Derivative-Free Model Reference Adaptive Control of a Generic Transport Model

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In this paper, we present both simulated and flight test results of a derivative-free model reference adaptive controller applied to a generic transport model, a high-fidelity transport aircraft model developed by NASA Langley Research Center. The derivative free form of adaptive control is expected to provide faster adaptation and smoother error transients, particularly for situations where aircraft dynamics undergo a sudden change. Moreover, it does not assume the existence of constant ideal weights, and does not require modification terms like $\sigma$- or $e$- modification, or a projection operator to guarantee that the error signals are uniformly ultimately bounded.

I. Introduction

Direct model reference adaptive controllers require less modeling information than robust controllers and can address system uncertainties and system failures. These controllers adapt feedback gains in response to system variations without requiring a parameter estimation algorithm. This property distinguishes them from adaptive controllers which employ an estimation algorithm to estimate the unknown system parameters and adapt the controller gains. This paper presents an adaptive control law that is particularly well suited for adaptation in the presence of sudden change in uncertain system dynamics, and that does require a parametrization for uncertainty that might occur in such situations. We present both simulated and flight test results of this adaptive control law applied to a generic transport model (GTM), which is a high-fidelity scaled transport aircraft model developed at NASA Langley Research Center.

There have been numerous model reference adaptive control (MRAC) approaches proposed in the last decades (see Refs. 2–11, and references therein) that deal with uncertain dynamical systems. All these methods have in common the underlying assumption that there exists a constant, but unknown, ideal set of weights. Although this assumption seems reasonable and these MRAC architectures work well on many systems, in some failure modes they may require the use of unrealistically high adaptation gain, or may fail to achieve the desired level of performance in terms of failure recovery. MRAC laws that require high gain can excite unmodeled dynamics, typically exhibit an excessive amount of control activity, amplify the effect of sensor noise, and are not sufficiently robust to time delay.

In this paper we present an evaluation of a derivative-free model reference adaptive control (DF-MRAC) design that does not assume the existence of constant ideal weights. The goal in removing this assumption is to improve performance with a lower adaptation gain. As a result, smoother transient characteristics can be obtained and faster adaptation can be achieved for systems that can undergo a sudden change in dynamics, either due to reconfiguration, deployment of a payload, or structural damage.

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The organization of the paper is as follows. Section 2 provides a brief background needed to explain a standard MRAC architecture. Section 3 describes the DF-MRAC law. Section 4 presents numerical example results for severe damage cases that are modeled in GTM. Section 5 presents flight test results. Finally, Section 6 summarizes the conclusions.

II. Model Reference Adaptive Control

This section states standard results for the MRAC problem. For this purpose, consider the controlled nonlinear uncertain dynamical system given by

\[ \dot{x}(t) = Ax(t) + Bu(t) + \Delta(x(t)) \]  

where \( x(t) \in \mathbb{R}^n \) is the state vector, \( u(t) \in \mathbb{R}^m \) is the control input, \( A \in \mathbb{R}^{n \times n} \) and \( B \in \mathbb{R}^{n \times m} \) are known matrices, and \( \Delta : \mathbb{R}^n \rightarrow \mathbb{R}^m \) is a matched uncertainty. Furthermore, we assume that the pair \((A,B)\) is controllable, the full state is available for feedback, and the control input \( u(t) \) is restricted to the class of admissible controls consisting of measurable functions.

The reference system is given by

\[ \dot{x}_m(t) = A_m x_m(t) + B_m r(t) \]  

where \( x_m(t) \in \mathbb{R}^n \) is the reference state vector, \( r(t) \in \mathbb{R}^r \) is a bounded piecewise continuous reference input, \( A_m \in \mathbb{R}^{n \times n} \) is Hurwitz, and \( B_m \in \mathbb{R}^{n \times r} \) with \( r \leq m \). It is convenient to think of \( A_m \) and \( B_m \) as having been obtained from nominal feedback and feed forward gains \( K_1 \in \mathbb{R}^{m \times n} \) and \( K_2 \in \mathbb{R}^{n \times r} \) such that \( A_m = A - BK_1 \) and \( B_m = BK_2 \). The following assumption is standard in the MRAC literature.

**Assumption 2.1.** The matched uncertainty in (1) can be linearly parameterized as

\[ \Delta(x(t)) = W^T \beta(x(t)) + \varepsilon(x(t)), \quad ||\varepsilon(x(t))|| \leq \varepsilon^* \]  

where \( W \in \mathbb{R}^{s \times m} \) is the unknown constant weight matrix, \( \beta : \mathbb{R}^n \rightarrow \mathbb{R}^s \) is a known vector of basis functions of the form \( \beta(x) = [\beta_1(x), \beta_2(x), \ldots, \beta_s(x)]^T \in \mathbb{R}^s \), and \( \varepsilon : \mathbb{R}^n \rightarrow \mathbb{R}^m \) is the residual error.

Consider the following adaptive control law

\[ u(t) = u_n(t) - u_{ad}(t) \]  

where \( u_n(t) \) is a nominal control law given by

\[ u_n(t) = -K_1 x(t) + K_2 r(t) \]  

and \( u_{ad}(t) \) is the adaptive feedback control component given by

\[ u_{ad}(t) = \hat{W}^T(t) \beta(x(t)) \]  

where \( \hat{W}(t) \in \mathbb{R}^{s \times m} \) is an estimate of \( W \) satisfying the weight update law

\[ \dot{\hat{W}}(t) = \gamma [\beta(x(t)) e^T(t) PB + \dot{\hat{W}}_m(t)] \]  

\( \gamma \) is a positive fixed gain, \( e(t) = x(t) - x_m(t) \) is the state tracking error, \( P \in \mathbb{R}^{n \times n} \) is a positive-definite solution of the Lyapunov equation

\[ 0 = A_m^T P + PA_m + Q \]  

for any \( Q = Q^T > 0 \), and \( \dot{\hat{W}}_m(t) \in \mathbb{R}^{s \times m} \) is a modification term, e.g. \( \dot{\hat{W}}_m(t) = -\sigma \hat{W}(t) \) for \( \sigma \)-modification.

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term, or $\dot{\hat{W}}_m(t) = -\sigma |e(t)||\hat{W}(t)$ for $e$- modification, where $\sigma$ is a positive fixed gain.

Theorems that highlight the uniformly ultimate boundedness (UUB) of the closed-loop system errors given by $e(t)$ and $\hat{W}(t)$, where $\hat{W}(t) \triangleq W - \hat{W}(t)$, for the $\sigma$- and $e$- modification cases can be found in Refs. 2–4, 16.

### III. Derivative-Free MRAC Architecture

The following assumption strengthens Assumption 2.1 by setting $\dot{e}(x(t)) = 0$, which can be justified under the assumption that time-variation is allowed in the unknown ideal weight matrix.

**Assumption 3.1.** The matched uncertainty in (1) can be linearly parameterized as

$$\Delta(x(t)) = W^T(t)\beta(x(t)) \quad (9)$$

where $W(t) \in \mathbb{R}^{s \times m}$ is an unknown *time-varying* weight matrix that satisfies $\|W(t)\| \leq w^*$ and $\beta : \mathbb{R}^n \to \mathbb{R}^s$ is a vector of known functions of the form $\beta(x) = [\beta_1(x), \beta_2(x), \ldots, \beta_s(x)]^T \in \mathbb{R}^s$.

**Remark 3.1.** Assumption 3.1 expands the class of uncertainties that can be represented by a given set of basis functions. That is, an adaptive law designed subject to Assumption 3.1 can be more effective than an adaptive law designed subject to Assumption 2.1 in canceling a wider class of uncertainties, due to the fact that time-variation is allowed in the unknown ideal weight matrix.

**Remark 3.2.** Assumption 3.1 does not place any restriction on the time derivative of the weight matrix.

The following theorem presents the main result.

**Theorem 3.1.** Consider the controlled nonlinear uncertain dynamical system given by (1) subject to Assumption 3.1. Consider, in addition, the feedback control law given by (4), with the nominal feedback control component given by (5) subject to Assumption 2.2, and with the adaptive feedback control component given by (6) that has a derivative-free weight update law in the form

$$\dot{\hat{W}}(t) = \Omega_1 \hat{W}(t - \tau) + \hat{\Omega}_2(t) \quad (10)$$

where $\tau > 0$, and $\Omega_1 \in \mathbb{R}^{s \times s}$ and $\hat{\Omega}_2 : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{s \times m}$ satisfy:

$$0 \leq \Omega_1^T \Omega_1 < \kappa_1 I, \quad 0 < \kappa_1 < 1 \quad (11)$$

$$\hat{\Omega}_2(t) = \kappa_2 \beta(x(t))e^T(t)PB, \quad \kappa_2 > 0 \quad (12)$$

with $P \in \mathbb{R}^{n \times n}$ satisfying (8) for any symmetric $Q > 0$. Then, $e(t)$ and $\hat{W}(t)$ are UUB.

**Proof.** See Ref. 17.

The proposed adaptive control architecture is shown in Figure 1.

**Remark 3.3.** Letting $\dot{\hat{W}}_m(t) = 0$ and using a 1st order Euler method for integration in (7), with $\tau_s$ being the step size results in:

$$\dot{\hat{W}}(t) = \gamma \tau_s (\beta(x(t))e^T(t)PB) + \dot{\hat{W}}(t - \tau_s) \quad (13)$$

This form of weight update law is identical to the DF-MRAC law in (10), if $\Omega_1 = I$, $\kappa_2 = \gamma \tau_s$, and $\tau = \tau_s$, with the exception that the choice $\Omega_1 = I$ is not permitted in DF-MRAC. In DF-MRAC, $\tau$ is not necessarily equal to $\tau_s$ and $\Omega_1$ can be chosen, for example, as $\varsigma I$ where $0 < |\varsigma| < 1$.

**Remark 3.4.** The derivative-free weight update law given by (10) subject to (11) and (12) does not
require a modification term to prove the error dynamics, including the weight errors, are UUB.

For the case of constant ideal weights in Assumption 3.1, Theorem 3.1 specializes to the following theorem. In this case, we assume that the uncertainty is structured. That is, the vector of known functions in (9) represent the vector of known basis functions.

**Theorem 3.2.** Consider the controlled nonlinear uncertain dynamical system given by (1) subject to Assumption 3.1, where $W \in \mathbb{R}^{s \times m}$ is an unknown constant weight matrix. Consider, in addition, the feedback control law given by (4), with the nominal feedback control component given by (5) subject to Assumption 2.2, and with the adaptive feedback control component given by (6) that has the derivative-free weight update law in the form (10) and (12), where $\Omega_1 = I$. Then, $e(t)$ and $\tilde{W}(t)$ approach a subspace in these error variables in which $e(t) = 0$ and $\tilde{W}^T(t)\beta(x(t)) = 0$.

**Proof.** See Ref. 17.

**Remark 3.5.** The system is said to be sufficiently excited if $r(t)$ is such that the conditions $e(t) = 0$, $\tilde{W}^T(t)\beta(x(t)) = 0$ admit only the solution $\tilde{W}(t) = 0$ in the limit $t \to \infty$. It is straightforward to show that this amounts to the standard MRAC condition for persistency of excitation\(^{18}\).

**IV. Generic Transport Model: Simulation Results**

This section presents a DF-MRAC design for the NASA GTM, and evaluates this design for several damage scenarios. GTM is a high-fidelity scaled transport aircraft model developed by NASA Langley Research Center\(^1\). It consists of a nonlinear aircraft model having two left and two right elevator controls, independent left and right aileron controls, upper and lower rudder controls, two left and two right spoiler controls, a stabilizer control, and left and right thrust controls. A linearized model for the controlled GTM
at an angle of attack of 2 degrees and 10^4 ft altitude is obtained in the form of (1). The primary sources of uncertainty are any one of a set of possible damage conditions that are included as a part of the modeling in GTM. A nominal controller is first designed for the linearized model using a robust servomechanism LQR approach that incorporates integral control\(^{20}\), with the objective of tracking roll rate, pitch rate, and yaw rate commands. Including the integral states, the linearized GTM model is 9th-order with the state vector \(x(t) = \begin{bmatrix} q_1(t) & p_1(t) & w_1(t) & u(t) & v(t) & w(t) & p(t) & q(t) & r(t) & \phi(t) & \theta(t) \end{bmatrix}^T\), where \(q_1(t), p_1(t), w_1(t)\) are the integrator states; \(u(t), v(t), w(t)\) are velocity components; \(p(t), q(t), r(t)\) are body angular rates about the roll, pitch and yaw body axes; and \(\phi(t)\) and \(\theta(t)\) are roll and pitch attitude, respectively. In this simulation study, tracking of roll and pitch rate commands are considered, and yaw rate command is set to zero. Roll and pitch attitude are not used in the design. Figure 2 shows the performance of the nominal controller under normal operating conditions.

For the adaptive design, neural network sigmoidal type functions are used in the form \(\beta(x(t)) = \begin{bmatrix} 1, \beta_1(x(t)), \beta_2(x(t)), \ldots, \beta_n(x(t)) \end{bmatrix}^T\), where \(\beta_i(x(t)) = (1 + e^{-x_i(t)})^{-1}, n = 9, \) and \(P\) in (8) is found by using \(Q = \text{diag}[10 10 10^{-4} 10^{-2} 10^{-3} 30 30 15]\). The DF-MRAC law considered in this example is in the form

\[
\dot{W}(t) = \varphi_1 \dot{W}(t - \tau) + \varphi_2 [\beta(x(t))e^T PB - \dot{\varphi}_2 \dot{\beta}_x(x(t))\beta^T_x(x(t))\dot{W}(t)]
\]

(14)

where \(\varphi_1 = 0.1I_6, \varphi_2 = \text{diag}[10 10 10 40 10]\), and \(\tau = 0.5\) seconds are associated with the proposed DF-MRAC design, and \(\dot{\varphi}_2 = 10I_6\) is associated with an adaptive loop recovery (ALR) modification term\(^{19}\) when it is employed, otherwise \(\dot{\varphi}_2 = 0\).

For comparison purposes, a standard MRAC law is also applied in the form of (7) with \(\gamma = \text{diag}[5 1 1 10 40 1]\) and without any modification term. Both designs were turned for optimum performance for a variety of failure modes.

### A. Missing Left Wing Tip

In the missing left wing tip damage scenario there is 25% loss of outboard left wing tip and the left aileron is missing, therefore available roll control effectiveness is reduced\(^1\). Figure 3 shows the degree of degraded performance when the nominal controller is evaluated for this damage case. Figure 4 shows the improvement in response obtained when DF-MRAC is employed. For comparison purposes, Figure 5 shows the performance when MRAC is employed. Note that the response is more oscillatory. In Figure 6, 0.01 seconds of time delay is added to the rudder channel. In this case, the performance using DF-MRAC alone is not satisfactory. Figure 7 shows that the addition of ALR modification significantly improves the robustness of the adaptive controller to time delay.

### B. Missing Vertical Tail

In the missing vertical tail damage scenario the entire vertical tail is missing, therefore there is a loss in directional stability and a complete loss in rudder control effectiveness\(^1\). Figure 8 shows that the nominal controller response for this damage case is unstable. Figure 9 shows that the DF-MRAC controller provides upset recovery and satisfactory tracking performance for this damage scenario. For comparison purposes, Figure 10 shows the performance when standard MRAC is employed. Note the high frequency oscillations that occur. Figure 11 shows a result for the same damage case in Figure 9 with 0.02 seconds of time delay in the right spoiler channel. The performance of the DF-MRAC controller is not satisfactory with this amount of time delay. Figure 12 shows that ALR modification improves the time delay margin of the DF-MRAC controller design for this failure case. Figure 14 shows that the time delay margin can be improved to at least 0.10 seconds in the right aileron channel with ALR modification.
Figure 2. GTM nominal control response for nominal operating conditions.

Figure 3. GTM nominal control response for the missing left wing tip case.
Figure 4. GTM DF-MRAC response for the missing left wing tip case.

Figure 5. GTM MRAC response for the missing left wing tip case.
Figure 6. GTM DF-MRAC response for the missing left wing tip case with 0.01 seconds of time delay in the rudder channel.

Figure 7. GTM DF-MRAC response with ALR modification term for the missing left wing tip case with 0.01 seconds of time delay in the rudder channel.
Figure 8. GTM nominal control response for the missing vertical tail case.

Figure 9. GTM DF-MRAC response for the missing vertical tail case.
Figure 10. GTM MRAC response for the missing vertical tail case.

Figure 11. GTM DF-MRAC response for the missing vertical tail case with 0.02 seconds of time delay in the right spoiler channel.
Figure 12. GTM DF-MRAC response with ALR modification term for the missing vertical tail case with 0.02 seconds of time delay in the right spoiler channel.

Figure 13. GTM DF-MRAC response for the missing vertical tail case with 0.10 seconds of time delay in the right aileron channel.
V. Generic Transport Model: Flight Test Results

This section presents recent flight test results for GTM, based on DF-MRAC design with ALR modification, and evaluates this design for several damage scenarios. The flight test vehicle is depicted in Figure 15. In-flight damage was represented by introducing additional feedback loops to change stability derivatives that represent unknown destabilizing changes in stiffness and damping terms in the longitudinal and lateral dynamics. In this section, we use the wording *adaptive controller* to denote DF-MRAC with ALR modification. Figures 16–21 present samples of the results. Each figure has four subfigures. The upper two subfigures show roll rate and pitch rate responses for a baseline, non-adaptive control design. The lower subfigures show the roll rate and pitch rate responses for the adaptive design. In all the figures, the dashed red line shows the pilot command and the solid blue line shows the actual GTM responses.

A. Case 1: Employing Baseline and Adaptive Controllers

Figure 16 compares the results when baseline and adaptive controllers are first engaged. Here, the responses of baseline and adaptive controllers in roll and pitch rates look nearly the same since there was no induced uncertainty, and pilot commands occurred primarily in the pitch axis.

B. Case 2: Roll and Pitch Rate Commands

Figure 17 compares the results when a roll rate doublet and a sinusoidal pitch rate are simultaneously commanded, without an induced uncertainty. This case shows improvement in pitch rate response with adaptive control most likely due to modeling error associated with cross axis coupling.

C. Case 3: Latency Emulation

Figure 18 compares the roll and pitch rate responses of baseline and adaptively controlled GTM, when a time delay up to 0.1 seconds is injected to all input channels. It is crucial to note here that DF-MRAC
Figure 15. Flight test vehicle.

with ALR modification preserves the time delay margin of the baseline controller. In addition, if we look at the magnitude of the commands, it can be seen that harsher commands were applied to the adaptively controlled GTM especially in roll rate. It should also be noted that the flight test results shown in this figure were taken on different days, and that the wind conditions for the results with DF-MRAC were considerably more variable and greater in magnitude. Unfortunately we do not have a flight test result that compares DF-MRAC with and without ALR when subjected to latency in order to validate that ALR modification does significantly improve time delay margin of an adaptive control design, as illustrated previously in the simulated results of Figures 6 and 7.

D. Case 4: $C_{m_{\alpha}}$ and $C_{l_{p}}$, Reduction by 50% and +0.2 under Pitch Rate Command

Figure 19 compares the roll and pitch rate responses of baseline and adaptively controlled GTM under parametric uncertainty (induced by introducing additional feedback of $\alpha$ and $p$), when a pitch rate doublet is commanded. Also near the end of the interval a pulse in the roll rate command is applied. The response with adaptive controller is better than the baseline controller’s response. The baseline controller’s roll rate response shown in upper left side of Figure 19 is much more oscillatory and varies approximately from -15 to +20. The adaptive controller’s roll rate response shown in the bottom left side of Figure 19 is better by more than a factor of 2. The responses in pitch rate look nearly the same.

E. Case 5: $C_{m_{\alpha}}$ and $C_{l_{p}}$, Reduction by 50% and +0.2 under Roll Rate Command

Figure 20 compares the roll and pitch rate responses of baseline and adaptively controlled GTM under parametric uncertainty, when a roll rate doublet is commanded. Roll rate tracking with the adaptive controller is again improved by roughly a factor of 2, and the pitch rate tracking is also improved, similar to what was observed in Figure 17.

F. Case 6: $C_{m_{\alpha}}$ and $C_{l_{p}}$, Reduction by 75% and +0.3 under Pitch Rate Command

Figure 21 compares the roll and pitch rate responses of baseline and adaptively controlled GTM under a harsher parametric uncertainty, when a pitch rate doublet is commanded. In this case the performance in terms of tracking error in both pitch and roll rate is reduced by at least a factor of 2 with the adaptive controller.
Figure 16. Comparison of baseline and adaptive controller responses for Case 1.

Figure 17. Comparison of baseline and adaptive controller responses for Case 2.
Figure 18. Comparison of baseline and adaptive controller responses for Case 3.

Figure 19. Comparison of baseline and adaptive controller responses for Case 4.
Figure 20. Comparison of baseline and adaptive controller responses for Case 5.

Figure 21. Comparison of baseline and adaptive controller responses for Case 6.
VI. Conclusions

A derivative-free model reference adaptive control architecture has been applied to the NASA generic transport model. Simulation results show that the approach is promising in applications in which there can be a sudden change in the plant parameters, such as might occur in the case of damage to the airframe and control surfaces of an aircraft. Limited flight test results have thus far validated the simulations. They have also demonstrated the ability of the adaptive design, when augmented with adaptive loop recovery, to maintain the margins of the reference model when subjected to time delay in all the control channels.

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