ adaptive control is that the adaptation gains are time varying and this results in reduced tuning effort, this fact was observed both during simulation runs and flight tests where in no tuning was needed for the adaptive gains.

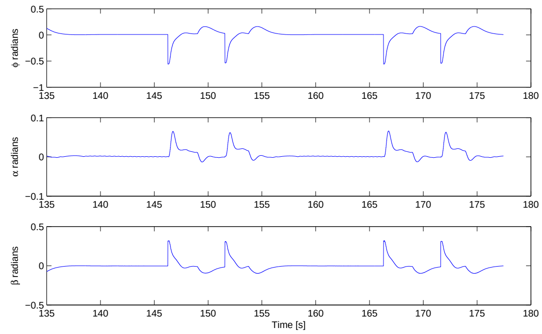

Figure 6. Innerloop errors in Simulation

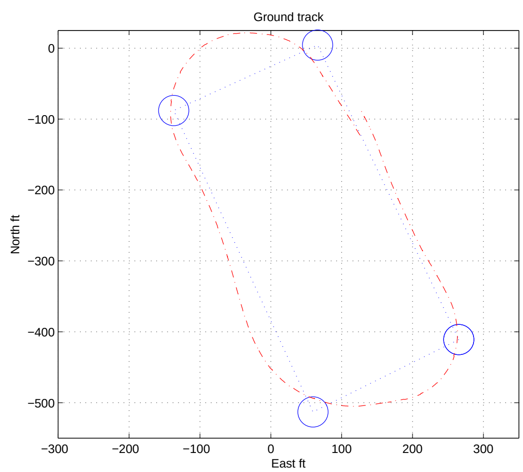

Figure 7. Twinstar Flight Test Trajectory

VI. Conclusions

In this paper a novel method of introducing constraints on the weights using kalman filter modification is extended to the nonlinear in the parameter neural networks. It is shown that the KF implementation of modification terms preserves the UUB property of standard adaptive laws. The performance of the proposed method for ALR modification that preserves loop properties is evaluated both in simulation and in flight test in the presence of time delays. In addition to KF- based ALR modification, flight test results of a novel model reference adaptive control called $\mathcal{H}_2$ have been evaluated in flight tests on the Twinstar. As observed from the flight test results, KF based ALR modification is tolerant to time delays present in the system. Only tracking performance of the $\mathcal{H}_2$ adaptive controller have been evaluated in flight tests. Future flight tests will be used to evaluate this method with actuator and/or structural failures.

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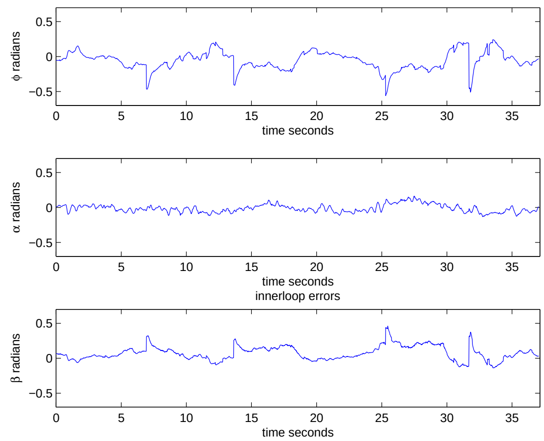

Figure 8. Innerloop errors in Flight Test

References

[1]Calise, A. J., Rysdyk, R., "Nonlinear adaptive flight control using neural networks," IEEE Control Systems Magazine, Vol. 18, No. 6, 1998.

[2]Calise, A. J., Lee, S., Sharma, M., *Develpment of a Reconfigurable Flight Control Law for the X-26 Tailless Fighter Aircraft*, J. Guidance Control and Decision, Sept., 2000.

[3]Ioannou, P., Kokotovic, P., "Instability analysis and improvement of robustness of adaptive control," Automatica, Vol. 20, No. 5, 1984, pp. 583-594.

[4]Narendra, K., Annaswamy, A., "A new adaptive law for robust adaptation without persistent excitation," IEEE Trans. on Automatic Control, Vol. 32, No. 2, 1987, pp. 134-145.

[5]Johnson, E. N., Oh, S. -M., "Adaptive control using combined online and background learning neural network," in AIAA Guidance, Navigation and Control Conference, 2004.

[6]Chowdhary, G., Johnson, E. N., "Theory and flight test validation of long term learning adaptive flight controller," in AIAA Guidance, Navigation and Control Conference, 2008.

[7]Kutay, A. T., Chowdhary, G., Calise A. J., Johnson, E. N., "A comparison of two novel direct adaptive control methods under actuator failure accommodation," in AIAA Guidance, Navigation and Control Conference, 2008.

[8]Volyanskyy, K. Y., Calise, A. J., Yang, B. -J., "A novel Q-modification term for adaptive control," Proc. IEEE American Contr. Conf., Minneapolis, MN, 2006.

[9]Volyanskyy, K. Y., Calise, A. J., Yang, B. -J., Lavretsky, E., "An error minimization method in adaptive control," AIAA Guidance, Navigation and Control Conference, 2006.

[10]Volyanskyy, K. Y., Haddad, W. M., Calise, A. J., "A new neuroadaptive control architecture for nonlinear uncertain dynamical systems: Beyond σ- and e- modifications,", submitted to IEEE Trans. on Neural Networks.

[11]Nguyen, N., Krishnakumar, K., Boskovic, J., "An optimal control modification to model-reference adaptive control for fast adaptation," AIAA Guidance, Navigation and Control Conference, 2008.

[12]Calise, A. J., Yucelen, T., Muse, J. A., Yang, B. -J., "An loop recovery method for adaptive control," AIAA Guidance, Navigation and Control Conference, 2009.

[13]Yucelen, T., Calise, A. J., "Enforcing a linear constraint in adaptive control: A Kalman filter optimization approach," AIAA Guidance, Navigation and Control Conference, 2009.

[14]Yucelen, T., Calise, A. J., "Kalman Filter Modification in Adaptive Control," accepted to J.Guidance Control and Dynamics.

[15]Pomet, J.B., Praly, L., "Adaptive Nonlinear Regulation: Estimation from Lyapunov Equation," IEEE Transactions on Automatic Control, Vol. 37, No. 6, 1992, pp. 729-740.

[16]Lewis, F. L., Yesilderick, A., Liu, K., *Multilayer Neural-Net Robot Controller with Guaranteed Tracking Performance*, IEEE Transactions on Neural Networks, Vol. 7, No. 2, March 1996, pp. 388-399.

[17]Khalil, H., *Nonlinear systems*, Prentice-Hall, Upper Saddle River, NJ, 1996.

[18]Johnson, E. N., Calise, A. J., *Neural Network Adaptive Control of Systems*, Proceedings of the American Control Conference, 2001.

[19]Yucelen, T., Calise, A. J.,Chandramohan, R., *$\mathcal{H}_2$ Adaptive Control*, IEEE American Control Conference, Baltimore, MD, 2010.

[20]T. Yucelen, K. Kim, A. J. Calise, $\mathcal{K}$- modification based $\mathcal{H}_2$ adaptive control," AIAA Guidance, Control and Navigation Conference, Toronto, Ontario (submitted).

[21]Chandramohan, R., Yucelen, T., Calise, A. J., Choudhary, G., Johnson, E.J., *Experimental Results for Kalman Filter Modification in Adaptive Control*, Infotech@Aerospace, Atlanta, GA, 2010.