

Adaptive SDRE Based Nonlinear Sensorless Speed Control for PMSM Drives

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Abstract — Field orientation or vector control is an advanced technique used for the development of high-performance permanent magnet synchronous motor (PMSM) drives. In this paper, a field-oriented adaptive nonlinear control for PMSM drives is presented for sensorless control of the speed and field of the motor, simultaneously. The proposed approach is based on a state-dependent Riccati equation (SDRE) control technique and its formulation utilizes a gradient-based neural-like system for online computation. The unknown parameters of the PMSM drive, that is, stator resistance and load torque, as well as the speed of the motor, are estimated using an extended Kalman filter (EKF) to eliminate the mechanical sensors. The resulting adaptive algorithm is simple and fast and is easily applicable to real-time control of PMSM drives. The efficacy of the proposed approach for sensorless control of PMSM drives is demonstrated through an illustrative simulation for the proof of concept.

NOMENCLATURE

List of symbols

i_d	Current in the direct axis
i_q	Current in the quadrature axis
ω	Mechanical speed of the motor
v_d	Input stator voltage in the direct axis
v_q	Input stator voltage in the quadrature axis
R	Stator resistance
L_d	Inductance in the direct axis
L_q	Inductance in the quadrature axis
P	Number of poles
F	Viscous friction coefficient
J	Moment of inertia
Φ	Rotor magnetic flux
T_L	Load torque

I. INTRODUCTION

Permanent magnet synchronous motors (PMSM) are widely used in high performance servo applications due to their high efficiency, high power density, and large torque to inertia ratio [1], [2]. However, PMSMs are nonlinear multivariable dynamic systems and, without speed sensors and under load and parameter perturbations, it is difficult to control their speed with high precision, using conventional control strategies.

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Linearization and/or high-frequency switching based nonlinear speed control techniques, such as feedback linearization control and sliding mode control, have been implemented for the PMSM drives [3]–[6]. However, it is more efficient to use a nonlinear control method that is based on minimizing a cost function and allows one to tradeoff between the control accuracy and control effort. One such nonlinear control strategy is the nonlinear quadratic optimal control method that is an extension of the well known quadratic optimal control for linear systems. The technique requires the solution of a state-dependent Riccati equation (SDRE) to achieve the desired optimal control performance [8]–[11]. Although there are a number of other methods for nonlinear control design problem, the SDRE-based control strategy is one of the few successful approaches that have important properties, such as applicability to a large class of nonlinear systems, and systematic formulation. Moreover, the SDRE formulation can be applied adaptively, especially for systems like the PMSM drives, which allows one to obtain the desired motor speed under perturbations and parameters variations, including the stator resistance and load torque changes.

In short, this paper presents the design of an adaptive nonlinear quadratic optimal control, based on the SDRE formulation, for the speed control of three-phase PMSM drives with uncertain stator resistance and load torque. This new control approach for PMSMs provides relatively fast stabilization, a wide operating range, and a good dynamic performance. The implementation of this control needs the knowledge of the mechanical speed of the motor, load torque, and the stator resistance, which varies due to temperature changes. As it is known, the conventional speed sensors reduce the reliability of the PMSM drives. Moreover, it is difficult to mount them on high speed drives. As a result, several sensorless control methods for PMSM drives are proposed in [14]–[16]. A disadvantage of these methods is that the variation of stator resistance introduces error in their motor speed estimation. Therefore, eliminating these errors requires simultaneous estimation of stator resistance and load torque along the motor speed estimation. In this paper, an extended Kalman filter (EKF) is used for simultaneous estimation of the motor speed, and the stator resistance and load torque. Fast microcontrollers and digital signal processors (DSP) allow for simultaneous implementation of the extended Kalman filter and the SDRE-based control. The proposed sensorless control strategy can enhance the performance and the reliability of the PMSM drives in industry.

The paper is organized as follows. Section II and III present PMSM modeling and the nonlinear quadratic control strategy. Extended Kalman filter estimation method and simulation results are given in sections IV and V, respectively. Finally, conclusions are given in section VI.

II. PMSM MODELING

In the d - q reference frame, the three-phase PMSM can be described as, [3]

$$\begin{aligned}\dot{x}_1 &= -\frac{R}{L_d}x_1 + P\frac{L_q}{L_d}x_2x_3 + \frac{1}{L_d}u_1 \\ \dot{x}_2 &= -P\frac{L_d}{L_q}x_1x_3 - \frac{R}{L_q}x_2 - P\frac{\Phi}{L_q}x_3 + \frac{1}{L_q}u_2 \\ \dot{x}_3 &= \frac{3}{2}P\frac{\Phi}{J}x_2 - \frac{F}{J}x_3 - \frac{T_L}{J}\end{aligned}\quad (1)$$

where $x = [x_1 \quad x_2 \quad x_3]^T = [i_d \quad i_q \quad \omega]^T$ and $u = [u_1 \quad u_2]^T = [v_d \quad v_q]^T$. In equation (1), the equilibrium values are not zero and are given by

$$x_2 = \frac{2}{3} \frac{1}{P\Phi} (F x_3 + T_L) \quad (2)$$

$$u_1 = R x_1 - P L_q x_2 x_3 \quad (3)$$

$$u_2 = P L_d x_1 x_3 + R x_2 + P \Phi x_3 \quad (4)$$

For a fixed desired speed, x_{3d} , and assuming fixed load torque, to satisfy field-orientation condition, we let $x_{1d}=0$. Then the desired values for x_2 , u_1 and u_2 would be

$$x_{2d} = \frac{2}{3} \frac{1}{P\Phi} (F x_{3d} + T_L) \quad (5)$$

$$u_{1d} = -P L_q x_{2d} x_{3d}$$

$$u_{2d} = R x_{2d} + P \Phi x_{3d}$$

The problem is now to find a suitable feedback control $u = [u_1, u_2]^T$ such that the regulation of the output speed x_3 to x_{3d} , without a speed sensor, can be achieved in the presence of uncertain stator resistance and load torque. Define the state and control errors as

$$\begin{aligned}\bar{x}_1 &= x_1 - x_{1d}, \quad \bar{x}_2 = x_2 - x_{2d}, \quad \bar{x}_3 = x_3 - x_{3d} \\ \bar{u}_1 &= u_1 - u_{1d}, \quad \bar{u}_2 = u_2 - u_{2d}\end{aligned}\quad (6)$$

where u_{1d} and u_{2d} defined in (5) are the necessary steady state values for the PMSM system to remain in the equilibrium state. Incorporating equations (5) and (6) in equation (1) results in the following steady state error equations for the PMSM speed control, as

$$\begin{aligned}\dot{\bar{x}}_1 &= -\frac{R}{L_d}\bar{x}_1 + P\frac{L_q}{L_d}(\bar{x}_3 + x_{3d})\bar{x}_2 + P\frac{L_q}{L_d}x_{2d}\bar{x}_3 + \frac{1}{L_d}\bar{u}_1 \\ \dot{\bar{x}}_2 &= -P\frac{L_d}{L_q}\bar{x}_1(\bar{x}_3 + x_{3d}) - \frac{R}{L_q}\bar{x}_2 - P\frac{\Phi}{L_q}\bar{x}_3 + \frac{1}{L_q}\bar{u}_2 \\ \dot{\bar{x}}_3 &= \frac{3}{2}P\frac{\Phi}{J}\bar{x}_2 - \frac{F}{J}\bar{x}_3\end{aligned}\quad (7)$$

The nonlinear quadratic optimal control design for the system given in (7) is explained in the following section.

III. NONLINEAR QUADRATIC OPTIMAL CONTROL

This section presents the proposed nonlinear quadratic optimal control approach for the sensorless PMSM drives based on state-dependent Riccati equation (SDRE). The idea behind the method is to express the nonlinear model of the system in a state-dependent linear form and consider an associated SDRE for the control design. An equivalent state-dependent linear model of the nonlinear system in (7) can be written as

$$\dot{\bar{x}} = A(\bar{x})\bar{x} + B\bar{u} \quad (8)$$

where the corresponding matrices $A(\bar{x})$ and B for the nonlinear PMSM model (1) are given as

$$A(\bar{x}) = \begin{bmatrix} -\frac{R}{L_d} & P\frac{L_q}{L_d}(\bar{x}_3 + x_{3d}) & P\frac{L_q}{L_d}x_{2d} \\ -P\frac{L_d}{L_q}(\bar{x}_3 + x_{3d}) & -\frac{R}{L_q} & -P\frac{\Phi}{L_q} \\ 0 & \frac{3}{2}P\frac{\Phi}{J} & -\frac{F}{J} \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \end{bmatrix}$$

However, one should be cautioned that, from many possible choices, the state-dependent matrices $A(\bar{x})$ and $B(\bar{x})$ must be selected in such a way that the pair $(A(\bar{x}), B(\bar{x}))$ is either controllable or stabilizable.

It should be noted that the pair $(A(\bar{x}), B(\bar{x}))$ is controllable if and only if the controllability matrix $\psi_C = [B(x) \quad A(x)B(x) \quad \dots \quad A^{(n-1)}(x)B(x)]$ has full rank for all x . It is stabilizable if the uncontrollable states of the system are stable [8].

Considering the above state-dependent linear model, a state feedback nonlinear quadratic regulator (NQR) design based on the SDRE technique is considered in the following for optimal control of the PMSM drives. In addition, we describe a neural-like computation methodology, for online implementation of the proposed techniques.

A. Nonlinear Quadratic Regulator

The objective is to find a state feedback control of the form $\bar{u} = -K(\bar{x})\bar{x}$ that minimizes a cost function given by

$$J(\bar{x}_0, \bar{u}) = \frac{1}{2} \int_{t_0}^{\infty} (\bar{x}^T Q \bar{x} + \bar{u}^T R \bar{u}) dt \quad (9)$$

where $Q \in \mathfrak{R}^{n \times n}$ is a symmetric positive semi-definite matrix, and $R \in \mathfrak{R}^{m \times m}$ is a symmetric positive-definite matrix. Moreover, $\bar{x}^T Q \bar{x}$ is a measure of control accuracy and $\bar{u}^T R \bar{u}$ is a measure of control effort [11]. It should be mentioned that the SDRE formulation allows one to tradeoff between the control accuracy and control effort.

It should be noted here that, in addition to controllability of the pair $(A(\bar{x}), B(\bar{x}))$, matrix $A(\bar{x})$ must be chosen in such a way that the pair $(A(\bar{x}), D)$ is observable or detectable, where D is a matrix such that $Q = D^T D$. Also note that the pair $(A(\bar{x}), D)$ is observable or detectable if the pair $(A^T(\bar{x}), D^T)$ is controllable or stabilizable, respectively.

The optimal control law is then given as

$$\bar{u}(\bar{x}) = -K(\bar{x})\bar{x} = -R^{-1}B^T(\bar{x})P(\bar{x})\bar{x} \quad (10)$$

where $P(\bar{x})$ is the symmetric, positive-definite solution of the state-dependent Riccati equation (SDRE) of the form

$$A^T(\bar{x})P(\bar{x}) + P(\bar{x})A(\bar{x}) + Q - P(\bar{x})B(\bar{x})R^{-1}B^T(\bar{x})P(\bar{x}) = 0 \quad (11)$$

The resulting closed loop system can then be written as

$$\dot{\bar{x}} = \underbrace{(A(\bar{x}) - B(\bar{x})R^{-1}B^T(\bar{x})P(\bar{x}))}_{A_{CL}(\bar{x})} \bar{x} \quad (12)$$

where the closed-loop system matrix $A_{CL}(\bar{x})$ satisfies the point-wise Hurwitz condition $\text{Re } \lambda_i(A_{CL}(\bar{x})) < 0$. The existence of a control law that guarantees local closed-loop stability is shown in [8].

Therefore, there is a need to solve the state-dependent Riccati equation (11) to realize an SDRE-based NQR law. The online neural-like computation method for solving the required SDRE is given in the following section.

B. Online Gradient Type Neural-Like Solution of State-Dependent Riccati Equations

It is well known that the solution of the SDRE could not be found analytically [8]–[12], but only for a very limited number of nonlinear systems. Therefore, one should require an online computation method for the solution of the SDRE.

In the literature, a few computational methods have been proposed to solve the SDRE, in real-time [9]–[12]. Most of these methods are numerically complex and depend on correct initial conditions. Here, we present a fast, computationally simple, and efficient online method for

solving the state-dependent Riccati equation (SDRE), using a gradient-based neural-like system for the purpose of nonlinear control synthesis. This technique is a state-dependent extension of the neural network approach, [12] and [13], which was developed for online solution of the algebraic Riccati equation (ARE).

For this purpose, consider (11) as a generic SDRE that needs to be solved. We know that the solution $P(\bar{x})$ must be positive definite and symmetric. We also know that $P(\bar{x})$ is positive-definite and symmetric if it has a Cholesky factorization [12]. Therefore, a constraint for $P(\bar{x})$ to be positive-definite and symmetric is added by requiring,

$$G_1(P(\bar{x}), L(\bar{x})) = [g_{1,jk}] = L(\bar{x})L^T(\bar{x}) - P(\bar{x}) = 0 \quad (13)$$

where $g_{1,jk}$ is the jk^{th} element of the objective function G_1 , $j, k = 1, \dots, n$, and L is a Cholesky factor for $P(\bar{x})$. Let us consider the following representation of (11) that we need to solve to obtain a robust stabilizing control law. That is,

$$G_2(P(\bar{x})) = [g_{2,jk}] = A^T(\bar{x})P(\bar{x}) + P(\bar{x})A(\bar{x}) + Q - P(\bar{x})B(\bar{x})R^{-1}B^T(\bar{x})P(\bar{x}) = 0 \quad (14)$$

where $N(x) = B(x)R^{-1}B^T(x)$, and $g_{2,jk}$ is the jk^{th} element of the objective function G_2 , $j, k = 1, \dots, n$. To solve $P(\bar{x})$ from (13) and (14), the following Lyapunov energy function is first derived [13],

$$E[G_1(P(\bar{x}), L(\bar{x})), G_2(P(\bar{x}))] = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n [g_{1,i,jk}^2 + g_{2,i,jk}^2] \quad (15)$$

Then, a matrix-oriented gradient algorithm is developed to find the update rule for $P(\bar{x})$ by changing the variables in the direction of the negative gradient of the energy function E to minimize (15), [12], [13], as

$$\frac{dP(\bar{x})}{dt} = -n_p \frac{\partial E}{\partial P(\bar{x})} = -n_p [A(\bar{x})\Psi_1 + \Psi_1 A^T(\bar{x}) + \Psi_2 - N(\bar{x})P(\bar{x})\Psi_1 - \Psi_1 P(\bar{x})N^T(\bar{x})] \quad (16)$$

$$\frac{dL(\bar{x})}{dt} = -n_L \frac{\partial E}{\partial L(\bar{x})} = -n_L [\Psi_2 L(\bar{x})] \quad (17)$$

where n_p and n_L are positive scalar learning rates, and

$$\Psi_1(P(\bar{x})) = \mathfrak{S} [A^T(\bar{x})P(\bar{x}) + P(\bar{x})A(\bar{x}) + Q - P(\bar{x})N(\bar{x})P(\bar{x})] \quad (18)$$

$$\Psi_2(P(\bar{x}), L(\bar{x})) = \mathfrak{S} [L(\bar{x})L^T(\bar{x}) - P(\bar{x})] \quad (19)$$

where \mathfrak{S} is a symmetric non-decreasing activation function. Typical examples of \mathfrak{S} is given in [12]. Here, without loss of generality, the activation function is chosen as

$$\mathfrak{I}(f(x)) = f(x) \quad (20)$$

The architecture of the gradient-based neural-like system seeking the positive definite and symmetric solution of (11) consists of two layers, where (18), (19) act as hidden layers, and (16), (17) act as output layers [12], [13]. A schematic of the SDRE neural-like system is shown in Fig. 1.

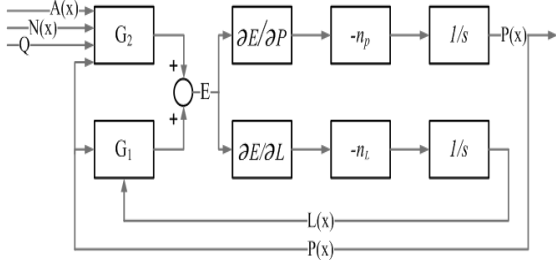


Fig. 1- Schematic of the SDRE neural-like system

The proposed online control strategy described above can be implemented with the knowledge of the system states (current components and speed) and motor parameters (stator resistance and load torque). The only measurable states, however, are current components. Hence, we need to design an observer that can correctly estimate the motor speed. In addition, we need to design a parameter estimator to estimate the rotor resistance and load torque.

An extended Kalman filter is used to estimate the unknown state and parameters of the PMSM, which is explained in the following section.

IV. EXTENDED KALMAN FILTER ESTIMATION

Here, the dynamic model of the PMSM described in section II augmented with that of the unknown parameters is considered for estimation. A continuous extended Kalman filter (EKF) is used for estimating the states of the augmented system. The augmented state variables are $z^T = [z_1, z_2, z_3, z_4, z_5] = [x_1, x_2, x_3, R, T_L]$. The dynamic model of the augmented system, consisting of the PMSM model and its unknown parameters (stator resistance and load torque), is given as

$$\begin{aligned} \dot{z} &= f(z, \bar{u}) + Gw \\ y &= Cz + v \end{aligned} \quad (21)$$

where y is the output vector (stator current components), $G = I_{5 \times 5}$, $w \in \mathfrak{R}^5$ and $v \in \mathfrak{R}^2$ are white noise vectors with

covariance matrices Ξ and Θ , and that $f(z, \bar{u}) = \begin{bmatrix} f_m(\bar{x}, \bar{u}) \\ 0 \end{bmatrix}$

such that

$$f_m = \begin{bmatrix} -\frac{z_4}{L_d} z_1 + P \frac{L_q}{L_d} z_2 z_3 + \frac{1}{L_d} \bar{u}_1 \\ -P \frac{L_d}{L_q} z_1 z_3 - \frac{z_4}{L_q} z_2 - P \frac{\Phi}{L_q} z_3 + \frac{1}{L_q} \bar{u}_2 \\ \frac{3}{2} P \frac{\Phi}{J} z_2 - \frac{F}{J} z_3 - \frac{z_5}{J} \end{bmatrix}, \quad C^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

The EKF estimator dynamics is then given as

$$\dot{\hat{z}} = F \hat{z} + G \bar{u} + K_f (y - C \hat{z}) \quad (22)$$

where $F = \left. \frac{\partial f(z, \bar{u})}{\partial z} \right|_{z=\hat{z}}$ and $G = \left. \frac{\partial f(z, \bar{u})}{\partial \bar{u}} \right|_{z=\hat{z}}$ are given by

$$F = \begin{bmatrix} -\frac{z_4}{L_d} & P \frac{L_q}{L_d} z_3 & P \frac{L_q}{L_d} z_2 & -\frac{z_1}{L_d} & 0 \\ -P \frac{L_d}{L_q} z_3 & -\frac{z_4}{L_q} & \left(-P \frac{L_d}{L_q} z_1 - P \frac{\Phi}{L_q} \right) & -\frac{z_2}{L_q} & 0 \\ 0 & \frac{3}{2} P \frac{\Phi}{J} & -\frac{F}{J} & 0 & -\frac{1}{J} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} \frac{1}{L_d} & 0 \\ 0 & \frac{1}{L_q} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

Selecting the matrices $\Xi \geq 0$ and $\Theta \geq 0$, and assuming the system is observable, the following filter differential Riccati equation (FDRE) provides the covariance matrix $N > 0$.

$$\dot{N} = FN + NF^T + \Xi - NC^T \Theta^{-1} CN \quad (23)$$

where $N(t_0) = N_0$. Then the time-varying gain K_f can be determined as

$$K_f = NC^T \Theta^{-1} \quad (24)$$

The unknown motor states and its parameters are then estimated using the EKF estimator (22) with the knowledge of K_f and the measurement of motor currents. Using the estimated states (the motor speed, currents, and its unknown parameters), the motor speed can be controlled using equations (5), (6), and the optimal control (10).

V. SIMULATION RESULTS

In this section, the proposed approach has been carried out for PMSM control in order to verify the performance of the proposed nonlinear optimal speed control scheme under the stator resistance and load torque variations, using MATLAB/Simulink software. Fig.2 shows the schematic diagram of the proposed control. For this purpose, the parameters of the PMSM d - q reference frame model are given in Table I [see 3], while the SDRE and gradient-based neural-like system parameters are given in Table II. The simulation results are shown in Fig. 3- Fig.7.

Table 1- Model Parameters

Model Symbols	Values
R	0.625 Ω
L_d	0.036H
L_q	0.051H
P	2
Φ	0.545 Wb
J	7.5×10^{-4} Nmsec ²
F	0.036 Nm/A

Table 2- Controller Parameters

Controller Symbols	Values
Q	diag(1, 1, 1)
R	diag(1, 10)
n_p, n_L	0.05, 0.5
$P(0), L(0)$	$3I, I$

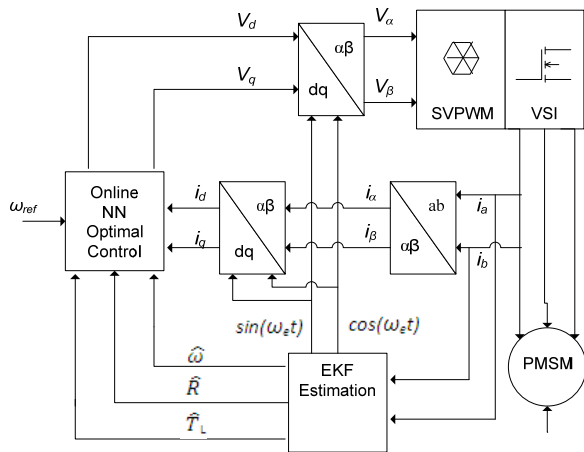


Fig. 2- The overall simulated block diagram of the proposed control

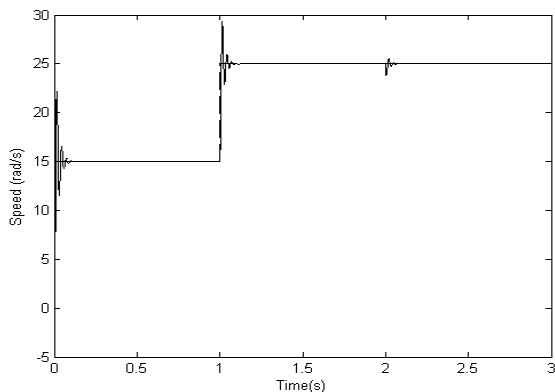


Fig.3- Actual motor speed (solid line), motor speed reference (dashed line)

Fig.3 shows the motor speed for a step change command. It is obvious that the motor speed can track its desired value in a short time for stator resistance of 3 Ω and load torque of 0.9Nm and also when these values change. Fig.4 and Fig.5 show the corresponding direct axis and quadrature axis currents, respectively.

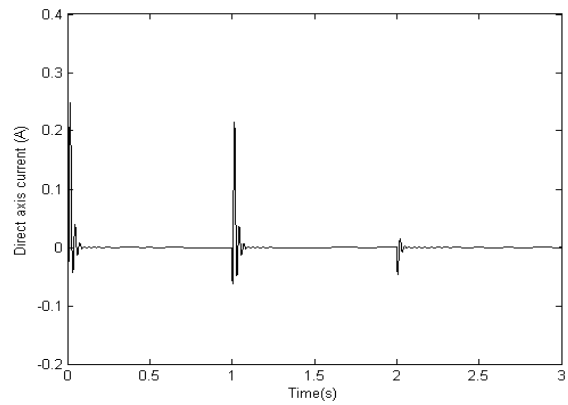


Fig.4- Direct axis current

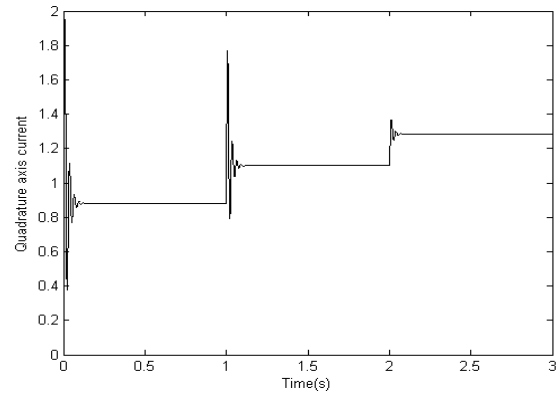


Fig.5- Quadrature axis current

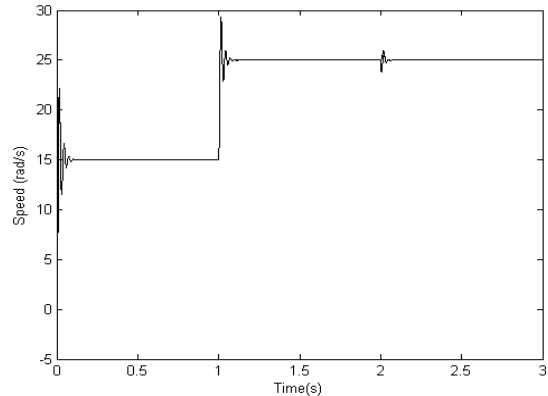


Fig.6- Rotor speed (solid line) and estimated speed (dashed line)

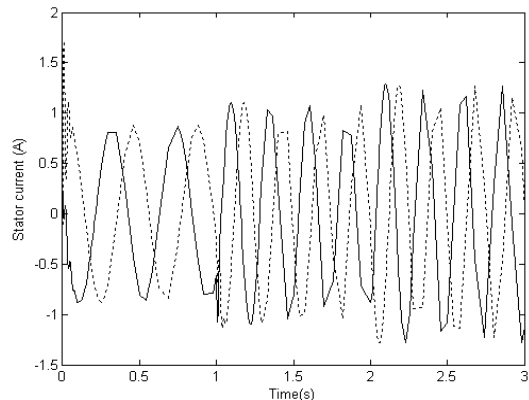


Fig.7- Three phase PMSM currents

A continuous time extended Kalman filter (EKF) is used to estimate the motor speed, stator resistance and load torque. The positive definite matrices Ξ and Θ are selected as $\Xi = \text{diag}[1e-6, 1e-6, 1e-6, 1e-3, 1e-5]$ and $\Theta = \text{diag}[0.09, 0.09]$. The initial value of the covariance matrix N_0 is selected as identity matrix. Fig.6 shows the rotor speed and the estimated speed with step change of 10 rad per sec. Also Fig.7 shows the three phase PMSM currents.

Furthermore, Fig.8 and Fig.9 show the estimation results for the stator resistance and the load torque, respectively. Clearly the EKF accurately estimates the parameters.

From the simulation results, it is evident that the proposed SDRE-based control design provides an efficient strategy for online sensorless speed control of the PMSM drives, which is also suitable for real-time implementation.

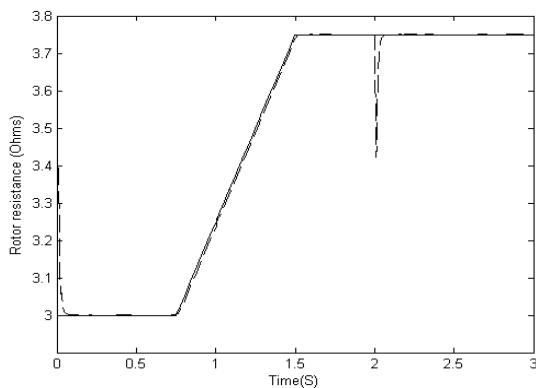


Fig.8- Stator resistance (solid line) and estimated resistance (dashed line)

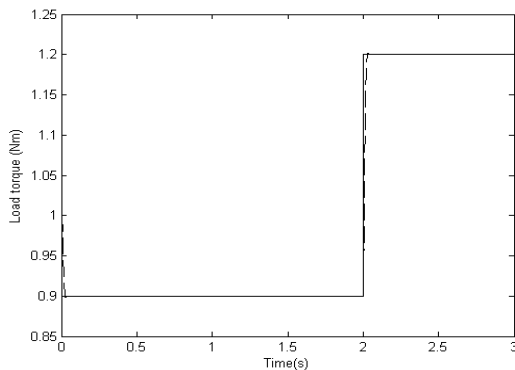


Fig.9- Load torque (solid line) and estimated load torque (dashed line)

VI. CONCLUSION

A new adaptive nonlinear optimal control is developed for online control of PMSM drives, without requiring speed sensors. The control provides a wide operating range with a good dynamic performance. An extended Kalman filter (EKF) algorithm is used to estimate the motor speed, as well as the stator resistance and load torque, simultaneously. The estimated motor speed and parameters are then used in the proposed nonlinear optimal control scheme. The control uses the solution of the state-dependent Riccati equation (SDRE) via a gradient-based neural-like computation algorithm. The neural-like system allows one to solve the required SDRE,

online, in an adaptive fashion. Successful simulation results verify that the proposed adaptive nonlinear control is suitable for real-time sensorless control of PMSM drives.

ACKNOWLEDGEMENT

Partial support for this work from the ECE department at SIUC is gratefully acknowledged.

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