OUTPUT FEEDBACK ADAPTIVE STABILIZATION AND COMMAND FOLLOWING FOR MINIMUM PHASE DYNAMICAL SYSTEMS WITH UNMATCHED UNCERTAINTIES

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Outline

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2 Nonminimal State-Space Realization
   - Formulation: Cayley-Hamilton Construction
   - Nonminimal Controllable Realization
   - Full State Feedback Equivalent Form
3 Adaptive Control Architecture
   - Stabilization and Command Following
   - Visualization of Adaptive Control Architecture
4 Illustrative Numerical Examples
   - Stable and Unstable Plants
5 Conclusion and Ongoing Research
Motivation and Goals

Nonminimal State-Space Realization
- Formulation: Cayley-Hamilton Construction
- Nonminimal Controllable Realization
- Full State Feedback Equivalent Form

Adaptive Control Architecture
- Stabilization and Command Following
- Visualization of Adaptive Control Architecture

Illustrative Numerical Examples
- Stable and Unstable Plants

Conclusion and Ongoing Research
Motivation

- Models do not adequately capture the physical system
  - Idealized assumptions and model simplifications
  - Dynamics are nonlinear and uncertain
- Many loops can be coupled (MIMO)
- Unknown disturbances and unmodeled dynamics
- Commands may not be known in advance
Motivation and Goals

**Goals**

- Develop an **adaptive control framework** for dyn systems
  - Matched and unmatched uncertainties and disturbances
  - Unstable dynamics
- Achieve system **stability** and **performance**
  - Without excessive reliance on system models
  - With easily verifiable assumptions and minimal tuning effort

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Adaptive Control for Linear Dynamical Systems
Motivation and Goals

Standard Adaptive Control Schemes

- Often restricted to (strictly) positive real dynamical systems
  - Relative degree restrictions
- Often require matched uncertainty
  - Matching conditions
- Require knowledge of the high frequency gain
- Might not be robust to
  - Unmodeled dynamics
  - Unmatched uncertainty
- Can burst
  - Adaptive parameters drift to large values
Can we develop an **output fdbk adaptive control architecture** that

- Requires **minimal knowledge** of the system
- Is (as **simple** as) a **full state** feedback design
- Is applicable to **systems** with **matched and unmatched uncs**
- Is **robust** to **unmodeled dynamics**
Nonminimal State-Space Realization

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Nonminimal state-space realization

- Involves an expanded system state that consists entirely of the filtered inputs and filtered outputs of the original system.

Why a nonminimal state-space realization?

- Allows us to cast an output feedback control problem as a full state feedback problem!
Consider the controllable and observable linear uncertain dynamical system $G_p$

$$\dot{x}_p(t) = A_p x_p(t) + B_p u(t), \quad x_p(0) = x_{p0}, \quad t \geq 0$$

$$y(t) = C_p x_p(t)$$

- $x_p(t) \in \mathbb{R}^n$, is an unknown state vector
- $(A_p, B_p, C_p)$ are unknown system matrices

What is $n$?

System order is unknown but bounded $\leq n$
An input-output equivalent nonminimal observer canonical state space model $G_o$ of $G_p$

\[
\dot{x}_o(t) = A_o x_o(t) + B_o u(t), \quad x_o(0) = x_{o0}, \quad t \geq 0
\]

\[
y(t) = C_o x_o(t)
\]

- $x_o(t) \in \mathbb{R}^{ln}$, $t \geq 0$, expanded state space

\[
A_o = \begin{bmatrix}
0 & I_l & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & I_l \\
-a_0I_l & -a_1I_l & \cdots & -a_{n-1}I_l
\end{bmatrix}, \quad B_o = \begin{bmatrix}
C_p B_p \\
C_p A_p B_p \\
\vdots \\
C_p A_p^{n-1} B_p
\end{bmatrix}
\]

\[
C_o = \begin{bmatrix} I_l & 0 & \cdots & 0 \end{bmatrix}
\]

- $A_o \in \mathbb{R}^{ln \times ln}$, $B_o \in \mathbb{R}^{ln \times m}$ expanded system model

- $a_i$'s are the characteristic polynomial coefficients of $A_p$
From IO equivalent nonminimal observer canonical model $G_o$

\[
\begin{align*}
a_0 y(t) &= a_0 C o x_o(t) \\
a_1 \dot{y}(t) &= a_1 [C o A o x_o(t) + C o B o u(t)] \\
& \vdots \\
a_{n-1} \ddots (n-1)(t) &= a_{n-1} [C o A_{n-1}^o x_o(t) + C o A_{n-2}^o B o u(t) + \cdots + C o B o u^{(n-2)}(t)] \\
y^{(n)}(t) &= C o A_n o x_o(t) + C o A_{n-1} o B o u(t) + \cdots + C o B o u^{(n-1)}(t)
\end{align*}
\]

Define

\[
\begin{align*}
\bar{B}_0 & \triangleq C o (a_1 I_n + a_2 A_o + \cdots + a_{n-2} A_{o}^{n-3} + a_{n-1} A_{o}^{n-2} + A_{o}^{n-1})B_o \\
\bar{B}_1 & \triangleq C o (a_2 I_n + a_3 A_o + \cdots + a_{n-1} A_{o}^{n-3} + A_{o}^{n-2})B_o \\
& \vdots \\
\bar{B}_{n-1} & \triangleq C o B_o
\end{align*}
\]
Adding the $n + 1$ equations

$$y^{(n)}(t) = - [a_0 I_1 a_1 I_1 \cdots a_{n-1} I_1] Y(t) + [\overline{B}_0 \overline{B}_1 \cdots \overline{B}_{n-1}] U(t)$$

$$+ C_0 [A_0^n + a_{n-1} A_0^{n-1} + \cdots + a_1 A_0 + a_0 I_{n_1}] x_0(t)$$

- $Y(t) \triangleq [y^T(t), \dot{y}^T(t), \ldots, y^{T(n-1)}(t)]^T$
- $U(t) \triangleq [u^T(t), \dot{u}^T(t), \ldots, u^{T(n-1)}(t)]^T$

Using the Cayley-Hamilton theorem

$$y^{(n)}(t) = - [a_0 I_1 a_1 I_1 \cdots a_{n-1} I_1] Y(t) + [\overline{B}_0 \overline{B}_1 \cdots \overline{B}_{n-1}] U(t)$$

Define expanded state vector

$$x_n(t) \triangleq [Y^T(t), U^T(t)]^T \in \mathbb{R}^{n_1}, \quad n_1 \triangleq (m + l) \times n$$
\[ \dot{x}_n(t) = A_n x_n(t) + B_n u^{(n)}(t), \quad x_n(0) = x_{n0}, \quad t \geq 0 \]
\[ y(t) = C_n x_n(t) \]

\[ A_n = \begin{bmatrix}
0 & I_l & 0 & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots \\
0 & \cdots & 0 & I_l & 0 & \cdots & \cdots & 0 \\
-a_0 I_l & \cdots & -a_{n-1} I_l & 0 & \cdots & 0 & \cdots & B_{n-1} \\
0 & \cdots & \cdots & 0 & I_m & 0 & \cdots & \cdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\vdots & \cdots & 0 & I_m & 0 & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & 0 & 0 & \cdots & \cdots & 0 \\
\end{bmatrix}, \quad B_n = \begin{bmatrix}
0 \\
\vdots \\
0 \\
\end{bmatrix} \]

\[ C_n = \begin{bmatrix} I & 0 & \cdots & \cdots & 0 \end{bmatrix} \]

- \( A_n \in \mathbb{R}^{n_f \times n_f} \), \( B_n \in \mathbb{R}^{n_f \times m} \), and \( C_n \in \mathbb{R}^{l \times n_f} \)
To eliminate differential input and output signals in $G_p$, filter $u(t)$ and $y(t)$ through $1/\Lambda(s)$.

- $\Lambda(s) = s^n + ns^{n-1}\lambda + \cdots + \lambda^n$ is a monic Hurwitz polynomial.
- $x_n(t) \sim x_f(t) \in \mathbb{R}^n$ is a known filtered expanded state.

$x_f(t) = [y_f^T(t), \ldots, y_f^{(n-1)}(t), u_f^T(t), \ldots, u_f^{(n-1)}(t)]^T$

$\Delta q_f^T(t) \quad \Delta q_f^T(t) \quad \Delta v_f^T(t) \quad \Delta v_f^T(t)$

Since

$$\mathcal{L}\{u_f^{(n)}(t)\} = \frac{s^n - (s + \lambda)^n + (s + \lambda)^n}{(s + \lambda)^n} \mathcal{L}\{u(t)\}$$
$$= \left[ s^n - (s + \lambda)^n \right] \mathcal{L}\{u(t)\} + \mathcal{L}\{u(t)\}$$

Alternative IO equi nonminimal controllable realization $G_f$ of $G_p$:

$$x_f(t) = A_f x_f(t) + B_f u(t), \quad x_f(0) = x_{f0}, \quad t \geq 0$$

$$y(t) = C_f x_f(t)$$
System Structure

\[
A_f = \begin{bmatrix}
0 & I_l & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \cdots & I_l & 0 & \cdots & 0 \\
-a_0 I_l & \cdots & -a_{n-1} I_l & B_0 & \cdots & B_{n-1} \\
0 & \cdots & 0 & I_m & 0 & 0 \\
\vdots & \ddots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & 0 & -\lambda^m I_m & \cdots & -n\lambda I_m \\
0 & \cdots & 0 & -\lambda^m I_m & \cdots & -n\lambda I_m \\
\end{bmatrix}, \quad B_f = \begin{bmatrix}
0 \\
\vdots \\
0 \\
\vdots \\
I_m \\
\vdots \\
I_m \\
\end{bmatrix}
\]

\[
C_f = \begin{bmatrix}
-a_0 I_l + \lambda^n I_l & \cdots & -a_{n-1} I_l + n\lambda I_l & B_0 & \cdots & B_{n-1}
\end{bmatrix}
\]

- \( A_f \in \mathbb{R}^{n_f \times n_f} \), \( B_f \in \mathbb{R}^{n_f \times m} \), and \( C_f \in \mathbb{R}^{l \times n_f} \)
- \( A_f = A_n - [0 \ B_n \lambda^T] \), \( \lambda = [\lambda^n, \ldots, n\lambda]^T \) - partially known
- \( B_f = B_n - \text{known!} \)
- \( C_f = \Phi + [\lambda^T \ 0] \) - Unknown but do not care since \( x_f \) is known!
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Assumption 1

\( \mathcal{G}_p \) is minimum phase and \( C_p A_p^{d-1} B_p = \bar{B} \Lambda \)

- \( d \): Smallest pos integer \( i \) s.t. \( i \)th Markov pm \( C_p A_p^{d-1} B_p \neq 0 \)
- \( \bar{B} \in \mathbb{R}^{l \times m} \) is known
- \( \Lambda \in \mathbb{R}^{m \times m} \) is unknown and satisfies
  \[
  \Lambda = \text{block-diag}[\Lambda_{m_1}, \ldots, \Lambda_{m_s}]
  \]
  \( \Lambda_{m_i} \in \mathbb{R}^{m_{i_1} \times m_{i_1}}, \ldots, \Lambda_{m_s} \in \mathbb{R}^{m_{s_1} \times m_{s_1}}, \) and \( m_1 + \cdots + m_s = m \)
- For \( i \in \{1, \ldots, s\} \), \( \Lambda_{m_i} \) is either PD or ND

- \( \bar{B}_{n-1} = \bar{B}_{n-2} = \cdots = \bar{B}_{n-d+1} = 0 \) and \( \bar{B}_{n-d} = C_p A_p^{d-1} B_p \neq 0 \)
- SISO: w.l.o.g. \( \bar{B} = 1 \) and \( \text{sgn}(C_p A_p^{d-1} B_p) \) is known
- MIMO: \( \Lambda = \text{id}(\Lambda) \text{pd}(\Lambda), \text{id}(\Lambda) \) known, \( \text{pd}(\Lambda) \) unknown
\[
\dot{q}(t) = A_0 q(t) + B_0 v_0(t) + B_1 \Lambda \phi(t), \quad q(0) = q_0, \quad t \geq 0
\]
\[
\dot{v}(t) = A_v v(t) + B_v u(t), \quad v(0) = v_0
\]

- \( q(t) \triangleq [q_1^T(t), \ldots, q_n^T(t)]^T \in \mathbb{R}^n \)
- \( v_0(t) \triangleq [v_1^T(t), \ldots, v_{n-d}^T(t)]^T \in \mathbb{R}^{m(n-d)} \)
- \( v(t) \triangleq [v_1^T(t), \ldots, v_n^T(t)]^T \in \mathbb{R}^{mn} \)

**Idea:**
\[
\phi(t) \triangleq v_{n-d+1}(t) \in \mathbb{R}^m \quad \text{— Virtual control signal}
\]

- \( A_0 \triangleq \begin{bmatrix}
0 & I_l & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
0 & \cdots & 0 & I_l \\
-a_0 I_l & -a_1 I_l & \cdots & -a_{n-1} I_l \\
0 & I_m & \cdots & 0
\end{bmatrix}, \quad B_0 \triangleq \begin{bmatrix}
0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0 \\
B_0 & \cdots & B_{n-d-1}
\end{bmatrix} \)

- \( A_v \triangleq \begin{bmatrix}
0 & \cdots & 0 \\
\vdots & \ddots & \vdots \\
0 & \cdots & 0 \\
-\zeta_1 I_m & \cdots & -\zeta_m I_m
\end{bmatrix}, \quad \zeta_1 \triangleq \lambda^n, \ldots, \zeta_m \triangleq n \lambda, \quad A_v \text{ is Hurwz} \)

- \( B_1 \triangleq [0 \cdots 0 B^T]^T, \quad B_v \triangleq [0 \cdots 0 I_m]^T \)
Assumption 2

There exist $K_q \in \mathbb{R}^{n \times m}$ and $K_v \in \mathbb{R}^{m(n-d) \times m}$ s.t. $A_m \triangleq A_0 + B_1 \Lambda K_q^T$ is Hurwitz and $B_0 = B_1 \Lambda K_v^T$ holds.

- If $q_0$ is square (i.e., $m = l$) and $\hat{B}$ is nonsingular, then Assumption 2 is automatically satisfied.

\[
\begin{align*}
\phi(t) &= \hat{K}_q(t)q(t) - \hat{K}_q(t)v_0(t) \quad - \text{Adaptive virtual control} \tag{1} \\
\dot{K}_q(t) &= -\Gamma_q q(t)q^T(t)P_mB_1 \text{id}(A), \quad \hat{K}_q(0) = \hat{K}_{q0}, \quad \Gamma_q > 0 \tag{2} \\
\dot{K}_v(t) &= \Gamma_v v_0(t)v_0^T(t)P_mB_1 \text{id}(A), \quad \hat{K}_v(0) = \hat{K}_{v0}, \quad \Gamma_v > 0 \tag{3} \\
0 &= A_m^T P_m + P_m A_m + R_m, \quad R_m > 0 \tag{4} \\
0 &= A_m^T P_m + P_m A_m + R_m, \quad R_m > 0 \tag{5} \\
\text{Then } (q(t), \hat{K}_q(t), \hat{K}_v(t)) \text{ is LS for all } (q_0, \hat{K}_{q0}, \hat{K}_{v0}) \in \mathbb{R}^m \text{ and } t \geq 0 \tag{6} \\
\text{And } q(t) \to 0 \text{ as } t \to \infty \tag{7}
\end{align*}
\]
$u(t) = \phi(t) + \zeta_1 \phi(t) + \zeta_2 \phi(t) + \cdots + \zeta_{n-d} \phi(t) + \zeta_{n-d+1} \phi(t)$

\[+ \zeta_{n-d} [\int_0^t \phi(\sigma_1) d\sigma_1] + \cdots + \zeta_2 \left[ \int_0^t \left( \int_0^t \phi(\sigma_1) d\sigma_1 \right) d\sigma_2 \right] + \cdots + \zeta_1 \left[ \int_0^t \left( \int_0^t \left( \int_0^t \phi(\sigma_1) d\sigma_1 \right) d\sigma_2 \right) \cdots d\sigma_{n-d-1} \right].\]

\[t \geq 0 \quad (*)\]

**Theorem**

Consider $G_f$ with control signal $(*)$ and assume Assumptions 1 and 2 hold. Then:

- $x_p(t), \ t \geq 0$, satisfying $G_f$ is bounded for all $x_p(0) \in \mathbb{R}^n$
- $y(t) \to 0$ as $t \to \infty$
Integrator state

\[ \dot{q}_{\text{int}}(t) = \mathcal{F}[r(t) - y(t)] = r_f(t) - y_f(t) = q_1(t) \]

Augmented dynamics

\[ q_a(t) = A_{a0} q_a(t) + B_{a0} v_0(t) + B_{a1} \Lambda \phi(t) + B_{am} r_f(t), \quad q_a(0) = q_{a0} \]

\[ q_a(t) \triangleq [q^T(t), q_{\text{int}}^T(t)]^T \in \mathbb{R}^{(n+1)} \]

\[ \dot{v}(t) = A_v v(t) + B_v u(t), \quad v(0) = v_0 \quad \text{– As before} \]

\[ A_{a0} \triangleq \begin{bmatrix} 0 & I_l & \cdots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & \cdots & 0 & I_l & \vdots \\ -a_0 I_l & -a_1 I_l & \cdots & -a_{n-1} I_l & 0 \\ -I_l & 0 & \cdots & 0 & 0 \end{bmatrix} , \quad B_{a0} \triangleq \begin{bmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \\ 0 & \cdots & I_l & \cdots & \cdots \end{bmatrix} \]

\[ B_{a1} \triangleq \begin{bmatrix} 0 & \cdots & 0 & B^T & 0 \end{bmatrix}^T , \quad B_{am} \triangleq \begin{bmatrix} 0 & \cdots & 0 & 0 & I_l \end{bmatrix}^T \]
Adaptive Control Architecture
Stabilization and Command Following

Assumption 2 for Command Following

Assumption 2′

There exist $K_{aq} \in \mathbb{R}^{l(n+1) \times m}$ and $K_{av} \in \mathbb{R}^{m(n-d) \times m}$ s.t.

$A_{am} \equiv A_{a0} + B_{a1} \Lambda K_{aq}^T$ is Hurwitz and $B_{a0} = B_{a1} \Lambda K_{av}^T$ holds.

If $G_p$ is square (i.e., $m = l$) and $\bar{B}$ is nonsingular, then Assumption 2 is automatically satisfied.

- Adaptive virtual control

\[
\phi_a(t) = \hat{K}_{aq}(t)q_a(t) - \hat{K}_{av}(t)v_0(t)
\]

\[
\dot{\hat{K}}_{aq}(t) = -\Gamma_{aq}q_a(t)e^T(t)P_{am}B_{a1}\text{id}(\Lambda), \quad \hat{K}_{aq}(0) = \hat{K}_{aq0}, \quad \Gamma_{aq} > 0
\]

\[
\dot{\hat{K}}_{av}(t) = \Gamma_{av}v_0(t)e^T(t)P_{am}B_{a1}\text{id}(\Lambda), \quad \hat{K}_{av}(0) = \hat{K}_{av0}, \quad \Gamma_{av} > 0
\]

\[
e(t) \triangleq q_a(t) - q_{am}(t)
\]

\[
q_{am}(t) = A_{am}q_{am}(t) + B_{am}r_e(t), \quad q_{am}(0) = q_{am0}
\]

\[
0 = A_{am}^TP_{am} + P_{am}A_{am} + R_{am}, \quad R_{am} > 0
\]
Adaptive Controller for Command Following

\[ u(t) = \phi_n^d(t) + \zeta_{n-1} \phi_n^{d-1}(t) + \zeta_{n-2} \phi_n^{d-2}(t) + \cdots + \zeta_{n-d+2} \phi_n(t) + \zeta_{n-d+1} \phi_n(t) \\
+ \zeta_{n-d} \left[ \int_0^t \phi_n(\sigma_1) d\sigma_1 \right] + \cdots + \zeta_2 \left[ \int_0^t \int_0^t \phi_n(\sigma_1) d\sigma_1 \right] d\sigma_2 + \cdots \\
\times d\sigma_{n-d-1} + \zeta_1 \left[ \int_0^t \int_0^t \int_0^t \phi_n(\sigma_1) d\sigma_1 \right] \times d\sigma_2 \cdots d\sigma_{n-d}, \quad t \geq 0 \quad (\ast) \]

**Theorem**

Consider \( G_f \) with control signal \((\ast)\) and assume Assumptions 1 and 2' hold. Then:

- \((e(t), K_q(t), K_a(t))\) is Lyapunov stable \(\forall (e_0, K_q(0))\) and \(t \geq 0\)
- \(x_p(t), t \geq 0\), satisfying \( G_p \) is bounded \(\forall x_p(0) \in \mathbb{R}^n\)
- \( e(t) \to 0 \) as \( t \to \infty \)

- \( \lim_{t \to \infty} e(t) = 0 \Rightarrow \lim_{t \to \infty} q_a(t) = q_{am}(t) \Rightarrow \lim_{t \to \infty} y(t) = r(t) \)
- \( \lim_{t \to \infty} y(t) = r(t) \)
Adaptive Control Architecture

Visualization of Adaptive Control Architecture

Visualization of Adaptive Controller

Adaptive Control for Linear Dynamical Systems
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Illustrative Numerical Examples
Stable and Unstable Plants

Illustrative Example 1

Asymptotically stable plant

Consider the uncertain plant $G_p$

\[
\begin{align*}
\dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ -50 & -2 \end{bmatrix} x(t) + \begin{bmatrix} -2 \\ 1 \end{bmatrix} u(t), \quad x(0) = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix} \\
y(t) &= \begin{bmatrix} -1 & 0 \end{bmatrix} x(t)
\end{align*}
\]

- Poles $\{-1.0 \pm 7.0j\}$ and zero $\{-1.50\}$
- Let $\lambda = 5$ and let the reference model system matrix be
  \[
  A_{am} = \begin{bmatrix} 0 & 1 & 0 \\ -0.69 & -1.22 & 0.15 \\ -1 & 0 & 0 \end{bmatrix}
  \]
- Let $R_{am} = 10I_3$, $\Gamma_{aq} = 50I_3$, and $\Gamma_{av} = 10$
- w.l.o.g. let $\bar{B} = 1$ and assume $\text{sgn}(C_p A_p^{-1} B_p) = 1$ is known
Closed-loop response of asymptotically stable plant

Adaptive controller with $\Gamma_{aq} = 50I_3$ and $\Gamma_{av} = 10$ tracks $r(t)$
Consider the uncertain plant $\mathcal{G}_p$

\[
\dot{x}(t) = \begin{bmatrix} 0.5 & 1 \\ -2 & 1 \end{bmatrix} x(t) + \begin{bmatrix} -0.1 \\ 1 \end{bmatrix} u(t), \quad x(0) = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}
\]

\[
y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(t)
\]

- **Poles** $\{0.75 \pm 1.39j\}$ and **zero** $\{-0.26\}$
- Let $\lambda = 5$ and let the reference model system matrix be

\[
A_{am} = \begin{bmatrix} 0 & 1 & 0 \\ -0.69 & -1.22 & 0.15 \\ -1 & 0 & 0 \end{bmatrix}
\]

- Let $R_{am} = 10I_3$, $\Gamma_{aq} = 50I_3$, and $\Gamma_{av} = 10$
- w.l.o.g. let $\bar{B} = 1$ and assume $\text{sgn}(C_pA_p^{-1}B_p) = 1$ is known
Closed-loop response of unstable plant
Adaptive controller with $\Gamma_{uq} = 50I_3$ and $\Gamma_{av} = 10$ tracks $r(t)$
Consider the uncertain plant $\mathcal{G}_p$

$$\dot{x}(t) = \begin{bmatrix} 0.5 & 5 \\ 2 & 0.5 \end{bmatrix} x(t) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(t), \quad x(0) = \begin{bmatrix} 0.5 \\ -0.5 \end{bmatrix}$$

$$y(t) = \begin{bmatrix} 1 & 2 \end{bmatrix} x(t)$$

- **Poles** $\{3.66, -2.66\}$ and **zero** $\{-2.75\}$
- Let $\lambda = 5$ and let the **reference model** system matrix be

$$A_{am} = \begin{bmatrix} 0 & 1 & 0 \\ -0.69 & -1.22 & 0.15 \\ -1 & 0 & 0 \end{bmatrix}$$

- Let $R_{am} = 10I_3$, $\Gamma_{aq} = 50I_3$, and $\Gamma_{av} = 10$
- w.l.o.g. let $\bar{B} = 1$ and assume $\text{sgn}(C_p A_p^{-1} B_p) = 1$ is known
Closed-loop response of unstable plant
Adaptive controller with $\Gamma_{aq} = 50/3$ and $\Gamma_{av} = 10$ tracks $r(t)$
Outline

1. Motivation and Goals
2. Nonminimal State-Space Realization
   - Formulation: Cayley-Hamilton Construction
   - Nonminimal Controllable Realization
   - Full State Feedback Equivalent Form
3. Adaptive Control Architecture
   - Stabilization and Command Following
   - Visualization of Adaptive Control Architecture
4. Illustrative Numerical Examples
   - Stable and Unstable Plants
5. Conclusion and Ongoing Research
Conclusion

- Developed an output feedback adaptive control architecture
  - Linear multivariable uncertain systems
  - Minimum phase zeros
- Predicated on a nonminimal state space realization
  - Known expanded set of states
- The controller does not require any information of
  - System zeros and poles
  - Structure of the uncertainty: Matched or unmatched

Ongoing Research

- Extensions to systems with unmatched disturbances
- Extensions to nonminimum phase systems
- Extensions to nonlinear uncertain dynamical systems
Conclusion and Ongoing Research

Thank You

QUESTIONS?